Observation of Large Deviations from the Bethe-Bloch Formula for Relativistic Uranium Ions

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By measuring the range deficit of 955-MeV/u ^{238}U ions in copper, the authors have identified very large deviations of the stopping power from the standard Bethe-Bloch formula. They show that this discrepancy can be accounted for by higher-order terms in the Mott cross section and by the relativistic Bloch correction. The significance of this result with regard to two high-energy astrophysics experiments is discussed.

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There have been several reports in the past[1-4] of deviations from the quadratic charge dependence of the Bethe-Bloch formula[5] for the stopping power of heavy charged particles in matter. Various contributions have been tentatively identified: (1) At low velocities a Z^3 correction has been observed[1,2] which is believed to be due to polarization of the atoms of the medium by the projectile.[6] (2) At low velocities a Z^4 correction has been measured[2] which is believed to be due to the finite lateral extent of those electron wave packets which participate in close collisions with the projectile[7,8] (this is the so-called Bloch correction). (3) At large velocities a correction to higher order in Z has been identified[3,4] by measuring range deficits of relativistic heavy nuclei in matter; these corrections have been explained as a combination of the Bloch correction[8] and the Mott correction[9] (which increases the energy lost as a result of the increase of the electron-ion scattering cross section over that determined in the lowest-order Born approximation).

Unfortunately, there are usually effects which complicate interpretation of the kinds of experiments referred to above. Accurate information regarding the average charge state of an ion, its primary energy, and the nature of its energy-loss density function are required to extract the fundamental dependence of its stopping power on its charge Z (which we regard here as the rms charge state of the ion in units of e, the magnitude of the electronic charge). For example, the observation of unusually large higher-order Z effects of the stopping power of channeled ions[10] has been explained as being due to asymmetric, Z-dependent, energy straggling.[11] The possibility of the other higher-order corrections being due to unidentified systematic effects has been a legitimate concern. To take a case in point, the extraction of stopping-power corrections from range deficits of heavy ions from the Lawrence Berkeley Laboratory Bevalac[3,4] relied on the calibration of particle energies in the Bevatron accurate to ~0.2%. Although the greatest care was taken in these calculations,[12] there was no way to verify their accuracy independently. In this Letter we report the results of an experiment to measure higher-order corrections which evades the problem of energy accuracy by increasing the size of the correction by nearly an order of magnitude. This softens our requirements on the energy resolution by a similar factor, although we have maintained our attention to this detail to the same degree as in earlier experiments.

The technique used to increase the higher-order effect consists simply of using the most energetic, heavily charged particle beam available. The capability of the upgraded Bevalac to accelerate relativistic uranium[13] provided us with the best possible beam for this application. We have studied the interactions of such a beam of particles with matter by measuring their range. This has been done by using copper blocks to degrade most of the energy of the ^{238}U nuclei and by determining the residual range with a stack of Lexan track etch detectors (each having an average thickness of 218 μm). The energy of the uranium beam was calculated to be $E_m = 955.7 \pm 2.0$ MeV/u by techniques similar to those used in the past.[4,12] The amount of material required to stop the beam was $[0.137 \pm 0.020$ g/cm^2 Al (beam window)$] + [0.0426 \pm 0.0020$ g/cm^2 air (flight path)$] + [8.3162 \pm 0.0010$ g/cm^2 Cu$] + [2.327 \pm 0.002$ g/cm^2 Lexan$]$. The range in the Lexan was determined by measuring the stopping positions of 96 particles, which comprise the peak of the stopping distribution shown in Fig. 1. The particles which populate this peak are predominantly ^{238}U. Fragments of atomic mass, A, produced via nuclear interactions in the upstream matter, will form broad distributions to the left or right of this peak if the ratio A/Z^2 is respectively less

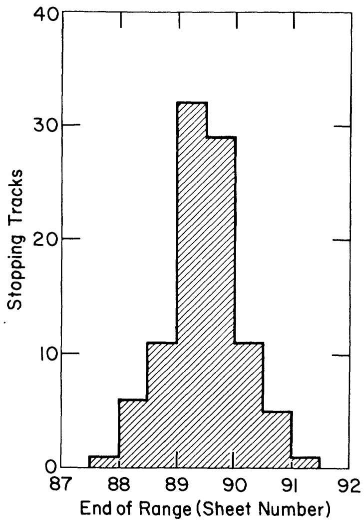

FIG. 1. Measured stopping distribution of U ions. 1 sheet ≈ 0.026 g/cm^2.

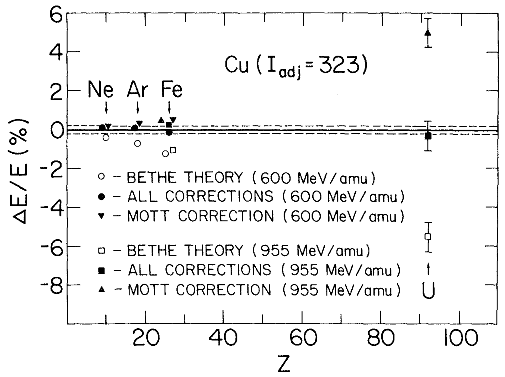

FIG. 2. Deviation of energies calculated from measured ranges, from those determined by time-of-flight methods. Data for 600-MeV/u Ne, Ar, and Fe were obtained from Ref. 4 while the 955-MeV/u Fe and U data are from this work. In all cases most of the range was in Cu. Large deviations from the Bethe theory are evident for U. See text for details.

or greater than that for ^{238}U. For example, a ^{238}Pa fragment produced by the loss of a single proton at the front of the Cu would stop ~10 sheets beyond the peak of Fig. 1. The standard deviation of the range histogram is 0.64 sheets which corresponds to the error in the average stopping position quoted for Lexan above.

To compare the observed range with various stopping-power theories we integrated the stopping power to the front of the Al beam pipe window in order to obtain a theoretical entrance energy E_t which was compared to the measured energy E_m. The parameter $\Delta E/E$ is determined by $(E_t - E_m)/E_m$. Rather than integrating from the projectile stopping position, which would have yielded questionable results because of the large uncertainties of stopping power for low-energy uranium due to electron capture and potentially very large higher-order corrections, we have used the measured range for 150-MeV/u ^{238}U from an earlier experiment[14] as our start point. Thus, in the comparisons we discuss below, we are actually focusing on the energy region from 150 to 956 MeV/u. For our first choice of a model to determine E_t we assumed the relativistic Bethe expression,[15,16] including the small density-effect correction. The large-velocity limit of the shell correction was included by using the adjusted mean ionization potential $I_{\rm adj}$, which was determined by using shell corrections and recommended values of the mean ionization potential I given in Ref. 16. The values used for $I_{\rm adj}$ were 166 ± 1 eV for Al, 85.4 ± 1.0 eV for air, 67.0 ± 1.0 eV for Lexan, and 323 ± 2 eV for Cu. To compensate for electron capture and loss by the uranium projectiles we used for Z the effective charge expression given by Pierce and Blann.[17] The result for E_t was 903.5 MeV/u. We checked the sensitivity to the choice of effective charge by calculating E_t according to the technique of Barkas and Berger[18] and found $E_t = 903.3$ MeV/u, quite close to the other value. The Pierce and Blann result, $\Delta E/E = -5.5\%$, is plotted in Fig. 2. For comparison, the error $\Delta E_m/E_m = \pm 0.2\%$ is shown as the dashed lines on either side of $\Delta E/E = 0$. The error bars on the data for U are due to uncertainties in electron-capture effects. These are extremely conservative error estimates. For example, even if the uranium ions were fully stripped for all energies greater than 150 MeV/u, the value for $\Delta E/E$ would shift only from -5.5% to -4.3%. Errors due to uncertainties in $I_{\rm adj}$, matter thickness, and angle of incidence are of the order of 0.2% or less in $\Delta E/E$. Thus, unless the beam energy is in error by ~5%, 25 times worse than estimated, the demonstration of deviations from Z^2 dependence is unavoidable.

We have ruled out any possibility of error in the beam energy accounting for the large value of $|\Delta E/E|$, by measuring the range in copper of iron ions, which were accelerated immediately after the uranium run under conditions *identical to those for uranium*. By identical, we mean that no changes in any of the Bevatron controls were made so that the field and frequency were literally identical for both beams. This situation was made possible by the similarity of the charge-to-mass ratios of $^{238}U^{68+}$ and $^{56}Fe^{16+}$ which were the charge states accelerated. The only difference between the beams was a slightly larger velocity for iron (957.7 MeV/u) due to the slight difference in the charge-to-mass ratios. The path lengths of the iron beam were measured to be $(0.137 \pm 0.020\ \mathrm{g/cm^2\ Al}) + (0.0356 \pm 0.0020\ \mathrm{g/cm^2\ air}) + (31.333 \pm 0.003\ \mathrm{g/cm^2\ Cu}) + (3.309 \pm 0.004\ \mathrm{g/cm^2}$ Lexan). The standard Bethe theory predicted an entrance energy of 947.3 MeV/u, only 1.1% less than E_m. This discrepancy is shown in Fig. 2 in comparison with earlier measurements.[4] For example the $\Delta E/E$ result for 600 MeV/u Fe was earlier observed to be -1.2%. When corrected for the various higher-order corrections (the Bloch[8] and Mott[9] corrections being the most important), $\Delta E/E$ becomes $+0.3\%$ for the present Fe experiment and -0.1% for the earlier Fe experiment. These values are completely consistent with estimates of beam-energy errors, which suggests that the Bevatron staff is quite capable of calculating beam energies to the stated accuracy and that there have been no substantial problems introduced in this regard by the dismantling and reassembly of the Bevatron to insert the vacuum liner.

To account for the large deviations observed in the uranium range we have chosen to calculate E_t in two additional ways: (1) by ignoring all Bloch corrections and assuming only the Mott correction; (2) by using all corrections (Mott, Bloch, and relativistic Bloch). In both cases, the Mott correction has been evaluated exactly from the tables of Mott cross sections calculated by Doggett and Spencer.[19] The Bloch corrections have been calculated according to the technique of Ref. 8. It should be noted that this employs relativistic electron wave functions valid only to the third Born approximation so that the results are less reliable than those for the Mott corrections. In accordance with Ref. 8 we have chosen θ_0 (the electron center-of-mass scattering angle above which the free, unlocalized electron approximation is valid) to be 0.1 rad. The results of these calculations are shown in Fig. 2. The errors presented for the uranium data are dominated by electron-capture uncertainties described previously. Additional errors are possible in the Bloch correction due to both the uncertainty in the third Born approximation and also uncertainties in θ_0. If these latter uncertainties are as large as 30%, as indicated in Ref. 8, the error bars for the point labeled "all corrections" for U would increase to $\sim\pm 2\%$. Using only the Mott correction we obtain a value for E_t of 1003.9 MeV/u, 5% larger than E_m whereas by using all corrections we obtain $E_t = 952.8$ MeV/u, only 0.3% smaller than E_m. By ignoring the Bloch corrections and using only Mott corrections we would be as much in error had we used no higher-order corrections at all.

The results presented here have an important impact on some experiments either already done or to be done involving the study of energetic nuclei in the cosmic rays. Binns *et al.*[20] have recently published the results of their HEAO-3 experiment to study actinide abundance in the cosmic rays. Their analysis was based on the assumption of a strict Z^2 dependence of Cherenkov radiation and of gas ionization. However, it has been shown[21] that if stopping power is modified by the Mott corrections, charge shifts in the HEAO-3 data of up to several charge units in the important platinum-lead region are possible. Our results indicate that the Bloch corrections significantly soften this effect for relativistic ultraheavy nuclei and such large corrections may not be required.

The demonstration of the validity of the Mott and Bloch corrections to the Bethe theory is also important with regard to an experiment which is now being built to search for heavy antinuclei in the cosmic rays.[22] This experiment relies on the charge asymmetry of the Mott correction to identify antinuclei by their reduced scintillation signal compared with nuclei having the same velocity. The agreement between experiment and theory over the entire periodic table strongly supports the premise of the proposed experiment and indicates that the calculations of antinucleus sensitivity for nuclei of lower charge ($Z \lesssim 28$) presented in Ref. 22 are valid.

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