EFFECT OF THE PION-PION RESONANCE ON THE NEGATIVE-POSITIVE RATIO

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It has been shown by Frazer and Fulco that a P-wave pion-pion resonance can account for the isotopic vector component of the nucleon electromagnetic structure.[1] The purpose of this note is to show that the contribution of this resonance to photoproduction is capable of producing a large effect in the π^- to π^+ ratio at threshold.

It is assumed that the Chew, Goldberger, Low, and Nambu invariant amplitudes,[2] $A^{\pm 0}$, $B^{\pm 0}$, $C^{\pm 0}$, and $D^{\pm 0}$, satisfy the Mandelstam representation.[3] The singularity produced by the 2π state is a branch cut running from $4\mu^2$ to ∞ in the complex plane for the momentum-transfer variable $t=-(q-k)^2=(\omega_q-k)^2-(\vec{q}-\vec{k})^2$.[4] Since G parity allows only the scalar part of the photon interaction in the process $\gamma+\pi\to\pi+\pi$, the 2π singularities will be present only in the photoproduction amplitudes representing the scalar part of the photon, namely the (0) amplitudes. It should also be noted that the (0) amplitudes have only the $I=1/2$ pion-nucleon phases in the physical region for $\gamma+n\to n+\pi$ and $\gamma+\bar{n}\to\bar{n}+\pi$. The strength of the singularities associated with these two channels is given by the imaginary part of the amplitudes. Therefore, since the $I=1/2$ phases are small in the low-energy region for photoproduction, it is a good approximation to neglect these singularities.

The (0) amplitudes may now be written in the form

$$A^{(0)}(s,t)=\text{Poles}+\frac{1}{\pi}\int_{4\mu^2}^{\infty}dt'\,\frac{\text{Im}_2A^{(0)}(s,t')}{t'-t},\qquad(1)$$

where Im_2 denotes the absorptive part of $A^{(0)}$ for the channel $\gamma+\pi\to n+\bar{n}$. The square of the total energy for this channel is t, and s is the square of the total energy for the photoproduction channel. Since the singularities are neglected for $s>(M+\mu)^2$, the physical region for photoproduction, a polynomial expansion may be used to continue $\text{Im}_2A^0(s,t')$ to $s>(M+\mu)^2$.

By means of the unitarity condition, it is now possible to obtain $\text{Im}_2A^0(s,t)$ expressed in terms of the helicity amplitudes for $\pi+\pi\to n+\bar{n}$, and $\pi+\gamma\to\pi+\pi$. Since the process $\pi+\gamma\to\pi+\pi$ contains only odd angular momentum, only the $J=1$ state is kept, and $J\geq 3$ is neglected. The $\pi+\pi\to n+\bar{n}$ states that enter into these imaginary parts are the same as those in the nucleon electromagnetic structure problem, and we find

$$\text{Im}A^0=-th(t)g_2^{\ v}(t),\qquad(2a)$$

$$\text{Im}B^0=h(t)g_2^{\ v}(t),\qquad(2b)$$

$$\text{Im}C^0=0,\qquad(2c)$$

and

$$\text{Im}D^0=h(t)g_1^{\ v}(t),\qquad(2d)$$

where

$$h(t)=\frac{3}{8\sqrt{2}e}\,\frac{m_1^{\ *}(t)}{F_\pi(t)},$$

$m_1(t)$ is the $J=1$ eigenamplitude for $\pi+\gamma\to\pi+\pi$ and g_1^v, g_2^v are the imaginary parts of the nucleon form factors given by FF. Here $F_\pi(t)$ is the pion form factor introduced by these authors.

Wong has shown that $m_1(t)$ is well represented for $t>4\mu^2$ by the form

$$m_1(t)=\frac{\Lambda}{F_\pi(1)}\left(\frac{1+a}{t+a}\right)F_\pi(t),\qquad(3)$$

where Λ is the new coupling constant required to describe $\pi+\gamma\to\pi+\pi$ and a is a parameter that depends on the pion-pion P-wave amplitude. If the FF pion-pion resonance is used, we have $a=5.7$ and $F_\pi(1)=1.08$.

Because of the simple form of $h(t)$, it is possible by making suitable subtractions in FF's Eq. (1) to write the dispersion-integral parts of A^0, B^0, C^0, and D^0 directly in terms of the nucleon isotopic vector form factors. Since the form factors are approximately linear in the region $-5<t<0$, the (0) amplitudes may be expressed in terms of $G_i^{\ v}(0)$ and

$$\frac{1}{G_i^{\ v}(0)}\,\frac{dG_i^{\ v}(t)}{dt}\bigg|_{t=0}=\tfrac{1}{6}\gamma^2=\frac{1}{t_\gamma^{\ '}},$$

where t_γ' is approximately the position of the pion-pion resonance. (Frazer and Fulco obtained $t_\gamma' \approx 12$.)[1] For the (0) amplitudes, we have

$$A^0 = \frac{e_\gamma g_\gamma}{2}\left[\frac{1}{s-M^2}+\frac{1}{\bar{s}-M^2}\right] + \lambda'\left(\frac{\mu_p'-\mu_n}{2}\right)\left[1+\frac{t-a}{t_\gamma'}\right], \quad (4a)$$

$$B^0 = \frac{e_\gamma g_\gamma}{(s-M^2)(\bar{s}-M^2)} - \lambda'\left(\frac{\mu_p'-\mu_n}{2}\right)\frac{1}{t_\gamma'}, \quad (4b)$$

$$C^0 = -\tfrac{1}{2}g_\gamma(\mu_{p_\gamma}'+\mu_{n_\gamma})\left[\frac{1}{s-M^2}-\frac{1}{\bar{s}-M^2}\right], \quad (4c)$$

$$D^0 = -\tfrac{1}{2}g_\gamma(\mu_{p_\gamma}'+\mu_{n_\gamma})\left[\frac{1}{s-M^2}+\frac{1}{\bar{s}-M^2}\right] - \lambda'\frac{e}{2t_\gamma'}, \quad (4d)$$

where $\bar{s} = -(p_2-k)^2$ and

$$\lambda' = \frac{3\Lambda(1+a)}{8\sqrt{2}\,eF_\pi(1)}.$$

Since the (0) amplitudes enter into $d\sigma(\pi^+)$ and $d\sigma(\pi^-)$ with opposite signs, the ratio $d\sigma(\pi^+)/d\sigma(\pi^-)$ at threshold will be quite sensitive to changes in the (0) amplitude. The threshold ratio becomes

$$R = \frac{d\sigma(\pi^-)}{d\sigma(\pi^+)} = \left(\frac{ef+(\mu_p+\mu_n)f-\frac{\lambda'}{8\pi}(\mu_p'-\mu_n)\left(1-\frac{1+a}{t_\gamma'}\right)}{ef-(\mu_p+\mu_n)f+\frac{\lambda'}{8\pi}(\mu_p'-\mu_n)\left(1-\frac{1+a}{t_\gamma'}\right)}\right)^2 \quad (5)$$

An upper limit on the magnitude of Λ can be obtained by considering π^+ production separately. It is observed that a change in f^2 of ±0.01 (which is roughly the current uncertainty) can be compensated for in the threshold π^+ cross section by giving Λ a value of $\mp1.75e$. The experimental errors in the present π^+ data, with $f^2 = 0.08$, would seem to allow $-1.8e < \Lambda < 1.8e$ (see Fig. 1). When combined with the uncertainty in f^2, this would constrain $|\Lambda|$ to be less than $2.5e$.

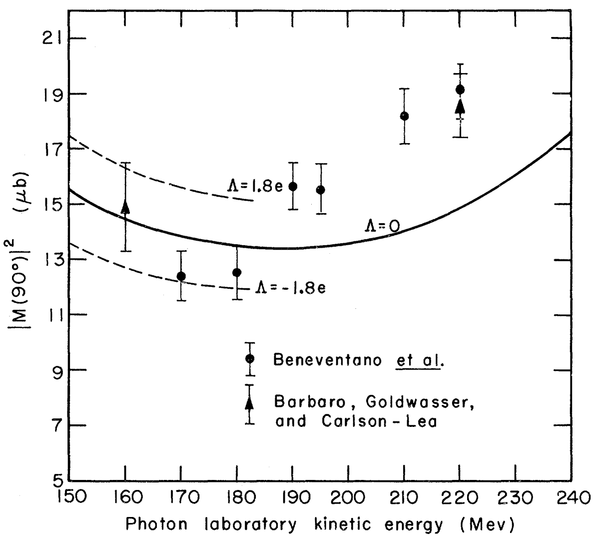

FIG. 1. The matrix element squared for π^+ photoproduction at $\theta = 90$ deg. The solid line represents the prediction for $\Lambda = 0$; the dashed lines, for $\Lambda = \pm1.8e$. The experimental points are those of Barbaro et al. and Beneventano et al. (see reference 6).

Wong[5] has estimated the size of Λ from the π^0 lifetime. His results are $|\Lambda| \approx e$. For $|\Lambda| = e$, formula (5) leads either to $R = 1.16$ or to $R = 1.44$, depending on the sign of Λ.

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[1]W. R. Frazer and J. R. Fulco, Phys. Rev. 117, 1609 (1960) (hereafter denoted FF).

[2]G. F. Chew, M. L. Goldberger, F. E. Low, and Y. Nambu, Phys. Rev. 106, 1345 (1957) (hereafter denoted CGLN).

[3]S. Mandelstam, Phys. Rev. 112, 1344 (1958); 115, 1741 and 1752 (1959).

[4]The notation of CGLN is used throughout.

[5]H. S. Wong, Bull. Am. Phys. Soc. 4, 407 (1959); and preceding Letter [Phys. Rev. Letters 5, (1960)].

[6]A. Barbaro, E. L. Goldwasser, and D. Carlson-Lee, Bull. Am. Phys. Soc. 4, 273 (1959); M. Beneventano, G. Bernardini, D. Carlson-Lee, G. Stoppini, and L. Tau, Nuovo cimento 4, 323 (1956).