Is Proton Decay Measurable?

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It is shown that nucleon collisions in a heavy nucleus may satisfy the conditions required for the inhibition of proton decay in the framework of the quantum mechanical theory of the decay of unstable systems.

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Almost all unified gauge theories predict the instability of the proton.[1] Attempts have been made during the past year to measure the proton decay. The Kolar Gold Field experiment[2] has established a lower bound on the lifetime of the order of 10^{31} yr; theoretical estimates[1,3] provide values ranging from 10^{27} to 10^{31} yr.

We wish to point out in this Letter that it is possible that the conditions for the quantum mechanical quenching[4] of proton decay are met in heavy nuclei, where it is considered possible to observe the proton decay because of the large concentration of nucleons.

The quantum mechanical description of a decaying system is based on the projection of a state, evolving in time under the action of the total Hamiltonian, back to the initial state (the survivial amplitude),

$$A_s(t) = (\psi, e^{-iHt}\psi), \tag{1}$$

where ψ is the initial state. If H is defined on ψ, then

$$d|A_s(t)|^2/dt\,|_{t=0} = 0. \tag{2}$$

The short-time behavior of the survival probability is, in fact, quadratic in t, up to a time T_1, when the pole contribution begins to become important.[5,6] If the evolution of the system is interrupted by an interaction, due to measurement or collision, then the calculation of the predicted survival amplitude must be started again with the new initial condition. Repetition of this phenomenon at sufficiently high frequencies can, in principle, have an appreciable effect on the observed lifetime.[4]

Proton decay is predicted primarily on the basis of quark-lepton transitions taking place inside the proton. The lifetime is predicted[1,3] from estimates of the superheavy boson mass, the quark wave function, and phase space. The microscopic processes studied in this way form the basic mechanism driving proton decay. For the proton decay actually to occur, however, the final-state decay products must emerge from the nucleon in order to avoid coherent recombination.

In any decaying system, short-time microscopic virtual decay processes can be followed immediately, and with the same amplitude, by coherent recombination. It is only after some time that incoherence in the recombination builds up, and the process begins to become effectively irreversible, i.e., the exponential mode becomes dominant. The short-range, short-time phenomenon of coherent recombination, due to the reversibility of the equations of motion, is reflected in the quantum mechanical description of decay discussed above.

The proton decay should be considered as the continuous evolution of the proton state to a state[7] that includes its decay products under the action of an effective Hamiltonian. If, during the course of this evolution, reflecting the imbalance of microscopic decay and recombination processes, an interruption occurs, for example, by a collision (or a measurement) which is sensitive to the identification of the proton in its initial state, then the development of incoherence in the final-state phases is destroyed, and the evolution must develop from the beginning. In the following, we shall base our arguments on the frequency of proton collisions in the nucleus. If the collisions are so fast that one must consider quark and gluon collision frequencies, then the mechanism for decay would have to be reconsidered,[8] i.e., the nucleus should then be considered as a system containing quarks and gluons as fundamental building blocks, and estimates based on quark wave functions in the proton would not be relevant.

A collision essentially constitutes a measurement, since the actual decay of the proton would create a new nucleus in an excited state, which can be understood according to the following mechanism. One can think of the stable state of the nucleus before proton decay as being maintained by nucleon collisions. After the proton decay, the distribution of collisions is altered, because of the absence of the proton, on the time scale of the individual collisions. The information about the proton decay is therefore distributed throughout the nucleus by means of collisions occurring after the decay, the first of which triggers the process of relaxation to an excited state. The excited state is then measurable by its decay processes, and therefore the proton decay is effectively measured by the nucleon collisions taking place in the nuclear ground state.

We now estimate the two times required for our analysis, the time limit of the short-time behavior of the decay amplitude, and the time between the effective measurements provided by the structure of the nucleus. Chiu, Sudarshan, and Misra[5] have proposed a resonance model for the decay of unstable systems for which the short-time behavior is consistent with Eq. (2). The limit of short times beyond which the pole term (exponential decay) becomes important, in this model, is given by

$$T_1 = 25/E_0, \tag{3}$$

where E_0 is the difference between the mass of the decaying system and the threshold of the continuous spectrum of the decay products. In the case of proton decay, among the important decay channels, $\pi^0 e^+$ leads to the shortest time estimate for T_1. In this case,

$$E_0 = M_p - m_\pi \tag{4}$$

and therefore

$$T_1 \cong 2\times10^{-23} \text{ sec.} \tag{5}$$

The time between nucleon-nucleon collisions in a nucleus can be roughly estimated from the average nucleon velocity and distance between the nucleons in the nuclear ground state. Nuclear forces seem to be reasonably well represented at low energies by a hard core of about $\frac{1}{2}$ fm radius, and a longer-range force with radius about 1 fm. Nucleons essentially overlap (close packed) in a nucleus, but they would have to travel about $\frac{1}{2}$ fm to suffer a hard-core collision. According to the uncertainty relation, $\Delta x \sim 1$ fm implies $\Delta p \sim 200$ MeV/c. Indeed, the average momentum of nucleons in a heavy nucleus such as Fe is about 250 MeV/c (velocity $\sim\frac{1}{4}c$), and in a lighter nucleus such as Ca, 225 MeV/c (velocity $\sim\frac{1}{5}c$).[9] The time between collisions would then be approximately

$$T \sim 0.8\times10^{-23} \text{ sec.} \tag{6}$$

A less direct estimate can be made by means of the time for decay of an excited nuclear state. In order for the excited state to make a transition to the nuclear ground state, the emission of, for example, a gamma ray must be accompanied by a redistribution of nucleons to achieve the almost degenerate Fermi-Dirac distribution of the ground state. Since many, perhaps 10^2–10^3, collisions per nucleon would be required to achieve the new equilibrium state, the fastest excited nuclear transition times provide a large upper bound on the interval between elementary collisions. Highly excited nuclear states (we do not consider excitations for which the nucleus breaks apart) typically decay in 10^{-20}–10^{-21} sec. If 10^2–10^3 elementary collisions are required in the course of this transition, we would obtain an estimate of the order of 10^{-23} sec for the time between collisions.

We finally remark that time scales associated with collective modes such as quadrupole and octupole vibration frequencies, and giant dipole transitions, are in the order of 10^{-21} sec.[9] Since these collective modes could involve 10^2–10^3 elementary collisions in each oscillation, we arrive at an estimate consistent with the other processes considered above.

Our (rather crude) estimates of the frequency of elementary collisions of nucleons in nuclei lie at the limit of times for which a decaying quantum mechanical system has not yet reached its exponential (Markovian) mode. Repetitive collisions at this frequency, corresponding to measurements verifying the persistence of the initial state, could conceivably have an appreciable effect on the observed lifetime. Although the predictions of grand unified theories could be correct, it may not be possible to observe proton decay with techniques involving heavy nuclei.

In view of the fact that our estimates do not rule out the applicability of decay-inhibiting phenomena of this type, some effort should be made to obtain improved estimates of the time between nuclear collisions. It is possible that effective collision times are so short that nucleon decay could be completely quenched[4] and would not be observed at all in heavy nuclei; if the collision times are longer, observed lifetimes may be affected differently in heavy and light nuclei.

If collision times are extremely short, the underlying quark-gluon structure of the nucleon may become important. In this case, possible alternative mechanisms offered by quark-gluon collisions, for which important modifications would have to be made in the framework for the calculation of proton decay, should also be studied.

Our discussion of elementary collision times is semiclassical. An estimate of the frequencies of the one-particle states involved in the construction of stationary or quasistationary nuclear many-body modes should also be made. We expect that these will agree with the semiclassical estimates.

We have, further more, argued qualitatively on the mechanism for the decay-inhibiting phenomenon from the point of view of the S-matrix amplitudes derived from quantum field theory, and we have assumed the results to be compatible with quantum mechanical models.[4,10] A more careful study should be made of this mechanism.

Nuclear β decay poses some serious problems for the formalism that we have applied to proton decay. A calculation similar to that made above (for example, for free nuetrons) indicates a value of T_1 of about 10^{-20} sec.[11] Such transitions, from nucleon to nucleon plus leptons, involve a small electromagnetic perturbation of the nuclear state which would not be detectable if isospin symmetry were exact. From this point of view of the arguments given above in terms of phase coherence, nucleon collisions in the final state, which are approximately isospin independent, would not be expected to destroy the growth of incoherence in the final-state phases since the initial nucleon component and the final nucleon component (with accompanying leptons) of the time dependent wave functions will produce essentially the same amplitude for scattering with the nucleon. Isospin symmetry breaking, however, could lead to a small probability that an effective measurement is carried out in each elementary nuclear collision during the time T_1. Since, according to our estimates, there could be of the order of 10^3 such collisions, one might be led to expect an observable effect on β-decay rates in nuclei beyond that of phase space. There are some features of nuclear β decay, however, which are essentially different from those of proton decay in a nucleus. The very strong effect of phase-space considerations (including the state of the entire nucleus) on nuclear β decay indicates a high degree of collectivity in this mode, i.e., a strong dependence on the total nuclear many-body wave function. The elementary collisions which we have interpreted as interrogations on the identity of nuclear constituents for the case of proton decay correspond, as mentioned above, to one-particle fluctuations in the essentially stationary many-body wave function describing the equilibrium nuclear state. In the case of nuclear β decay, the transition seems to be more closely associated with an evolution of the entire many-body wave function of the nuclear state. It may not, therefore, be appropriate to compare the T_1 of β decay to nucleon-nucleon collision frequencies, since in this case they would not act as effective measurements (external collisions with the nucleus could play this role). It is our feeling that the latter situation should prevail in the case of nuclear β decay, but the answer to this question can be provided only by a more complete anaylsis.

The difficulty of verifying the existence of the quenching of decay through repetitive measurement experimentally has been discussed by Sudarshan and Misra.[4] It is hard to imagine an accessible physical system other than the atomic nucleus for which one might hope to find it.

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[1]J. Ellis, CERN Report No. LAPP-TH-48, Ref. TH-3174 CERN, 1981 (to be published), and references given there.

[2]Indian-Japanese collaboration, in Proceedings of the Twenty-First International Conference on High Energy Physics, Paris, 1982 (to be published). See also M. R. Krishnaswamy *et al.*, Phys. Lett. 106B, 339 (1981). For a review of the experiments, see L. Sulak, in Proceedings of the Les Houches Summer School, August 1981 (to be published).

[3]P. Langacker, University of Pennsylvania Report No. UPR-0186T (to be published).

[4]B. Misra and E. C. G. Sudarshan, J. Math. Phys. (N.Y.) 18, 756 (1977); Y. Aharonov *et al.*, Phys. Rev. D 21, 2235 (1980).

[5]C. B. Chiu, E. C. G. Sudarshan, and B. Misra, Phys. Rev. D 16, 520 (1977).

[6]N. Bleistein, H. Neumann, R. Handelsman, and L. P. Horwitz, Nuovo Cimento 41A, 389 (1977).

[7]For example, see diagrams (7.4) and (7.6) in Ref. 1.

[8]These times are very difficult to estimate since the mechanism for penetration to the bound constituents (via strong interaction) is not well understood.

[9]J. M. Eisenberg and Walter Greiner, *Nuclear Models* (North-Holland, Amsterdam, 1970).

[10]V. Weisskopf and E. Wigner, Z. Phys. 63, 54 (1930); and 65, 18 (1930). For more recent work, see, for example, L. P. Horwitz, J. A. LaVita, and J.-P. Marchand, J. Math. Phys. (N.Y.) 12, 2537 (1971); B. Simon, Ann. Math. 97, 247 (1973); L. P. Horwitz and I. Sigal, Helv. Phys. Acta 51, 685 (1978).

[11]This estimate agrees with the characteristic weak interaction T_1 found in computations with a model for (one-channel) K_0 decay by N. Bleistein, R. Handelsman, L. P. Horwitz, and H. Neumann, Nuovo Cimento 41A, 389 (1977).