Quarkonium Bound States and Coupling to Hadrons

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An inconsistency is found involving heavy quarkonia energy levels in the usual single-channel potential picture. Coupling to the nearest hadronic channel resolves the difficulty.

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The potential model for bound states of a heavy quark and antiquark has been generally quite satisfactory.[1] It is important for any scheme that as more elaborate calculations are made, better agreement with experimental quantities should result. We shall show, in a rather model-independent way, that this is not the case for a single-channel confined quark system. Even for the sharp states below open-flavor threshold we should take into account virtual hadron coupling to the quark pair. When this is done a calculationally accurate model is achieved.

We first consider the interquark power potential[2]

$$V(r) = A + Br^{\nu}. \tag{1}$$

Even though this potential is not QCD motivated and does not agree completely with all of the experimental data,[3] it has certain compelling advantages. The energy-level predictions and ratios of leptonic widths, within a given flavor, are quite accurately predicted and the power potential closely approximates the actual interquark potential in the intermediate distance range[4] $0.2 < r < 5$ GeV^{-1}. Rescaling the Schrödinger equation shows that the energy levels can be expressed as[2]

$$E_{n,l} = 2m_q + A + (B^2/m_q^{\ \nu})^{1/(2+\nu)}\, C(n, l, \nu). \tag{2}$$

From this it is easy to see that the ratio of two energy differences

$$(E_2 - E_1)/(E_3 - E_1) = f(\nu, \text{quantum numbers}) \tag{3}$$

depends only on the power ν and is independent of the quark mass m_q and the potential parameters A and B.

For charmonium there are three spin-averaged states[4] below open-charm threshold. These are the $1S$, $1P$, and $2S$ levels having energies of 3067, 3523, and 3663 MeV, respectively.[4,5] The ratio of differences $r_\psi \equiv (E_{1P} - E_{1S})/(E_{2S} - E_{1S})$ as a function of ν is shown by the solid curve in Fig. 1. The experimental value for this ratio is also shown, implying that the effective $c\bar{c}$ power is

$$\nu_{c\bar{c}} \simeq -0.10. \tag{4}$$

For bottomonium we use the three S-wave states below $B\bar{B}$ threshold. These are the $1S$, $2S$, and $3S$ levels having energies of 9439, 9999, and 10 328 MeV.[4] The ratio of differences $r_\Upsilon \equiv (E_{2S} - E_{1S})/(E_{3S} - E_{1S})$ as a function of ν is given by the dashed curve of Fig. 1. The experimental value for this ratio implies an effective $b\bar{b}$ power of

$$\nu_{b\bar{b}} \simeq +0.12. \tag{5}$$

We feel that the difference $\nu_{b\bar{b}} - \nu_{c\bar{c}} \simeq 0.22$ is significant.[6]

Because of their greater masses, the $b\bar{b}$ states have smaller average radii than the $c\bar{c}$ states. A

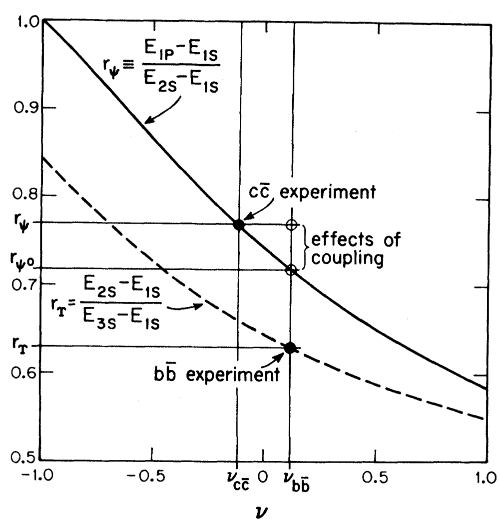

FIG. 1. The ratios r_ψ and r_Υ depend only on the power ν for a power potential $V = A + Br^\nu$. The solid circles represent the observed values of these ratios with corresponding powers $\nu_{c\bar{c}}$ and $\nu_{b\bar{b}}$. Coupling to hadronic channels primarily changes r_ψ so that the common power with hadronic coupling is nearly $\nu_{b\bar{b}}$ and the contribution to r_ψ due to $D\bar{D}$ coupling is given by the two open circles.

realistic potential with an attractive Coulomb-like singularity would then imply that the effective $b\bar{b}$ power should be less than the $c\bar{c}$ effective power. This is opposite to the experimental result of Eqs. (4) and (5). As pointed out in Ref. 4, a similar inconsistency is observed for a wide class of potentials. In all cases studied,[4] a potential which fits the three lowest S-wave Υ states and the two S-wave ψ states predicts a charmed χ center of gravity about 25 MeV low. Comparison with Fig. 1 shows that this is equivalent to the problem encountered with the power potential.

An initial reaction to the above difficulty might be that the charmed system is more relativistic than the bottom system so that consistency would be regained after the proper relativistic corrections are made. This turns out to be incorrect. For the three lowest $c\bar{c}$ states the net relativistic shift is positive and is smallest for the P-wave[7,8] state by about 10 MeV. This means that a potential which fits the $1S$ and $2S$ will make an even lower prediction for the $1P$ state when relativistic corrections are included. We conclude that there is no simple potential which can account for the spin-averaged states.[9]

The proximity of a coupled hadronic channel, however, resolves this problem. Coupling of the closed quark channel to a hadronic channel results in a depression of all quarkonium levels lying below continuum threshold. Above threshold the level energies become complex. The energy shift is largest for those states closest to the hadronic threshold. Of the states we have considered we see from Table I that only the charm $2S$ state is less than 200 MeV from threshold. The $2S$ state lies just 63 MeV below the open-flavor $D\bar{D}$ threshold. Since only one state is strongly shifted, we anticipate that the open channel will raise the predicted value for the $c\bar{c}$ ratio of Fig. 1, thereby increasing the effective power $\nu_{c\bar{c}}$.

To explore the effect of an open channel, we use a simple model.[10] A single hadronic channel communicating with the confined quark channel satisfies the system of Schrödinger equations

$$H_q\psi_q + V\psi_h = E\psi_q, \tag{6a}$$

$$V\psi_q + H_h\psi_h = E\psi_h. \tag{6b}$$

When the transition potential V vanishes, the discrete eigenvalues of Eq. (6a) give the usual confined quarkonia levels. When $V\neq 0$, the quark states above the hadronic threshold are immersed in a continuum and their energies become complex. Making the simplest assumptions, we take V constant and H_h to be force-free and we neglect quark and hadron spin effects. With the usual definitions for central potentials for the wave functions $\psi(\vec{r}) = [u(r)/r]\,Y_{lm}(\theta,\varphi)$ and for the wave numbers

$$k_q^{\,2} \equiv m_q(E-2m_q), \quad k_h^{\,2} \equiv m_h(E-2m_h), \tag{7}$$

we have, for a given partial wave l, the formal solution to Eq. (6b),

$$\begin{aligned} u_h(r) &= \int_0^\infty G(r,r')m_h V u_q(r')dr', \\ G(r,r') &= -ik_h r r' j_l(k_h r_<)h_l^{\,1}(k_h r_>). \end{aligned} \tag{8}$$

Here $r_<$ ($r_>$) is the smaller (larger) of r and r' and j_l and $h_l^{\,1}$ are the spherical Bessel and first Hankel functions.

Solving Eq. (6a) by perturbation using the solution of Eq. (8), we find that

$$E = E_0 + m_h V^2 \int_0^\infty dr\, u_{q0}^*(r)\int_0^\infty dr'\, G(r,r')u_{q0}(r'), \tag{9}$$

where the unperturbed quark energy is E_0 and the unperturbed quark wave function is given by $u_{q0}(r)$. Below flavor threshold[11] it is easy to see that all states are shifted downwards independent of the sign of V.

Our calculational scheme is straightforward. For a given set of energy states (e.g., $1S$, $1P$, and $2S$ for $c\bar{c}$) we solve for the potential parameters A, B, and ν of Eq. (1) fixing the c-quark mass at the value 1.40 GeV found in a more complete analysis.[4] The resulting wave functions then are used along with a D meson mass of 1863 MeV to compute the energy shift $E-E_0$ from Eq. (9). First-order changes in the wave functions as usual result in only second-order changes in

TABLE I. Level shifts expected due to coupling to the lowest-lying hadronic states having the open flavor. The transition potential V of Eq. (9) is assumed to be 50 MeV.

State	Energy shift (MeV) (with V = 50 MeV)	Energy below flavor threshold (MeV)
ψ states		
$1S$	-2	659
$1P$	-5	203
$2S$	-29	63
Υ states		
$1S$	-2	1114
$2S$	-3	544
$3S$	-8	225

the energy shift. The energy shifts for $V=50$ MeV are given in Table I. A similar analysis for the upsilon states (with $m_b=4.790$ MeV, a B mass of 5277 MeV, and $V=50$ MeV) is also given in Table I.

We first conclude from Table I that all of the energy shifts are relatively small except the $2S$ shift. The $b\bar{b}$ ratio of Fig. 1 will thus not appreciably change and the common power must thus be nearly $\nu_{b\bar{b}} \simeq 0.12$. From Fig. 1, with use of this power, the uncoupled ratio $r_{\psi 0}$ must be about 0.718, so that by Table I

$$r_\psi^{-1} = r_{\psi 0}^{-1} - \frac{29}{456}\left(\frac{V}{50}\right)^2. \tag{10}$$

Solving for V gives

$$V \simeq 60 \text{ MeV}. \tag{11}$$

Since r_Υ is roughly independent of V, the result given in Eq. (11) is the transition potential of $c\bar{c}$ to $D\bar{D}$, although one would expect V to be flavor independent. A more careful analysis gives $\nu = 0.145$ and $V=69$ MeV.

Although the specific model calculation presented here is considerably oversimplified,[12] we only wish to point out the qualitative effects produced by a coupled hadronic channel. Independent of the sign of the transition potential, levels below the hadronic threshold will be depressed and the greatest depression occurs for levels nearest threshold. This type of shift is needed to reconcile the spin-averaged spectra of the ψ and Υ families within the potential model. Hadronic effects must be allowed for in any scheme where relativistic corrections are made to a potential model, especially for those states near a hadronic threshold.

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[1]Some reviews of the development and status of the subject are given by T. Appelquist, R. M. Barnett, and K. D. Lane, Annu. Rev. Nucl. Part. Sci. 28, 387 (1978); V. A. Novikov, L. B. Okun, M. A. Shifman, A. I. Vainshtein, M. B. Voloshin, and V. I. Zakharov, Phys. Rep. 41C, 1 (1978); M. Krammer and H. Krasemann, Acta Phys. Austriaca Suppl. 21, 259 (1979); K. Gottfried, Comments Nucl. Part. Phys. 9, 141 (1981).

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[5]The $1P$ mass quoted is the center of gravity of the spin-triplet χ states. The ${}^1P_1$ state has not been observed but would probably not change the overall center of gravity significantly because of its low statistical weight. A recent prediction that the ${}^1P_1$ lies above the χ center of gravity somewhat strengthens our argument [K. J. Miller and M. G. Olsson, "On the Nature of the Short-Distance $Q\bar{Q}$ Interaction with Implications Concerning Confinement" (to be published)].

[6]The uncertainty in the experimental values of the two ratios due to measurement errors or to various reasonable assumptions on as yet unobserved hyperfine partners implies changes of $\nu < 0.04$.

[7]The relativistic corrections for static electric confinement (see Ref. 5) are given by K. J. Miller and M. G. Olsson, University of Wisconsin Report No. MAD/PH/57, May 1982 (to be published).

[8]An independent calculation of relativistic corrections to the charmonium states agrees with our result (Paul MacKenzie, private communication).

[9]A fit to the vector ψ and ψ' states and the χ center of gravity is much easier since by the neglect of S-wave hyperfine splitting the χ state has effectively been shifted downward in mass.

[10]A more detailed coupled hadron model has been discussed by E. Eichten *et al.*, Phys. Rev. D 21, 203 (1980). The mathematical problem of a discrete system coupled to a continuum is as old as quantum mechanics and is discussed in detail by R. F. Dashen, J. B. Healy, and I. J. Muzinich, Phys. Rev. D 14, 2773 (1976). Application to duality has been considered by Bernice Durand and Loyal Durand, Phys. Rev. D 23, 1531 (1981).

[11]Below flavor threshold $k_h \to i\kappa_h$. The depression of levels below the hadronic threshold is an example of level repulsion.

[12]There are two main oversimplifications. The inclusion of higher thresholds such as DD^*, etc., will enhance the threshold effect. A less naive treatment of angular momentum conservation gives a reduced threshold effect since for most states the $D\bar{D}$ orbital angular momentum is nonzero. The qualitative conclusions are unchanged. A more complete analysis is in preparation.