NEUTRINO EMISSION FROM BLACK-BODY RADIATION AT HIGH STELLAR TEMPERATURES

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That neutrino emission processes may affect the properties of matter at high temperatures, and hence stellar evolution, was first pointed out by Gamow and Schoenberg[1] and by Pontecorvo.[2] We list the known important neutrino emission processes as follows:

(i) The so-called "urca" process, in which neutrinos are emitted by nuclear transitions.[1,3,4] This process depends only upon the observed nucleon-neutrino coupling, while all those which follow take place only with a universal Fermi interaction.

(ii) Neutrino bremsstrahlung[2,5]:

$$e^- + (Z,A) \to e^- + \nu + \bar{\nu} + (Z,A).$$

(iii) The photoneutrino or neutrino Compton effect[4]:

$$\gamma + e^- \to e^- + \nu + \bar{\nu}.$$

We add two new processes, which are the subject matter of this note[6]:

(iv) Pair-annihilation neutrinos:

$$e^- + e^+ \to \nu + \bar{\nu}.$$

(v) Photon-photon neutrinos:

$$1.\ \gamma + \gamma \to \nu + \bar{\nu},$$

$$2.\ \gamma + \gamma \to \gamma + \nu + \bar{\nu}.$$

The energy lost from finite regions due to processes (i), (ii), and (iii) is not important for temperatures below $10^{10}\,^\circ$K. For most densities, and for temperatures around $10^9\,^\circ$K, processes (iv) and (v) are considerably more rapid.

These last two processes involve direct neutrino emission from black-body radiation itself, mediated either by virtual electron-positron pairs or by the real pairs present in thermal equilibrium. In spite of the more complex graphs which describe processes (v), they reach importance comparable with the simple process (iv) since the available energy in the center-of-mass system is typically higher, at least in the region of $kT/mc^2 \ll 1$. The graphs for these processes are shown in Fig. 1; process (v, 1)

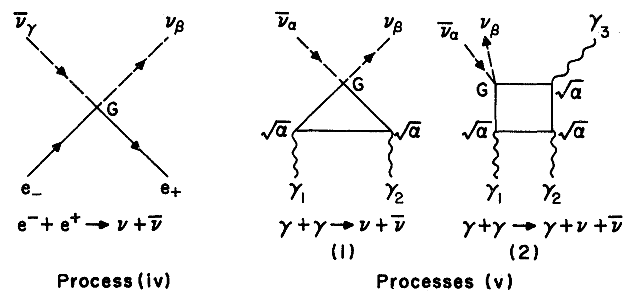

FIG. 1. Feynman diagrams for the processes considered here.

requires renormalization before it can be interpreted. An estimate of process (v, 2) confirms the idea that it contributes to the energy loss rate about like process (iv) even though it is proportional to an additional three powers of 1/137.

Only the simple process (iv), with its unique Feynman diagram, has been calculated exactly. The transition amplitude is proportional to $(\bar{\nu}e)(\bar{e}\nu)$ as in the theory of Feynman and Gell-Mann,[7] from which we take the Fermi coupling constant, G. Calculation of the cross section for the pair process in the center-of-mass system is elementary, and gives

$$\sigma_{iv} = 1.5\times10^{-45}(c/v_{\mathrm{rel}})(E_T{}^2 - 1)\ \mathrm{cm}^2,$$

where E_T is the total c.m. energy in units of m_0c^2, and v_{rel} is the relative velocity in the c.m. system. The number of electron pairs present at equilibrium is well known.[8] In the pure vacuum radiation field, where the initial proper mass density is zero, the positron density becomes

$$n_+{}^v = (1/\pi^2)(\hbar/m_0c)^{-3} f(\beta), \quad \beta = (m_0c^2/kT) \qquad (1)$$

and

$$f(\beta) = \int_0^\infty \exp[-\beta(1+x^2)^{1/2}]x^2dx$$

$$= \frac{1}{\beta}\int_0^\infty \exp[-\beta\cosh\theta]\cosh2\theta d\theta$$

$$= K_2(\beta)/\beta. \qquad (2)$$

$K_2(\beta)$ is the modified Hankel function of second order.[9] The presence of nondegenerate matter expresses the density of positrons present at equilibrium, but does not affect the rate of production of positrons. The energy loss, however, depends only upon the production rate (and the equal annihilation rate), and is therefore independent of matter density, until degeneracy begins to affect the production of positron-electron pairs. For the nondegenerate case, then, the electron density does not matter; in the degenerate case, if also $n_- \gg n_+$, we have

$$n_+^{(d)} = n_+^{v} \exp(-E_F/kT);$$

E_F is the electron Fermi energy, including m_0c^2.

Note that

$$(1+\tfrac{1}{2}x^2) > (1+x^2)^{1/2} > x.$$

We find the approximations to (2):

$$f_\infty(\beta) = e^{-\beta}\int_0^\infty \exp[-\tfrac{1}{2}x^2\beta]x^2dx = (\tfrac{1}{4}\sqrt{\pi})(2/\beta)^{3/2}e^{-\beta}, \quad \beta \gg 1,$$

$$f_0(\beta) = \int_0^\infty \exp[-x\beta]x^2dx = 2/\beta^3, \quad \beta \ll 1. \qquad (3)$$

The rate of loss of energy density is given by the integral

$$-\frac{dU}{dt} = \iint (E_- + E_+)\sigma_{iv} v_{\mathrm{rel}} n_- n_+ d^3p_- d^3p_+,$$

where the subscripts refer to electron and positron quantities. Making such rough estimates as

$$E_- \cong E_+ \cong m_0c^2,$$

the integral can be approximated. We obtain

$$-dU/dt \cong 7.5\times10^{20}[f(\beta)]^2 \text{ ergs/cc-sec,} \qquad \text{(nondegenerate)}$$

$$\cong 2.5\times10^{14}(\bar{Z}/\bar{A})\exp(-E_F/kT)f(\beta), \qquad \text{(degenerate)}$$

where $\bar{Z}$ and $\bar{A}$ are mean values of atomic number and atomic mass.

In stellar problems the energy loss rate is usually expressed not in terms of unit volume, but per gram of matter, for which the relations above must be modified to read

$$\epsilon = (1/\rho)(dU/dt) \text{ ergs/g-sec.}$$

We list in Table I a few values of ϵ for various processes of energy loss in the relevant temperature and density region. It is to be observed that processes of type (v), so far only roughly estimated, are not only independent of density but even of degeneracy of the material. Both processes (iv) and (v) are independent of chemical composition, apart from the effect of electron density on degeneracy.

The thermal energy content of matter at $T = 10^9$ °K is 10^{17} ergs/gram. Therefore the relaxation time for cooling by neutrino loss due to the processes we consider lies between 10^9 sec and 10^{15} sec. In any case, by the time T has reached 2×10^9 °K, the relaxation time is only of the order of 10^4 to 10^5 sec.

Temperatures of 2×10^9 °K or more are required by the current theories of element synthesis for elements beyond iron, and by the ideas of supernova collapse.[10-13] It seems that neutrino-loss processes change the means by which such temperatures may be reached, but

Table I. Rates of energy loss by neutrino emission.[a] Energy loss in ergs/gram-second, at a density of 10^6 g/cc.

Temperature (°K)	Process: Photoneutrino (iii)	Annihilation (iv)	Photon-photon (v, 2) (approximate)	t_{cool} (sec)
5×10^8	10^2	$10^{0.7}$	$10^{3.5}$	10^{13}
1×10^9	$10^{4.2}$	$10^{7.4}$	10^7	10^9
2×10^9	$10^{8.6}$	$10^{11.6}$	10^{11}	10^5
2.5×10^9	$10^{10.4}$	$10^{12.6}$	10^{12}	10^4

[a]In the last column is entered the relaxation time in seconds for losing the full thermal energy content of the material by neutrino emission.

do not modify their effects.

The matter and thermal radiation in a stellar interior will remain in thermal equilibrium in spite of neutrino emission, provided that the energy loss rate is low enough to permit readjustment of the local near-equilibrium conditions. The virial theorem relates the thermal energy strictly to the gravitational energy while this remains true. Neutrino loss will therefore not result in reduced temperature but only in the contraction of the star, the gravitational energy making up for the neutrinos radiated. The central temperature will rise at a rate limited by neutrino emission, until the time for gravitational readjustment—roughly measured by the free-fall times of 1 to 100 seconds (depending on density)—becomes longer than the characteristic time of neutrino energy loss. Until central temperatures of about 3×10^9 °K are reached, the star which has begun to emit neutrinos will everywhere remain in a state of near equilibrium, and show increasingly rapid gravitational contraction. This final stage of stellar evolution is thus controlled by the consequences of the universal Fermi interaction, together with gravitational effects; nuclear processes do not play the determining role until the "phase change" from iron to helium is reached at 5×10^9-7×10^9 °K.

In view of the high neutrino emission rate, the first mark of supernovae might be high neutrino flux while the surface of the star has yet shown no sign of this sudden internal collapse. At $T=5\times10^9$ °K, the neutrino flux from such stars will amount to 10^{53} neutrinos of average energy 1 Mev per second. At a distance of 100 light years, the flux is around 10^{13}/cm^2-sec. This is detectable.[14] Therefore the establishment of a neutrino monitor station in terrestial or spatial laboratories may help us predict forthcoming supernovae.

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