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INTERACTION OF THE SOLAR PLASMA WITH THE EARTH'S MAGNETIC FIELD*

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Recent satellite observations of the interplanetary magnetic field[1,2] and the belts of trapped radiation surrounding our planet have focused attention on the shape and extent of the terrestrial magnetic field. The terrestrial magnetic field, if it occurred in a vacuum, could be well approximated by a simple magnetic dipole field. Many authors, including Bierman[3] in particular, however, have pointed out that the sun perpetually emits an intense time-varying neutral flux of energetic protons and electrons. It is the purpose of this Letter to show how the magnetic field surrounding the earth is reshaped and terminated by the incident current of what amounts to a diamagnetic medium.

The charged particles in a flow of neutral plasma will follow different paths on encountering a magnetic field depending on their charge and mass. The electrons having negligible momentum are deflected by the magnetic field in an infinitesimal distance when compared with the proton behavior creating a large electric field due to charge separation. The electric field decelerates the protons and accelerates the electrons and their combined motion results in a very thin current layer beyond which the total magnetic field is zero. By adapting Rosenbluth's[4] treatment of the magnetic pinch to the problem, the thickness of the layer was found to be of the order of $(8\pi N_0 e^2/m_e c^2)^{-1/2}$, where N_0 is the particle density in the plasma, e the particle charge, c the velocity of light, and m_e the mass of the electron. For $N_0 = 50$/cc the sheath thickness amounts to 0.3 kilometer.

Under such circumstances it has been shown that the hydromagnetic approximation,

$$p + B^2/8\pi = \text{constant} \tag{1}$$

(where p is the proton pressure, a tensor, and B is the magnetic field), is valid except for a small region very close to the magnetic poles. For proton mass m_i and initial plasma velocity $\vec{v}$,

$$p = 2N_0 m_i v^2 \cos^2\psi, \tag{2}$$

where ψ is the angle between $\vec{v}$ and the normal to the surface of the current sheath. In a spherical coordinate system where the $\theta = 0$ axis is parallel to the earth's dipole and $\vec{v}$ is in the plane of $\phi = \pi/2$,

$$\cos\psi = (\vec{v}\cdot \mathbf{grad}F)/(v\,|\mathbf{grad}F\,|) = a\left\{\frac{\partial F}{\partial r}[\sin\phi\ \sin\theta\ \cos\lambda - \cos\theta\ \sin\lambda] + \frac{1}{r}\frac{\partial F}{\partial \theta}[\sin\phi\ \cos\theta\ \cos\lambda + \sin\theta\ \sin\lambda] + \frac{1}{r\sin\theta}\frac{\partial F}{\partial \phi}[\cos\phi\ \cos\lambda]\right\}, \tag{3}$$

Table I. Values of α and β as functions of θ, the latitude angle, where α and β are the coefficients in a series expansion of r of the plasma sheath surface as a function of η, the longitudinal angle from the plane containing the earth's dipole and the earth-sun line. $r=1+\alpha\eta^2+\beta\eta^4$ in units of $r_0=(M^2 8\pi N_0 m_i v^2)^{1/6}$, and η is expressed in radians.

θ	90°	75°	60°	45°	30°	15°
α	0.104	0.102	0.096	0.086	0.068	0.031
β	0.011	0.011	0.010	0.008	0.007	0.004

where a is a normalizing factor involving partial derivatives, λ is the solar latitude position, and $F(r, \theta, \phi)=\text{constant}$ is the equation for the surface of the current sheath.

In the case of a plane current sheath, only the local surface current would contribute to the component of the magnetic field tangential to the sheet. In the case of the curved surface derived in this work, neglecting the nonlocal currents is only an excellent first approximation. The surface derived here should be used to derive a closer approximation to the exact magnetic field in higher approximation. The magnetic field resulting from a hemispherical current sheath whose current density is an appropriate function of angular position has been calculated and found to be twenty to forty percent of the total magnetic field at the sheath, depending on the position. Since the earth's magnetic field decreases as the cube of the distance, the surface is brought in 6% closer to the earth. The shape of the surface is altered slightly; the radial distance of the sheath everywhere being less, relative to the sheath intersection with the earth-sun line, than calculated in first approximation.

The component of the earth's magnetic field parallel to the sheath surface is doubled on the interior by the surface current and is canceled on the exterior of the surface. The component of the earth's field perpendicular to the surface is canceled on both sides of the sheath surface in the neighborhood of the sheath by the changes in surface current that occur along a magnetic field line. The component of the earth's field which lies in the surface sheath is given by

$$B_S=(Mb/r^3)[\sin\theta+2\cos\theta(dr/d\theta)], \tag{4}$$

where b is another normalizing factor, M is the earth's dipole moment, and the derivative is taken in the plane of the sheath surface.

The differential equation for the surface of the sheath resulting from substituting Eqs. (2), (3), and (4) into Eq. (1) has been solved approximately. For the daylit side of the earth and $\lambda=0$ the solution is well approximated between $\phi=\pi/2$ and $\phi=\pi$ by a power series in η, where $\eta=\phi-\pi/2$:

$$r=1+\alpha\eta^2+\beta\eta^4; \tag{5}$$

α and β are functions of θ. r is expressed in units of $r_0=(M^2 8\pi N_0 m_i v^2)^{1/6}$. r_0 represents a scaling factor dependent on the solar plasma pressure. If $N_0=50/\text{cc}$ and $v=2\times10^8$ cm/sec, r_0 would be about five earth radii which is approximately the position of the sudden change in the earth's field reported by Sonnett et al.[1] from satellite observations. Values of α and β as a function of θ are presented in Table I. Table II presents values of r and $|r\cos\phi|$ as a function of ϕ for $\theta=90°$. Table III presents approximate values of r as a function of θ for $\phi=3\pi/2$, the night side of the earth where the sheath current is reversed (at $\phi=3\pi/2$) over the daylit current direction. A more detailed discussion and complete calculation will be presented elsewhere.

The predictions of infinite r on the night side of the earth should not be taken seriously since sidewise pressure from a radial solar magnetic field, or especially sidewise proton pressure from a nonzero temperature plasma, would close the "tail" of the sheath as expected from Eq. (1). The axis of the sheath surface is shifted roughly one third of λ for $\lambda\neq0$; otherwise the surface is affected little by changes in λ.

Table II. r of the plasma sheath surface (in units of r_0) and $|r\cos\phi|$ as functions of ϕ, the longitudinal position in the equatorial plane of the earth's magnetic dipole. $180°<\phi<270°$ corresponds to the night side of the earth.

ϕ	90	105	120	135	150	180	210	240	270
r	1.000	1.009	1.031	1.068	1.126	1.342	1.842	3.472	∞
$\|r\cos\phi\|$	0	0.261	0.515	0.755	0.976	1.342	1.595	1.736	1.781

Table III. r of the plasma sheath surface (in units of r_0) on the night side of the earth and $|r \sin\theta|$ as functions of polar angle in the plane of the earth's dipole and earth-sun line.

θ	15.5°	19°	23°	37.5°	55°	69°	90°
r	~1.10	1.26	1.50	2.00	3.00	5.00	∞
$\lvert r \sin\theta \rvert$	~1.06	1.19	1.38	1.58	1.72	1.77	1.785

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[2]M. Neugebauer, Phys. Rev. Letters 4, 6 (1960).
[3]L. Bierman, Observatory 77, 109 (1957).
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EVIDENCE FOR A CONFIGURATIONAL EMF IN A CONDUCTING MEDIUM*

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There is a pressure drop associated with an increase in the flow rate of a moving fluid. The venturi meter and the aspirator are familiar devices which derive from this phenomenon. This paper concerns itself with the detection of an analogous effect in the electron gas in a metal.

The usual analysis for the pressure drop in classical fluids requires the conservation of energy. Associated with a higher flow rate is an increase in kinetic energy. The pressure is a measure of potential energy. It must be lower in the region of higher flow velocity where the kinetic energy is higher. If the higher flow velocity is brought about by a narrowing of the flow channel then the phenomenon is called the Bernoulli effect.

In the electron gas of a metal the dissipative effects dominate over the inertial ones. Unlike the case for a viscous fluid, the dissipation comes from the transfer of momentum to the lattice vibrations and imperfections rather than to the walls of the container. Regardless of the nature of the collisions, however, the two fluids have much in common. They both have need of the presence of a force merely to maintain the flow. A force maintains the current in the face of the scattering of forward momentum into random motion by collisions. In the electron fluid this force is the applied electric field. By Ohm's law the force is proportional to the flow velocity. The electric field in the electron gas is the analog of the pressure gradient in viscous fluid flow.

The electron gas in a metal sees a background of positive charge from the ion cores. These keep the electron density uniform on penalty of building up high internal fields in the metal. The electron gas is, therefore, relatively incompressible.

Suppose that an electric current flow is constricted in cross section as in Fig. 1. Continuity and incompressibility demand that the flow velocity or drift of electrons be greater in the constricted region. Inspired by analogy with the hydrodynamic problem, the following question is posed: What accelerates the electrons in the region of changing flow rate so that they acquire the increased speed necessary for drift through the constriction? Is there an additional field present from which the electrons get the extra accelerating kick which they must have in order to move faster in the constriction? The experi-

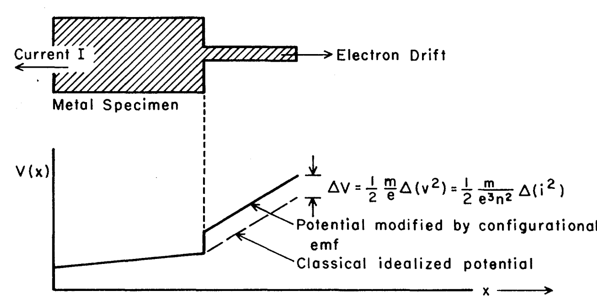

FIG. 1. Expected potential vs distance due to constricted current flow in the conducting specimen shown.