Time Dependence in Rayleigh-Bénard Convection with a Variable Cylindrical Geometry

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Studies of the first time dependence in Rayleigh-Bénard convection in ^{4}He are reported. The aspect ratio Γ could be varied continuously. Data at 31 Γ values with $4 < \Gamma < 13$ show that near onset the first time dependence is always periodic and associated with changes in the heat transport. The dominant periodicity has a nonzero amplitude, A, and vanishing frequency at onset. Changes in A with Γ show wave-number effects. Except for the time dependence, the transition is consistent with predictions for the skew-varicose instability.

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Rayleigh-Bénard convection is one of the most extensively studied nonlinear systems. Among the important related problems is the onset of time dependence, which is the main subject of this work. The relevant parameters[1] are the Rayleigh number R with critical value R_c, the Prandtl number P, and the aspect ratio $\Gamma = D/2d$. Here D and d are the diameter and height of the cylindrically confined layer.

Experiments[2-6] using normal liquid ^{4}He, as the present ones do, identified Γ as an important parameter in determining the route to chaos, revealing many time-dependent flows depending on Γ and the method of flow preparation. Even fairly small changes, $\Delta\Gamma \simeq 1$, had substantial effects on the time-dependent states. Although provocative, existing experiments have not produced a cohesive picture of the prevailing time dependence.

To better understand such effects, we constructed an apparatus, to be described elsewhere,[7] in which Γ can be varied continuously. Measurements with this apparatus were made for 31 values of Γ in the range $4 \lesssim \Gamma \lesssim 13$ with $0.54 \leq P \leq 0.69$. This range in Γ was chosen because larger Γ's are likely to be dominated by orientational disorder of the convective rolls, whereas smaller Γ's are influenced by modes which may be atypical of more open geometries. As in previous work,[2,4-6] we observed time dependence by measuring fluctuations δT in the bottom temperature with the heat current Q and top temperature fixed. This provides considerably more precise data than can be obtained from heat-flux measurements with a fixed temperature difference across the layer. Convenient scales are the critical temperature difference ΔT_c for δT, and $t_v = d^2/D_T$ for time, where D_T is the thermal diffusivity.

We denote the onset Rayleigh number and heat current for the first steady time dependence by R_1 and Q_1. The salient features of this time dependence are the following: (1) The motion is always periodic, (2) there exist two different kinds of periodic states, and (3) the qualitative features of these oscillatory states are periodic in Γ, with period ~ 1.

The first kind of periodic state, a normal bifurcation to periodic flow, occurs when $\Gamma = I + 0.6 \pm 0.15$, where I is an integer. An example is given in Fig. 1(a) for $\Gamma = 8.741$, where we show A, the peak-to-peak amplitude in $\delta T/\Delta T_c$, vs $\epsilon_1 \equiv (Q - Q_1)/Q_1$. The solid curve is a fit by the form $A = a\epsilon_1^{1/2} + b\epsilon_1^{3/2}$ with $a = 4.04\times10^{-3}$ and $b = 9.82\times10^{-3}$.

The second kind of periodic state shows relaxation oscillations with a nearly constant amplitude and vanishing frequency, f, as R approaches R_1 from above (inverted bifurcation). We will call this type-II periodicity to distinguish it from the first case, type-I periodicity. Most of the aspect ratios yielded type-II behavior. One example is for $\Gamma = 9.002$: Fig. 1(b) shows f vs ϵ_1 and Fig. 2(a) shows several runs in detail, where the number beneath each curve represents the value of ϵ_1. We find that for this case f has the form $ft_v = a\epsilon_1^{1/2}$ with $a = 7.45\times10^{-2}$. Another type-II example, for $\Gamma = 8.275$, is shown on an expanded scale in Fig. 2(b).

To illustrate the variations between the types, we show in Fig. 3(a) the peak-to-peak amplitude $A(\epsilon_1 = 0)$ of the oscillations at onset; for type I, $A(0) = 0$. The corresponding variations in $R_1(\Gamma)/$

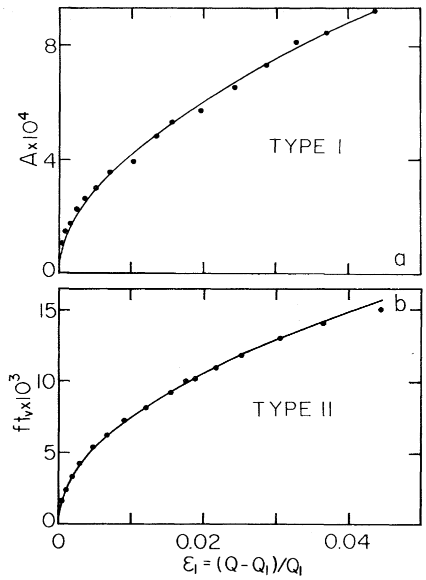

FIG. 1. (a) The peak-to-peak amplitude A in $\delta T/\Delta T_c$ vs ϵ_1 for $P = 0.54$ and $\Gamma = 8.741$. The results have been multiplied by 10^4. (b) The frequency f normalized by t_v and multiplied by 10^3 for $P = 0.54$ and $\Gamma = 9.002$.

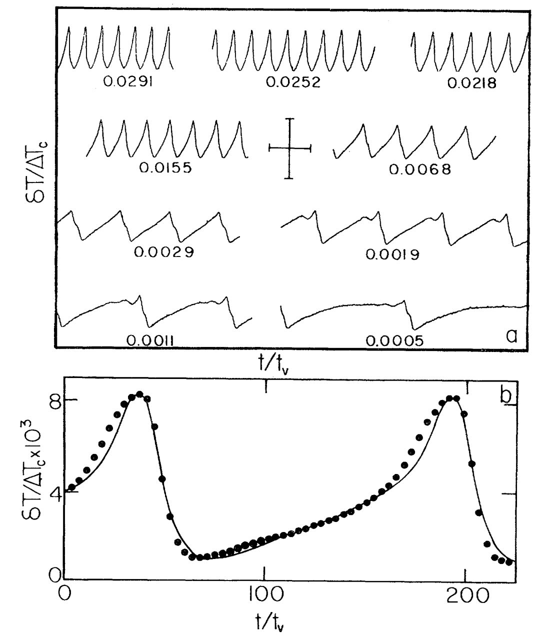

FIG. 2. (a) A series of runs showing the vanishing of f as $\epsilon_1 \to 0^+$. These data are for $\Gamma = 9.002$, $P = 0.54$ and provide some of the frequency data in Fig. 1(b). The number beneath each run is the value of ϵ_1. The vertical bar corresponds to a change in $\delta T/\Delta T_c$ of 5×10^{-3} and the horizontal bar corresponds to a time of $t/t_v = 200$. (b) The solid symbols show data points for another example of type-II oscillations ($P = 0.54$, $\Gamma = 8.275$). The line was obtained by adjusting the parameters of a simplified equation of motion due to Busse.

$R_c(\Gamma)$ are shown in Fig. 3(b). Clearly the transition depends significantly on the mean wave number of the flow. Associated with these effects are distinct changes in N, the dimensionless heat transport, as shown in Fig. 4. There is a single curve for all $R < R_1(\Gamma)$ and a nearly universal curve for $R > R_1(\Gamma)$.

An additional feature is that while type-II periodicity shows critical slowing down for $R < R_1$, for $R > R_1$ a similar slowing down is not observed. Rather, following a change in the heat current, δT undergoes a rapid response and locks into the periodic state.

One final topic involves the time dependence occurring as R is increased well above R_1. Although a more detailed study will be made in the future, one point is worth mentioning here. For the lower aspect ratios, $\Gamma \lesssim 6$, type-II time dependence exists over only a relatively narrow band, $R_1 \leq R \leq R_1^*$ with $(R_1^* - R_1)/R_c \lesssim 0.05$, and for observation times of the order of $500 t_v$ there is a band of time-independent states for $R_1^* < R < R_2$. At R_2 there is a bifurcation to another periodic state. Sample runs above R_2 are similar to run j of Ref. 4, Fig. 2, or run m of Ref. 5b, Fig. 2.

At low Γ, the evolution of the time-dependent states involves a complex process not observed for higher Γ. For $\Gamma \lesssim 6$ the type-II periodicity is not easily accessible if the heat current is gradually increased through the transition at R_1. Rather, this technique produces complex aperiodicity. If R_1^* is exceeded the time-independent states occur; then if the heat current is slowly decreased type-II periodicity occurs.

We now turn to the physical origin of the time dependence. Busse and Clever[8] have made detailed linear stability calculations for $\Gamma = \infty$. On the basis of their work, the most likely origin of the transition is the skew-varicose instability (SVI). The SVI has been shown by Gollub and Steinman[9] to be important for higher Prandtl numbers and has been associated[3,6] with transitions in liquid helium. The variations in $R_1(\Gamma)$ (Fig. 3) are consistent with the predicted wave-number dependence.

An apparent contradiction in applying the SVI to the time-dependent data arises because the predictions show a real growth rate. However, the time dependence above R_1 need not rule out the SVI, since the amplitude envelope (Fig. 3) of the

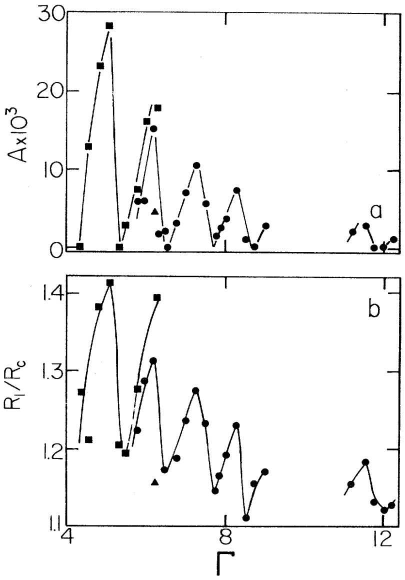

FIG. 3. (a) Values of the peak-to-peak amplitude A of $\delta T/\Delta T_c$ vs aspect ratio Γ. Here A, which is shown multiplied by 10^3, is the limiting value as $\epsilon_1 \rightarrow 0^+$. Squares correspond to $P = 0.69$ and circles to $P = 0.54$. The triangle is from Ref. 6. Lines have been added to guide the eye. (b) Values of the onset Rayleigh number $R_1(\Gamma)$, normalized by $R_c(\Gamma)$, vs Γ. The minima in A correspond with the minima in R_1/R_c.

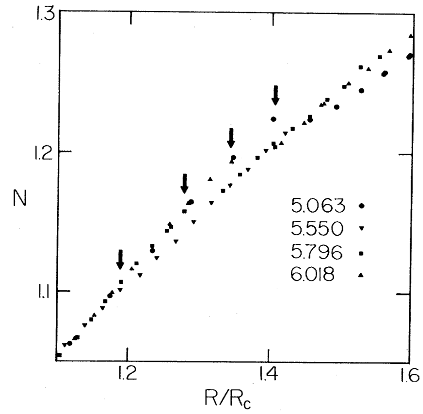

FIG. 4. The heat transport N vs R/R_c for $\Gamma = 5.063$, 5.550, 5.796, and 6.018, all with $P = 0.69$. The arrows indicate, from left to right, the transition values for $\Gamma = 5.550$, 5.796, 6.018, and 5.063, respectively. For $R < R_1(\Gamma)$ there is a universal curve within experimental error. For $R > R_1(\Gamma)$ the data fall close to but not uniformly on a lower curve.

observed time dependence becomes smaller as Γ increases. Furthermore, Busse[10] has shown that oscillations will occur when there is a downward discontinuity in N with increasing R, and the heat flux from below is fixed, conditions which clearly apply here. Using a simplified version of Busse's theory[10] we have found substantial agreement with our data. An example is given in Fig. 2(b), where the solid symbols are data points and the line is obtained by numerically integrating the equations of motion. To our knowledge, values of the heat transport above the onset of the SVI have not been calculated, but we expect them to be different from those for two-dimensional flow. We note that the observed changes in N cannot be caused by large departures from the Boussinesq approximation. For instance, the values of $\overline{Q}(R_c)$, Busse's[11] parameter giving the departure from a Boussinesq flow, are $-0.104 \leq \overline{Q}(R_c) \leq -0.015$ for the four aspect ratios shown in Fig. 4. Studies by Walden and Ahlers[12] indicate that for such $\overline{Q}$'s non-Boussinesq behavior is irrelevant.

On the basis of the present data, it is possible to place much of the previous experimental work in perspective. Similar measurements have been reported for $\Gamma = 4.72$[2,4] and $\Gamma = 7.57$.[5] The initial time dependence for both of these aspect ratios has been generally nonperiodic, yielding at higher Rayleigh numbers to a quiescent state, a periodic state at still higher Rayleigh numbers, and ultimately a turbulent state. We expect that the observed initial time dependence is associated with the transition at R_1, and the quiescent state with the region $R_1{}^* < R < R_2$. Neither of these experiments were made with a small enough mesh in R to identify the periodic regime near R_1. Measurements with $\Gamma = 6.22$ have been reported[5] which show type-II periodicity, and Walden[6] has found relaxation oscillations as the lowest occurring time dependence for $\Gamma = 2.93$. For $\Gamma = 2.08$ a decrease in N at $R/R_c = 3.2$ with associated relaxation oscillations was reported.[2]

To conclude, we have shown the existence of a well-defined transition over a broad range of Γ's. Structural changes associated with the transition are manifested in the heat transport, and should significantly affect the route to turbulence. In the neighborhood of the transition, the flow is periodic. For some Γ's there is a normal bifurcation, but for the majority there is an inverted bifurcation. Many features of the periodic behavior can be reproduced by the predictions of Busse, and stability calculations by Busse and Clever indicate that the transition is associated with the skew-varicose instability. Existing data, on a coarser grid in R than the present work, now fit into a more coherent picture.

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