pion, through further inelastic collisions, is captured from the continuum in about 0.9×10^{-12} sec. A π^-p atom with $n\approx30$ is formed, and de-excites to $n\approx15$ by collisions in which H_2 molecules are dissociated, in $\approx0.2\times10^{-12}$ sec. At $n\approx15$, the predominant mode of energy loss becomes Auger transitions in collisions, which can de-excite the π^-p atom to $n\approx4$ in $\approx1.0\times10^{-12}$ sec. At this point the Stark effect mixing and consequent nuclear capture from s states happens[5] in $<\approx10^{-12}$ sec, a much shorter time than would be required for further de-excitation via Auger collisions or via radiation.

The present data are in reasonable agreement with the above picture[6] of pion absorption from s states via Stark effect mixing. They therefore imply the absence of mesonic x rays from low-lying levels of the π^-p atom.

We wish to thank John Deahl and Gale Pewitt for help in running the chamber, and Anna Marie Bankowski for scanning the pictures.

*This work was supported in part by the U. S. Atomic Energy Commission.

[1]T. Fields, G. B. Yodh, M. Derrick, and J. Fetkovich, Bull. Am. Phys. Soc. 4, 401 (1959).

[2]A. S. Wightman, Phys. Rev. 77, 521 (1950); and Princeton University thesis, 1949 (unpublished).

[3]T. B. Day, G. A. Snow, and J. Sucher, Phys. Rev. Letters 3, 61 (1959).

[4]T. B. Day, invited paper at 1960 American Physical Society Washington Meeting, and University of Maryland Physics Department Report No. 175 (unpublished).

[5]J. E. Russell and G. L. Shaw, Phys. Rev. Letters 4, 369 (1960).

[6]A crucial part of the above picture is the rapid de-excitation of the π^-p atom to $n\approx4$. Wightman's (reference 2) original upper limit for the de-excitation time was 5×10^{-11} sec. An investigation of the implications of this value in view of the present experimental result was reported by T. B. Day, G. A. Snow, and J. Sucher, Phys. Rev. 118, 864 (1960). However, the rapid de-excitation (1.2×10^{-12} sec) calculated in reference 4 seems to allow fairly good agreement with this experiment without invoking other absorption mechanisms.

EFFECTIVE-RANGE FORMULA FOR PHOTOPION PRODUCTION FROM PIONS*

How-Sen Wong
Lawrence Radiation Laboratory, University of California, Berkeley, California
(Received June 27, 1960)

In the Chew-Mandelstam[1] approach to the strong-interaction problem at low energies, the amplitude for $\gamma+\pi\rightarrow2\pi$ plays a central role in any phenomena involving photons. Recently, Ball[2] has found that this process produces an additive correction to the formulas of Chew et al.[3] for photopion production from nucleons. Compton scattering by pions and by nucleons also involves the $\gamma+\pi\rightarrow2\pi$ reaction. A simple and reasonably accurate formula for the $\gamma+\pi\rightarrow2\pi$ amplitude which can be used in any of these applications is given here.

We assume that the simple invariant transition amplitude, $M(s,t,u)$, has the Mandelstam representation[4]

$$M(s,t,u)=\frac{1}{\pi^2}\int_4^\infty\int_4^\infty \rho(x,y)\left[\frac{1}{(x-s)(y-t)}+\frac{1}{(x-t)(y-u)}+\frac{1}{(x-u)(y-s)}\right]dx\,dy. \quad (1)$$

It is defined through the relation,

$$T_{fi}=\frac{i(2\pi)^4\delta^4(K+p_1-p_2-p_3)\sum[(-i/\sqrt{2})\epsilon_{\alpha\beta\gamma}\epsilon_{\lambda\sigma\mu\nu}(p_1)_\lambda(p_2)_\sigma K_\mu\epsilon_\nu]M(s,t,u)}{(16p_{10}p_{20}p_{30}K)^{1/2}}, \quad (2)$$

where α, β, and γ are the isotopic indices of the pions, and $\epsilon_{\alpha\beta\gamma}$ and $\epsilon_{\lambda\sigma\mu\nu}$ are the conventional antisymmetric tensors of third and fourth rank, respectively, Here s, t, and u are defined[5] as (see Fig. 1)

$$s=-(K+p_1)^2,$$

$$t=-(K-p_2)^2,$$
$$u=-(K-p_3)^2, \quad (3)$$

and are related by the condition

$$s+t+u=3.$$

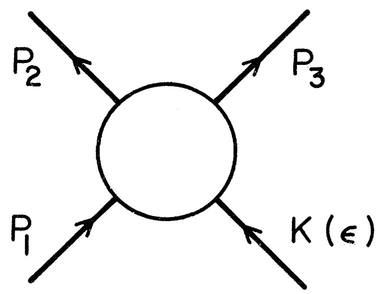

FIG. 1. Diagram of $\gamma+\pi\rightarrow 2\pi$.

When the photon K and meson p_1 are the incoming particles, the variables s, t, and u are related to barycentric system variables,

$$s=4(1+p^2),$$
$$t=1-2kE+2kp\cos\theta,$$
$$u=1-2kE-2kp\cos\theta,$$

where p and E are the magnitudes of the outgoing pion momenta and energy, respectively, k is the energy of the photon, and we have $\cos\theta=(\vec{p}\cdot\vec{k})/pk$. The formula for the differential cross section in the barycentric system is

$$\frac{d\sigma}{d\Omega}=\sum\frac{p}{k}\left|\frac{1}{8E}\frac{T_{fi}}{2\pi}\right|^2.$$

From the Mandelstam representation, we can locate the singularities of the partial-wave amplitudes, defined through the relation

$$M(s,z)=\sum_{\text{odd}\,l}M_l(s)P_l'(z),$$

where $z=\cos\theta$. The M_l's are analytic in the whole complex s plane except for cuts on the real axis. The right-hand cut runs from 4, the physical threshold for two pions, to ∞; the left cut runs from 0 to $-\infty$. Using these analytic properties, the unitary condition, and the Omnès[6]-Frazer-Fulco[7] method, we find

$$M_l(s)=\frac{1}{\pi D_l(s)}\int_{-\infty}^{0}\frac{D_l(s')\operatorname{Im}M_l(s')}{s'-s}ds', \quad (4)$$

where $D_l(s)$ is the π-π denominator function introduced by Chew and Mandelstam.[1]

If we assume that for $s\geqslant 4$, we have $\operatorname{Im}M\approx\operatorname{Im}M_1$, we can show that

$$M(s,t,u)\approx\frac{1}{\pi}\int_4^\infty\operatorname{Im}M_1(x)\left[\frac{1}{x-s}+\frac{1}{x-t}+\frac{1}{x-u}\right]dx. \quad (5)$$

We see that all partial waves for $l>1$ can be obtained once M_1 is known. Formula (5) is the basis of an independent calculation that has been carried out by Gourdin and Martin.[8]

Using crossing symmetry, we can obtain a formula for $\operatorname{Im}M_1(s)$ for $s<0$ in terms of $\operatorname{Im}M_1(s)$ for $s>4$. We do not give this formula here because the effect-range approach does not require such detailed information. In any event, it is clear that for $l=1$, the integral of Eq. (4) is homogeneous, so that its solution is not unique, at least with respect to a multiplicative factor. Therefore an arbitrary parameter must be introduced. In the calculation below, we shall fix this parameter Λ as the value of $M_1(s)$ at $s=1$. At present, we do not know how to relate Λ to fundamental constants, and for the time being it must be determined from experiment.

To proceed further, we need the denominator function, D_1, for p-wave pion-pion scattering. The pion-pion calculations of Chew and Mandelstam have not yet reached a conclusive stage but these authors have given an approximate form for D_1 which corresponds to the replacement of the unphysical branch cut in the pion-pion amplitude by a finite number of poles.[9] Further, they have shown that two poles lead to an accurate approximation in the physical region up to $s\sim 40$. Once the parameters of the two-pole formula have been determined, it will be a straightforward problem to incorporate the information into the amplitude $\gamma+\pi\rightarrow 2\pi$. The determination of the pion-pion parameters is still in progress,[10] but we outline here, for future use, the form of solution of our problem that corresponds to the pole approximation of Chew and Mandelstam.[9] We shall illustrate the method with the one-pole pion-pion amplitude, for which parameters have been given by Frazer and Fulco.[7]

We propose to replace the left-hand cut of the amplitude M_1 by poles with appropriate positions and residues. The "n-pole formula" thus obtained contains $2n$ parameters. One of these may be identified with Λ; the rest will be determined from crossing relations.

By representing $\operatorname{Im}M_1$ on the left-hand cut by one or two delta functions, we obtain from Eq. (4) the one-pole formula

$$M_1(s)=\frac{\Lambda(1+a)D_1(1)}{(s+a)D_1(s)}, \quad (6)$$

or the two-pole formula

$$M_1(s)=\frac{\Lambda D_1(1)}{D_1(s)}\left[\frac{D_1(-b)}{s+b}+\frac{\Lambda_1 D_1(-c)}{s+c}\right]\times\left[\frac{D_1(-b)}{1+b}+\frac{\Lambda_1 D_1(-c)}{1+c}\right]^{-1}, \quad (7)$$

where Λ and Λ_1 are real, and a, b, and c are real and positive. The parameters Λ_1, a, b, and c may be determined from the following crossing relations which are derived from Eq. (5):

$$M_1(1) - \tfrac{3}{2} M_1^L(1) = 0, \tag{8a}$$

$$\frac{\partial}{\partial s} M_1(1) = 0, \tag{8b}$$

$$\frac{\partial^2}{\partial s^2} M_1(1) - 6 \frac{\partial^2}{\partial s^2} M_1^L(1) = 0, \tag{8c}$$

where

$$M_1^L(s) = \frac{1}{\pi} \int_{-\infty}^{0} \frac{\mathrm{Im} M_1(s')ds'}{s'-s} = \frac{\Lambda(1+a)D_1(1)}{(s+a)D_1(-a)}$$

for the one-pole formula, and

$$M_1^L(s) = \Lambda D_1(1)\left[\frac{1}{s+b} + \frac{\Lambda_1}{s+c}\right]\left[\frac{D_1(-b)}{1+b} + \frac{\Lambda_1 D_1(-c)}{1+c}\right]^{-1}$$

for the two-pole formula.

If the Frazer-Fulco one-pole $D_1(s)$ function is used for a resonance at $s=10$, we find $a \cong 5.7$ from the crossing condition (8a). The parameters Λ_1, b, and c in the two-pole formula may be determined from (8a) to (8c). It turns out that no choice of Λ_1, b, and c can satisfy all three conditions (8a) to (8c) if the Frazer-Fulco one-pole formula is used. However, the first two conditions of Eqs. (8) can be satisfied with the pair of poles in the range between 0 and -4.93. The p-wave amplitude in the physical region not only is insensitive to the positions of the two poles so long as (8a) and (8b) are satisfied, but also is not much different from the one-pole solutions, which satisfies (8a) alone. Thus we may be confident of the accuracy of the two-pole results once the parameters of the two-pole π-π scattering formula are known. The one-pole formula already obtained is probably adequate for most purposes.

We can estimate the order of magnitude of Λ by considering the decay of neutral pions, assuming that the $\gamma+\pi\rightarrow 2\pi$ reaction plays a prominent role in the intermediate state. We find $|\Lambda| \approx 1.3e$ ($e^2 = 1/137$) if the π^0 lifetime is 4×10^{-16} sec.[11] Our calculation is based on the assumption that only the least massive (two-pion) intermediate state contributes to the dispersion integral, but there is no good reason to neglect the 3π contribution, especially if there is a resonance or a bound state at roughly the same energy as the two-pion resonance.[12] A better estimate of Λ may come from photopion production on nucleons, where Ball has shown that $\gamma+\pi\rightarrow 2\pi$ makes an important and characteristic contribution.[2]

The author wishes to thank Professor G. F. Chew for suggesting this investigation and for his guidance. He also wishes to thank Dr. D. Y. Wong and Dr. A. Martin for helpful discussions.

*A preliminary account of this work was given at the 1959 Thanksgiving meeting of the American Physical Society, November 27-28, 1959 [Bull. Am. Phys. Soc. 4, 407 (1959)]. This work was performed under the auspices of the U. S. Atomic Energy Commission.

[1]G. F. Chew and S. Mandelstam, Phys. Rev. 119, 467 (1960).

[2]J. S. Ball, University of California Radiation Laboratory Report UCRL-9172 (unpublished); and J. S. Ball, following Letter [Phys. Rev. Letters 5, 73 (1960)].

[3]G. F. Chew, M. L. Goldberger, F. E. Low, and Y. Nambu, Phys. Rev. 106, 1337 (1957).

[4]S. Mandelstam, Phys. Rev. 112, 1344 (1958); 115, 1741 and 1752 (1959).

[5]We use the fundamental metric tensor such that $p_1 \cdot p_2 = \vec{p}_1 \cdot \vec{p}_2 - p_{10} p_{20}$. Units are used in which $\hbar = c = 1$ and $\mu = 1$, where μ is the mass of the pion.

[6]R. Omnès, Nuovo cimento 8, 316 (1958).

[7]W. R. Frazer and J. R. Fulco, Phys. Rev. Letters 2, 365 (1959); Phys. Rev. 117, 1909 (1960).

[8]M. Gourdin and A. Martin, CERN Report No. 7687 TH/67 (unpublished).

[9]G. F. Chew and S. Mandelstam, University of California Radiation Laboratory Report UCRL-9126 (unpublished).

[10]J. S. Ball and D. Y. Wong, Lawrence Radiation Laboratory (private communication).

[11]G. Harris, J. Orear, and S. Taylor, Phys. Rev. 106, 327 (1957).

[12]G. F. Chew, Phys. Rev. Letters 4, 142 (1960). The author wishes to thank Professor R. J. Eden of Cambridge University for indicating this point to him.