Effect of Gap Distortion on the Field Splitting of Collective Modes in Superfluid ^{3}He-B

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The field splitting of the real squashing, $J=2$, mode in ^{3}He-B is shown to become highly nonlinear at large fields as a result of the ellipsoidal deformation of the energy gap. This leads to crossings of the $J_z=+1$ and $J_z=0$ levels with the $J_z=+2$ level. The crossing points depend sensitively on the couplings between the $J=2$, 1, and 0 modes. The theory is in good agreement with the observed field evolution and level crossing as measured recently by Shivaram *et al.*

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The condensate of superfluid ^{3}He-B consists of p-wave Cooper pairs in spin-triplet states $S_z=\pm1$ and $S_z=0$. The fluctuations of the corresponding order-parameter components about their equilibrium values give rise to eighteen order-parameter collective modes.[1] These eighteen modes can be classified in terms of total angular momentum $J=0$, 1, and 2. The $J=2$ modes, i.e., the so-called squashing (sq) and real squashing (rsq) modes, have ideal frequencies (without Fermi-liquid corrections) equal to $\omega=(\frac{12}{5})^{1/2}\Delta_0$ and $\omega=(\frac{8}{5})^{1/2}\Delta_0$ where Δ_0 is the BCS gap parameter. The unique nature of the $J=2$ modes is that in a magnetic field they split into five components.[2]

Field splitting of the sound-absorption peak due to the rsq mode into five components has been observed first by Avenel, Varoquaux, and Ebisawa.[3] For low fields (up to about 50 mT) this splitting is linear with magnetic field H in accordance with the theoretical prediction.[2] This linear splitting arises solely from the $S_z=0$ component of the spin-triplet Cooper pairs which leads to (1) anisotropic spin polarization, and (2) a dynamic spin-singlet pairing component. Both effects give rise to contributions proportional to $(\vec{H}\cdot\vec{d})(\vec{d}\times\delta\vec{d})$ in the equation of motion for the fluctuation $\delta\vec{d}$ of the order parameter vector $\vec{d}$.

In recent acoustic impedance measurements Shivaram *et al.*[4] have observed a strong nonlinear behavior of the levels of the rsq-mode quintuplet and even a crossing of levels as the field increases to about 0.2 T. In the following we shall show that the previous theory of Ref. 2, which is valid for all fields, is also capable of explaining the observed nonlinear field behavior and the crossing of levels at higher magnetic fields. It turns out that the ellipsoidal deformation of the energy gap by the field[2,5] is responsible for the crossing of levels (this effect was not noticed in Ref. 2 because gap distortion was neglected in the numerical calculations and figures). This is similar to the effect of quadrupolar deformation of nuclei on the hyperfine spectrum. We find that in large fields the nonlinear effects of gap distortion on the $J_z=\pm1$ and 0 states become smaller if coupling between $J=2$ and $J=1$ or 0 is taken into account. This means that we are dealing with a field regime lying between the Zeeman regime (J is a good quantum number) and the Paschen-Back regime (L_z and S_z are good quantum numbers[6]). J_z is a good quantum number for all fields.

The importance of the gap distortion effect in ^{3}He-B is demonstrated by the fact that the magnetic susceptibility increases as the field is increased.[7] The reason is that the transverse gap parameter Δ_1 increases with respect to the BCS gap Δ_0, and the longitudinal gap parameter Δ_2 decreases as the field is increased.[2] The parameters Δ_1^2 and Δ_2^2 are proportional to the fractions of the $S_z=\pm1$ and $S_z=0$ Cooper pair populations. Since Δ_2 and thus the fraction of $S_z=0$ pairs is suppressed, the susceptibility is enhanced as H is increased. The self-consistent equations determining the gap parameters and the effective field have been extended[8] to include, in addition to the effect of the Landau parameter F_0^a, the effect of F_2^a. The calculated susceptibility is found to be in good agreement with that measured in Ref. 7.

Let us reformulate now the theory of Ref. 2 in terms of quantum numbers J, J_z of total angular momentum. The equations for the order-parameter fluctuations have been stated by two of us[9] in the form of eigenvalue equations for the order-parameter fluctuations: $[(3v_1)^{-1}+R']\,\delta d'=0$. Here $3v_1$ is the BCS coupling constant for p-wave pairing, R' is a 9×9 matrix, and $\delta d'$ is a nine-component column vector of the real parts of the order-parameter fluctuations $\delta d_{j\mu}$ (j, $\mu=x, y, z$). An analogous equation holds for the vector $\delta d''$ of the imaginary parts of the $\delta d_{j\mu}$. The small coupling of $\delta d'$ and $\delta d''$ by particle-hole asymmetry has been discussed elsewhere[10] and will be neglected in the following. A unitary transformation by means of a 9×9 matrix U leads from the nine Cartesian basis vectors for the direct product of orbital and spin space to the nine basis vectors $|J, J_z\rangle$ ($J=0, 1, 2$) which are eigenvectors of total angular momentum J. The transformed equation of motion becomes $A'(U\,\delta d')=0$ where $A'\equiv[(3v_1)^{-1}+UR'U^\dagger]$. The eigenfrequencies of the real collective modes are given by $\det(A')=0$. It turns out[6] that $\det(A')$ factorizes into five secular determinants corresponding to $J_z=\pm 2$, $J_z=\pm 1$, and $J_z=0$. These secular determinants are found to be identical with those given in Ref. 2 as it should be because they are invariant with respect to unitary transformation. The zeros of the $J_z=\pm 2$ determinant [see the factor in square brackets in Eq. (63) of Ref. 2] yield the frequencies of the $J_z=\pm 2$ components of the rsq mode. The zeros of the $J_z=\pm 1$ determinant [see Eq. (51) of Ref. 2] yield the frequencies of the $J_z=\pm 1$ components of the rsq mode *and* the frequencies of the transverse NMR modes. In this expression there occur off-diagonal matrix elements of A' which lead to couplings between $J=2$ and $J=1$. The zeros of the $J_z=0$ determinant [see the factor in curled brackets in Eq. (63) of Ref. 2] yield the eigenfrequencies of the longitudinal NMR mode, the mode at 2Δ, and the $J_z=0$ component of the rsq mode. Here occur off-diagonal matrix elements of A' which couple $J=2$, 1, and 0. An analogous factorization holds for the secular equation of the sq-mode quintuplet [see Eq. (80) of Ref. 2 for $J_z=\pm 2, 0$ and Ref. 6 for $J_z=\pm 1$].

From the expressions in Refs. 2 and 9 we obtain to lowest order in the field and gap distortion the following frequency shifts for the rsq-mode quintuplet:

$$\omega-\omega_0=-J_z\Omega g_2'(\omega_0)+J_z^2\omega_0\Delta_0^{-2}(\Delta_1^2-\Delta_2^2)\left[\tfrac{1}{8}-\tfrac{1}{7}g_2'(\omega_0)-\tfrac{1}{3}g''(\omega_0)\right]$$
$$+\,\omega_0\Delta_0^{-2}\left\{(\Delta_1^2-\Delta_0^2)\left[\tfrac{1}{2}+\tfrac{2}{7}g_2'(\omega_0)-\tfrac{8}{3}g_2''(\omega_0)\right]+(\Delta_2^2-\Delta_0^2)\left[\tfrac{5}{8}-\tfrac{2}{7}g_2'(\omega_0)-\tfrac{7}{3}g_2''(\omega_0)\right]\right\}. \tag{1}$$

Here $\omega_0=(\frac{8}{5})^{1/2}\Delta_0$, and $g_2'(\omega_0)$ and $g_2''(\omega_0)$ are the temperature-dependent Landé factors of Ref. 9 for the rsq and sq modes taken at $\omega=\omega_0$. For $F_2^a=0$ the effective Larmor frequency Ω is given by[2,8]

$$\Omega=-\gamma H[1+F_0^a\chi_{\mathrm{BW}}^0(\Omega)/\chi_n^0]^{-1}. \tag{2}$$

For consistency of the approximation one must take the zero-field ideal susceptibility $\chi_{\mathrm{BW}}^0/\chi_n^0=(2+Y)/3$ where Y is the Yosida function. The nonlinear frequency shifts in Eq. (1) which arise from gap distortion are proportional to $\Delta_1^2-\Delta_2^2$, $\Delta_1^2-\Delta_0^2$, and $\Delta_2^2-\Delta_0^2$. To lowest order in Ω^2 one has[8] $(\Delta_1^2-\Delta_0^2)=\Omega^2/4$ and $(\Delta_2^2-\Delta_0^2)=-c\Omega^2$ where $c=\frac{1}{2}$ for $T=0$ and $c\to 1$ for $T\to T_c$: $c=\frac{1}{2}+(\Delta_0/4)(\partial/\partial\Delta_0)\ln(1-Y)$.

We have calculated numerically the gap parameters Δ_1 and Δ_2 and the rsq-mode frequencies from the exact equations of Refs. 8 and 2. For $\Delta_0(T)$ we take the BCS gap. The m^*/m values of Greywall and Busch[11] have been used to determine F_0^a, and F_2^a is set equal to zero. In Fig. 1 we show the field evolution of the rsq-mode frequencies for a pressure of 1.35 bars and a fixed temperature $T_0=0.77T_c$ (solid lines). For comparison we have included in Fig. 1 the frequencies which have been calculated by taking $\Delta_1=\Delta_2=\Delta_0$ (dotted lines). One sees clearly that gap distortion is the main reason for the strong deviation from linear field splitting. For about $H=0.16$ T

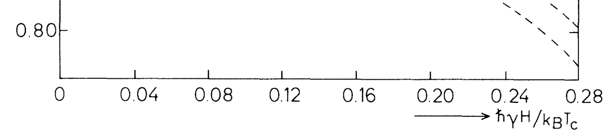

FIG. 1. Field evolution of the rsq-mode frequencies ω for $J_z=+2, \ldots, -2$, at $p=1.35$ bars, $F_0^a=-0.712$, and $T_0=0.77T_c$ (solid lines). The dotted lines have been calculated by neglecting gap distortion ($\Delta_1=\Delta_2=\Delta_0$). The dashed lines for $J_z=0, \pm 1$ have been obtained by neglecting coupling between $J=2$ and $J=1$ or 0.

we obtain crossing of the $J_z = +2$ and $+1$ levels, and for $H = 0.19$ T there occurs crossing of the $J_z = +2$ and 0 levels. It is interesting to notice that the levels for $J_z = -2$ and -1 disappear above a threshold field. The reason is that the effective pair-breaking edge (equal to $2\Delta_2 - \Omega$ for all fields) decreases as the field is increased.

We have evaluated also the zeros of the exact diagonal matrix elements $\langle 2, 0|A'|2, 0\rangle$ and $\langle 2, \pm1|A'|2, \pm1\rangle$ neglecting the coupling between $J=2$ and $J=1, 0$. The resulting frequencies of the $J_z = 0$ and ±1 modes are given by the dashed lines in Fig. 1. Comparison between the solid and dashed lines shows that the mixing of different J at given $J_z = 0, \pm1$ leads to a substantial increase of these frequencies.

In Fig. 2 we show the measured[4] and calculated frequency shifts $\Delta\omega$ (abscissa) of the rsq-mode states as a function of H (ordinate) for $p = 1.35$ bars and $T_0 = 0.77 T_c$. One sees that the shapes of our theoretical curves are very similar to curves which can be drawn through the experimental bars (level widths). The experimental field for the crossing point of the $J_z = +2$ and $+1$ levels is about 0.14 T while the calculated field is about 0.16 T. One notices that the measured nonlinear field behavior of the $J_z = +2$ and $+1$ levels is even more pronounced than the calculated one.

We have calculated also the frequency shifts $\Delta\omega = \omega - \omega_0$ from Eq. (1) and find very good agreement with the exact results over the whole range of fields shown in Fig. 2. Since this holds for all temperatures and pressures we conclude that Eq. (1) yields a good account of linear as well as nonlinear field splitting.

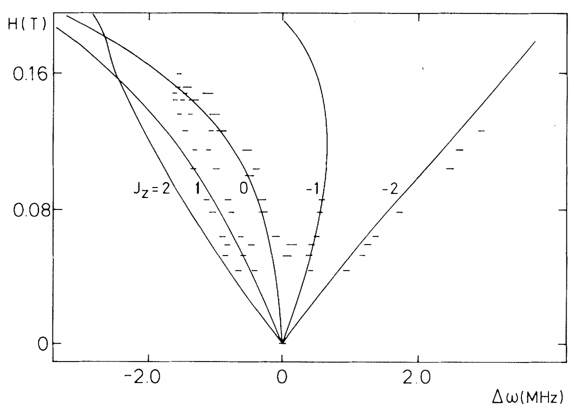

FIG. 2. Frequency shifts $\Delta\omega$ (abscissa) of the rsq-mode states $J_z = +2, \ldots, -2$, vs field H (ordinate), at $p = 1.35$ bars, $F_0^a = -0.712$, and $T_0 = 0.77 T_c$. The experimental bars (level widths) are taken from Ref. 4.

Shivaram *et al.*[4] have fitted their observed frequency shifts by the expression

$$\Delta\omega = \alpha(T,p)J_zH + \beta(T,p)J_z^2H^2 - \Gamma(T,p)H^2. \quad (3)$$

This is just the form of $\Delta\omega = \omega - \omega_0$ given by Eq. (1) together with Eq. (2): α is proportional to the Landé factor $g_2'(\omega_0)$, β is proportional to the factor multiplying J_z^2, and $-\Gamma$ is proportional to the last term in Eq. (1). Figures 3(a) and 3(b) show plots of our theoretical β and Γ vs T/T_c for $p = 1.35$ and 7.3 bars. These β and Γ have been determined by fitting our exact curves for $\Delta\omega(H)$ (like those in Fig. 2) by Eq. (3). The agreement between the measured[4] and calculated values of β and Γ is seen to be reasonably good. We have also calculated β and Γ directly from Eq. (1)

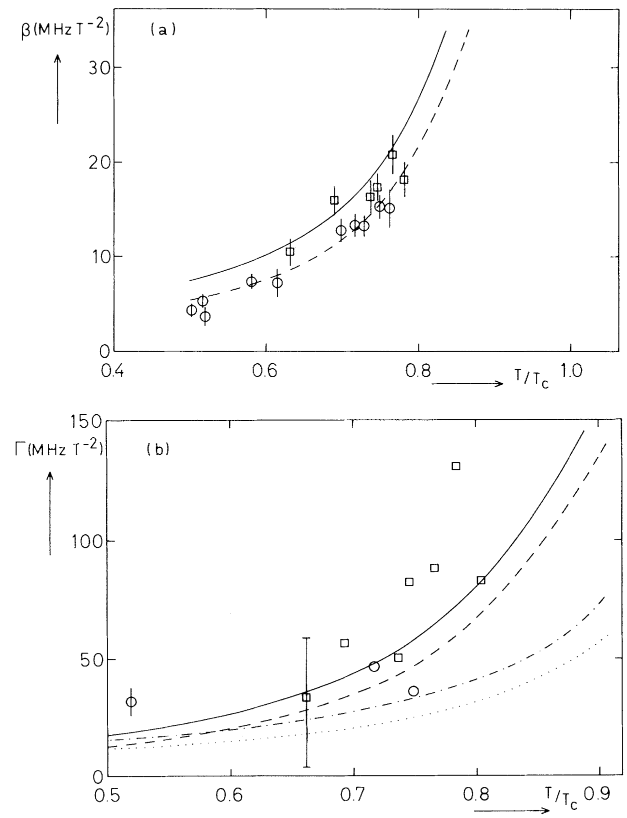

FIG. 3. The quadratic field terms β and Γ of the rsq-mode frequency shift [see Eq. (3)] are plotted in (a) and (b) as functions of T/T_c for $p = 1.35$ bars, $F_0^a = -0.712$ (solid lines), and $p = 7.3$ bars, $F_0^a = -0.740$ (dashed lines). Open squares (circles) represent data of Ref. 4 taken at 38.24 MHz around 1 bar (63.77 MHz around 7 bars). The dashed-dotted (1.35 bars) and dotted (7.3 bars) lines in (b) represent the Γ of the squashing mode.

and find very good agreement with the curves shown in Fig. 3. Thus we conclude that the relatively simple expression for $\Delta\omega$ in Eq. (1) constitutes a very good approximation for the whole range of experimentally accessible fields.

One sees from Fig. 3 that the quadratic field parameters β and Γ increase as T increases towards T_c. This behavior can be explained with the help of Eq. (1). For instance, comparison of Eqs. (1) and (3) shows that β is proportional to

$$(\Delta_1^2 - \Delta_2^2)/(\Delta_0 H^2) = (\tfrac{1}{4} + c)(\Omega/H)^2/\Delta_0.$$

Thus β rises like $(T_c - T)^{-1/2}$ as T increases towards T_c, and the same is true for Γ. Further one sees from Eq. (2) that a decrease of F_0^a leads to an enhancement of Ω. We find better agreement between the measured[4] and calculated values of α, β, and Γ by employing the F_0^a values due to Ref. 11 rather than those given by Alvesalo, Haavasoja, and Manninen.[12]

The effect of F_2^a on the rsq-mode frequencies has been considered so far only to first order in the field.[13,14] The inclusion of F_2^a in the nonlinear theory of Ref. 2 will be carried out[15] self-consistently in analogy to Ref. 8. The sensitivity of the crossing point should enable one to determine bounds for F_2^a by fitting the experimental level widths.

Finally we remark that for the squashing-mode quintuplet we have obtained a similar nonlinear field evolution.[6] However, mode crossing occurs only between the $J_z = +2$ and $+1$ states. The Γ_{sq} which describes the frequency shift of the $J_z = 0$ component [see Eq. (3)] is shown in Fig. 3(b).

In conclusion we can say that the observed nonlinear field evolution and crossing of rsq-mode levels can be explained convincingly by the theory of Ref. 2 if the ellipsoidal deformation of the energy gap, that is, the suppression of $S_z = 0$ pairs, is taken into account.

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1For a review, see P. Wölfle, Physica (Utrecht) 90B, 96 (1977).

2L. Tewordt and N. Schopohl, J. Low Temp. Phys. 37, 421 (1979).

3O. Avenel, E. Varoquaux, and H. Ebisawa, Phys. Rev. Lett. 45, 1952 (1980).

4B. S. Shivaram, M. W. Meisel, Bimal K. Sarma, W. P. Halperin, and J. B. Ketterson, following Letter [Phys. Rev. Lett. 50, 1070 (1983)], and private communication.

5S. Engelsberg, W. F. Brinkman, and P. W. Anderson, Phys. Rev. A 9, 2592 (1974).

6M. Warnke, N. Schopohl, and L. Tewordt, to be published.

7R. F. Hoyt, H. N. Scholz, and D. O. Edwards, Physica (Utrecht) 107B, 287 (1981).

8N. Schopohl, J. Low Temp. Phys. 49, 347 (1982).

9N. Schopohl and L. Tewordt, J. Low Temp. Phys. 45, 67 (1981).

10N. Schopohl, H. Streckwall, and L. Tewordt, J. Low Temp. Phys. 49, 319 (1982).

11D. S. Greywall and P. A. Busch, Phys. Rev. Lett. 49, 146 (1982).

12T. A. Alvesalo, T. Haavasoja, and M. T. Manninen, J. Low Temp. Phys. 45, 373 (1981).

13J. A. Sauls and J. W. Serene, Phys. Rev. Lett. 49, 1183 (1982).

14Y. Hasegawa and H. Namaizawa, Prog. Theor. Phys. 67, 389 (1982), and 68, 345 (1982).

15N. Schopohl, to be published.