Chu Responds: It is well known that Maxwell's equations are invariant under rotation and space inversion. However, the solutions of Maxwell's equations do not necessarily possess the same invariant properties. This situation is similar to that of the one-dimensional harmonic oscillator problem in which the Schrödinger equation itself is invariant under space inversion but allows solutions of both even and odd parity. The $\vec{E} \parallel \vec{B}$ waves discussed in Ref. 1 are analogous to the odd-parity solutions in a harmonic oscillator. In addition to satisfying Maxwell's equations, these $\vec{E} \parallel \vec{B}$ waves also satisfy the equation $\nabla \times \vec{B} = k\vec{B}$ which is not invariant under space inversion.

In the Comment[2] on our work, Lee makes two points: (1) $\vec{B}$ being a pseudovector is inconsistent with $\nabla \times \vec{B} = k\vec{B}$ except for $\vec{B} = 0$. (2) Using an example, he shows that $\vec{E} \parallel \vec{B}$ only at selected points where $\vec{E}$ and $\vec{B}$ are zero vectors.

The confusion in point (1) arises from the fact that he is trying to use this noninvariant equation $\nabla \times \vec{B} = k\vec{B}$ to discuss vector properties which are only meaningful for invariant equations. As a matter of fact, his example of a left-handed circularly polarized electromagnetic (EM) wave serves perfectly as a counterexample to this point. Namely, a nonzero pseudovector solution $\vec{B}_l$ of Maxwell's equations satisfies $\nabla \times \vec{B}_l = k\vec{B}_l$ and a nonzero vector $\vec{E}_l$ satisfies $\nabla \times \vec{E}_l = k\vec{E}_l$. Furthermore, in obtaining $\vec{E} \parallel \vec{B}$ solutions of Maxwell's equations, we never replace Ampere's law $\nabla \times \vec{B} = (1/c)\partial \vec{E}/\partial t + (4\pi/c)\vec{j}$ by $\nabla \times \vec{B} = k\vec{B}$. We use the full set of Maxwell's equations plus certain conditions to get a subset of solutions which have the unusual property $\vec{E} \parallel \vec{B}$.

In regard to point (2), I agree with Lee that his example of a linearly polarized traveling wave (which he erroneously labeled as a standing wave) does not have the property $\vec{E} \parallel \vec{B}$ everywhere (in fact, nowhere). However, I would like to point out that in Ref. 1 we did not claim that an arbitrary wave can have $\vec{E} \parallel \vec{B}$. All we said is that there exist certain waves which have the counterintuitive property of $\vec{E} \parallel \vec{B}$. To demonstrate this once more, let us just use the circularly polarized wave in Lee's example to construct a wave with $\vec{E} \parallel \vec{B}$. Following Lee, we have a left-handed polarized (LHP) EM wave propagating in the $\hat{z}$ direction described by

$$\vec{E}_l = kA[\hat{x}\cos(kz-\omega t) - \hat{y}\sin(kz-\omega t)],$$

$$\vec{B}_l = kA[\hat{x}\sin(kz-\omega t) + \hat{y}\cos(kz-\omega t)].$$

Let us also have a right-handed polarized (RHP) wave propagate in the opposite $(-\hat{z})$ direction,

$$\vec{E}_- = kA[-\hat{x}\cos(kz+\omega t) + \hat{y}\sin(kz+\omega t)],$$

$$\vec{B}_- = kA[\hat{x}\sin(kz+\omega t) + \hat{y}\cos(kz+\omega t)].$$

Notice that here we use $\vec{E}_-$ and $\vec{B}_-$ to describe the RHP wave propagating in the $-\hat{z}$ direction in order not to confuse the $\vec{E}_r$ and $\vec{B}_r$ notations Lee used for RHP propagating in the $+\hat{z}$ direction (which he erroneously claimed to be propagating in the $-\hat{z}$ direction). Since Maxwell's equations are linear in vacuum, the superposition of these two waves gives a standing-wave solution of Maxwell's equations,

$$\vec{E} = \vec{E}_l + \vec{E}_- = 2kA[\hat{x}\sin kz \sin\omega t + \hat{y}\cos kz \sin\omega t],$$

$$\vec{B} = \vec{B}_l + \vec{B}_- = 2kA[\hat{x}\sin kz \cos\omega t + \hat{y}\cos kz \cos\omega t].$$

Obviously, the $\vec{E}$ and $\vec{B}$ of this wave are parallel to each other at all points in space. The phases in time for $\vec{E}$ and $\vec{B}$, however, are different by $\pi/2$. This wave is exactly the same wave that we used as an example in Ref. 1. As was pointed out by Evtuhov and Siegman,[3] this standing wave has a uniform field density.

In summary, the cause of confusion in the symmetry argument by Lee is pointed out. A counterexample against his argument is given. By using the more familiar circularly polarized waves, an $\vec{E} \parallel \vec{B}$ wave solution is constructed. The existence of the $\vec{E} \parallel \vec{B}$ wave is once again demonstrated.

The author would like to thank Dr. S. K. Wong for an illuminative discussion on the invariant properties of fields. This work is supported by Phillips Petroleum Company and GA Technologies Inc. under a Joint Research and Development Agreement (GA Ref. No. 01341).

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Received 19 November 1982
PACS numbers: 03.50.De, 41.10.Hv, 52.35.Hr

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