Role of the Phonon in the Quantum-Chromodynamical Plasma

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It is observed that a color-neutral scalar phonon mode of velocity $c/\sqrt{3}$ should exist in the QCD plasma. The energy density of the phonons is $3\sqrt{3}/16$ that of noninteracting massless gluons and compensates to a considerable extent for the suppression of gluon modes due to collective plasma effects.

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The properties of QCD matter at finite temperature have attracted increasing attention in recent times.[1] Most work has been devoted to the gluon plasma, though attention is now turning to inclusion of the effect of quarks. Lattice calculations indicate the existence[1,2] of a phase transition (at least one) for $SU(3)^c$ and also a residue of asymptotic freedom in the validity of the Stefan-Boltzmann law[3] at large T. The full potential with regard to phase structure is no doubt very rich, though experimental access to the theoretical possibilities is likely to remain difficult. In the first part of our discussion we shall be concerned with the gluon plasma.

The analysis of quasiparticles in the gluon plasma remains incomplete. Early work[4] revealed the existence of a plasmon, in analogy to that in familiar Coulomb plasmas. The screening associated with this effect modifies the interparticle force and gives an effective mass to the longitudinal gluon propagation and evidently also the transverse gluon[5] (though disputes surround the latter). These masses are responsible for a substantial suppression of the thermodynamic contributions compared to the free massless gas of gluons. (Such suppression is already evident in the perturbative evaluation of the free energy,[6] i.e., without infinite summations required to exhibit collective behavior.)

For small momentum the plasma frequency is given[4] by

$$\omega_p = (N/9)^{1/2} gT, \tag{1}$$

where $N=8$ for $SU(3)^c$ and $g^2(T)/4\pi = \alpha$ is the running coupling constant. For $\alpha \approx 1$ we find $\omega_p \sim 3.3T$; since the most probable momentum is $\sim 2.8T$, the contribution of these degrees of freedom is roughly half that expected in the massless case. A similar conclusion follows[5] for the transverse modes.

There is, however, another contribution to the dynamics (and thermodynamics) which seems to have escaped attention: The hydrodynamic phonon. The gluon-gluon cross section is fairly sizable,[7] even when cut off at the plasma mass, or the temperature. Averaging over angles, using known perturbative QCD results, one finds something of order[8]

$$l^{-1} \approx 30\alpha^2(T)T \tag{2}$$

for the mean free path l. For a minimally interesting T of m_π and $\frac{1}{2} < \alpha < 1$ we find $\frac{1}{7}f < l < \frac{1}{42}f$, which is substantially smaller than typical systems of interest created in hadronic collisions. (The maximum T of current interest is of order 1 GeV, so that the logarithmic variation of α is not of enormous significance.) Hence $p_{max} \sim 2\pi/l$ for the phonon is essentially infinite in the computation of thermodynamic quantities.

Apart from anomalies and quark masses discussed below, the equation of state

$$\langle \theta_\mu{}^\mu \rangle = \epsilon - 3p = 0, \tag{3}$$

where the angular brackets denote thermal average (for a uniform system), is that of a perfect relativistic fluid. In the collision regime mentioned above, the perfect classical energy-momentum tensor

$$T_{\mu\nu} = (\epsilon + p)u_\mu u_\nu - g_{\mu\nu} p \tag{4}$$

holds, with color-singlet density fluctuations propagating with sound velocity[9]

$$c_s = (\partial p/\partial \epsilon)^{1/2} = 1/\sqrt{3}. \tag{5}$$

ϵ is the energy density, p the pressure scalar, and u_μ the unit four-velocity.

The contribution of the phonons associated with this excitation to the energy density is

$$\epsilon_{ph} = \frac{1}{(2\pi)^3} \int_0^{p_{max}} \frac{(p/\sqrt{3})d^3p}{\exp(p/\sqrt{3}\,T) - 1}. \tag{6}$$

Comparing with free massless glue (two helicities, eight colors) we see

$$\epsilon_{ph}/\epsilon_{free\ glue} \cong 3\sqrt{3}/16 \tag{7}$$

for $T < p_{max}$, i.e., $T/\Lambda \lesssim \exp(6\pi)^{1/2} \sim 76.8$, a safe margin. Evidently the phonons make up for much of the suppressed gluon energy density, and must be considered in a complete dynamical description of the medium. Because of this one may expect "precocious" onset of the Stefan-Boltzmann law.

Next we include quarks in our considerations. Even in the most optimistic laboratory conditions for creation of QCD plasma, degeneracy is not expected to be significant. The exact evolution depends very much on the initial configuration. The perturbative inequalities[7]

$$\sigma_{gg\to gg} \gg \sigma_{gQ\to gQ} \gg \sigma_{gg\to Q\bar{Q}} \tag{8}$$

dictate the kinetic evolution of the plasma. Beginning with a pure gluon plasma, as envisioned in Ref. 10, it will take a significant time to create an equilibrated quark-antiquark population. On this time scale, existing quarks will reach equilibrium. For the energy-momentum tensor trace[11] we have

$$\begin{aligned}\langle\theta_\mu{}^\mu\rangle &= \epsilon - 3p \\ &= \textstyle\sum_i m_i\langle Q_i Q_i\rangle - (b\alpha/8\pi)\langle F_{\mu\nu}{}^a F^{\mu\nu a}\rangle.\end{aligned} \tag{9}$$

We have assumed canonical dimensions for the quark fields Q_i. In (9) m_i are the quark masses; $F_{\mu\nu}{}^a$ is the gluon field tensor and $b = \frac{11}{3}N_c - \frac{2}{3}N_f$ measures the strength of the anomaly. Since the anomaly is of order α we shall neglect its effect here. The quark mass term on the right-hand side of (9) is essentially of kinematical origin; in the absence of interactions it is easily seen to agree with the elementary thermodynamic relation for a single fermion species j

$$\epsilon_j - 3p_j = \frac{m_j{}^2}{\pi^2}\int_{m_j}^{\infty} \frac{d\epsilon\,(\epsilon^2 - m_j{}^2)^{1/2}}{\exp(\epsilon-\mu_j)/T+1}, \tag{10}$$

where μ_j is the appropriate chemical potential. Apart from the strange-quark mass, the right-hand side of (10) is negligibly small. Hence we expect the ideal equation of state, Eq. (3), to be approximately valid in the presence of quarks for a variety of phase-space distributions.

The quarks themselves will acquire a shifted mass due to interactions with the medium. A first-principles description of the phonon will require calculation of the density-density correlation. It is interesting to know whether the corrected phonon dispersion relation is concave or convex since in the former case on-shell processes such as one phonon→two phonons can occur. Finally as in condensed matter physics the phonon will modify the effective interaction between the other quasiparticles. These questions will be addressed in more detail elsewhere.

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