the deuteron has spin 1. In addition, the conservation of isotopic spin observed in the inelastic scattering of deuterons, alpha particles, or more recently C^{12} ions from N^{14} can be understood as arising from the improbability of the deuteron in the target nucleus undergoing an internal spin flip to form its $T=1$ state.

It would appear then that at energies where stripping is expected to be important, one does not require the introduction of the formal concept of isotopic spin conservation to describe the inhibition observed in reactions leading to such 0+, $T=1$ levels. That isotopic spin selection rules are experimentally satisfied at high energies has also been pointed out by Wilkinson,[7] who suggests that the reason for their validity is the overlapping of many levels in the compound system which break up before isotopic spin mixing occurs. In this connection it would be of interest to examine the situation in a reaction leading to a higher $T=1$ level where $J\neq 0$, to see whether the transition is allowed or not. It will also be of interest to carry out a study of the reaction $B^{10}(Li^6,d)N^{14*}(T=1)$ and $B^{10}(Li^7,t)N^{14*}(T=1)$ to verify further the results obtained here. It can be pointed out that a study of the latter reaction in conjunction with the reaction $B^{10}(Li^7,He^3)C^{14}$(g.s.) should also provide a sensitive test of the relations between the cross sections predicted by the isotopic spin selection rules.

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SEARCH FOR RESONANCE IN π-π INTERACTION IN π-N SCATTERING AT 0.96 Bev

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Photographs of interactions in a hydrogen bubble chamber exposed to the 0.96-Bev negative pion beam from the Brookhaven Cosmotron were kindly supplied by Professor Steinberger and have been measured and analyzed.[1] This Letter reports a detailed examination of 110 events of the type

$$\pi^- + p \to p + \pi^- + \pi^0$$

for evidence of a resonance in the pion-pion interaction.

Other photographs of the same exposure have been measured by Alles-Borelli, Bergia, Ferreira, and Waloschek[2] at Bologna, and some by Pickup, Ayer, and Salant[3] at Brookhaven. Center-of-mass momuntum spectra of the π^- and π^0 mesons from the above reaction were found[3,4] to be in reasonable accord with the extended isobaric nucleon model of Lindenbaum and Sternheimer,[4] which includes contributions from the $T'=\frac{1}{2}$ isobaric state (corresponding to the $T=\frac{1}{2}$ resonance in the π-N system at $T_\pi = 600$ Mev) as well as from the $T'=3/2$ state.

Laboratory kinetic energy spectra of protons were found to possess a low-energy peak which has been interpreted by Bonsignori and Selleri[5] and Derado[6] as evidence for a strong pion-pion interaction. This interpretation is based on an estimate of the "pole part" of the pion-nucleon scattering cross section by the method of Chew and Low.[7] Their formula, valid when the square of the four-momentum transfer Δ^2 to the spectator nucleon approaches the unphysical limit $-\mu^2$, where μ is the pion mass, becomes in the case of the reaction considered,

$$\frac{\partial^2\sigma}{\partial\Delta^2\partial\omega^2} = \frac{f^2}{2\pi}\,\frac{\Delta^2/\mu^2}{(\Delta^2+\mu^2)^2}\,\frac{1}{q_{1L}^{\;2}}\,\omega(\tfrac{1}{4}\omega^2-\mu^2)^{1/2}\sigma_{\pi\pi}(\omega), \tag{1}$$

where f^2 is the renormalized pion-nucleon coupling constant, q_{1L} the laboratory momentum of the incident pion, and $\sigma_{\pi\pi}(\omega)$ the total cross section for scattering of the two pions at total energy ω (in their barycentric system). Bonsignori and Selleri argue that if $\sigma_{\pi\pi}$ is large enough the pole may be expected to dominate the first part of the physical zone, and suggest that formula (1) constitutes a reasonable approximation to the true matrix element in the region $\Delta^2 < 0.3M^2$, where M is the nucleon mass.

Upon integration over ω^2, formula (1) becomes

$$\frac{\partial\sigma}{\partial\Delta^2}=\frac{f^2}{2\pi}\frac{\Delta^2}{(\Delta^2+\mu^2)^2}\frac{1}{q_{1L}^2}\times\int_{4\mu^2}^{\omega_{\max}(\Delta^2)}\sigma_{\pi\pi}(\omega)(\omega^2/2\mu^2)(1-4\mu/\omega^2)^{1/2}d\omega^2, \quad (2)$$

where $\omega_{\max}(\Delta^2)$ is the upper limit obtained from the phase space diagram in the (Δ^2, ω^2) plane. Figure 1 shows experimental spectra of our results separately, and combined with those of the Bologna group, compared with predicted curves of[8] $\partial\sigma/\partial T$ from the statistical and isobar models,[9] and from Eq. (2) on the assumption that $\sigma_{\pi\pi}$ is constant. The striking low-T peak confirms that interactions of the incoming pion with a virtual meson emitted by the nucleon are significant in the low-Δ^2 region.

Qualitatively one would expect the observed low-energy spectrum to be only partially explained by this simple mechanism. Peierls[10] has recently predicted the inelastic $\pi^- - p$ branching ratios on the basis of a model in which final states are dominated by two types of final-state interaction: (a) between the two final-state pions in a state with $t_{\pi\pi}=1$, and (b) between a pion and the nucleon through the 33 resonance. According to this model, about two-thirds of the $(p\pi^-\pi^0)$ cross section involves mechanism (a). This can be regarded as an upper limit to the contribution of the simple picture described above. This agrees qualitatively with the point of view of Selleri,[11] who suggests that the reaction is initiated wholly by pion-pion interaction in the cloud, with a certain probability of their having no further interaction with the nucleon, which in the $(p\pi^-\pi^0)$ case is probably greater than 50%.

It is much more difficult to infer the presence of a resonance in $\sigma_{\pi\pi}$ from available data. Such a resonance in $\sigma_{\pi\pi}$ would be expected to produce a hump in a spectrum of events against ω^2/μ^2. A strong peak at $\omega^2/\mu^2 \approx 22$ is evident in the spectrum of all our events (solid histogram of Fig. 2), in disagreement with the predicted curves of the statistical and isobar models. Since isobar formation is known to occur,[1-3] it should be noted that this would yield a general background in ω^2/μ^2, above which our peak is still significant. Pickup, Ayer, and Salant[3] report a strong peak in their Q distribution for the $(\pi^-\pi^0)$ system at $Q=325$ Mev $(\omega^2/\mu^2=19)$ after considering events with protons going backwards in the center-of-mass system.

It is proposed that valuable information on the existence and position of a resonance in $\sigma_{\pi\pi}$ could be obtained by resolving the available data into a number of separate regions of ω^2/μ^2, and considering spectra over Δ^2/μ^2 in each. This has been done and Fig. 3 compares experimental re-

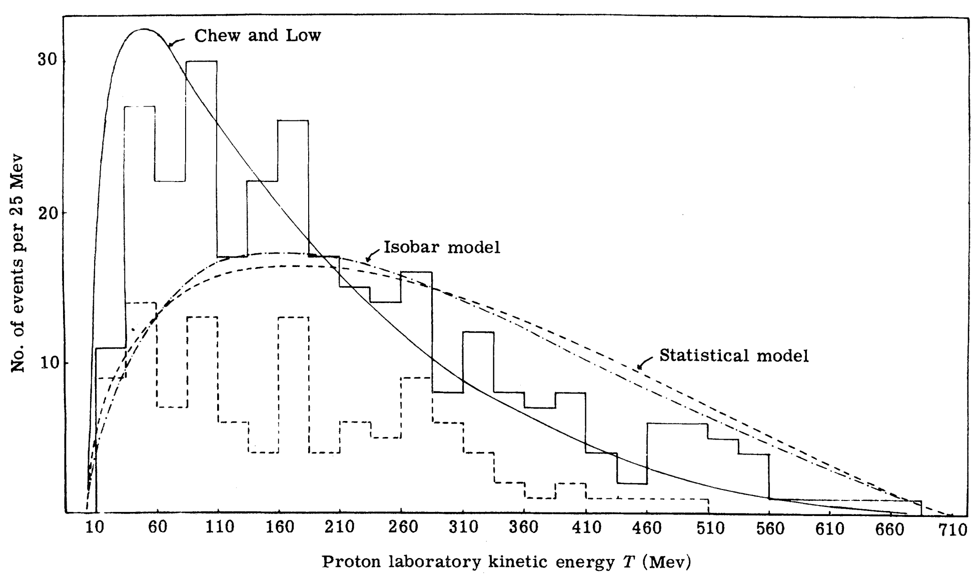

FIG. 1. Laboratory kinetic energy spectrum of the protons from the reaction $\pi^- + p \rightarrow \pi^- + \pi^0 + p$ at incident π^- kinetic energy 0.96 Bev. The histograms consist of Cambridge results alone (broken lines) and Cambridge and Bologna results combined (solid lines). The solid curve is obtained from the formula of Chew and Low [Eq. (2)], and the dashed curves give the results of the isobar model and the statistical model. The theoretical curves and the solid histogram are normalized to the same area.

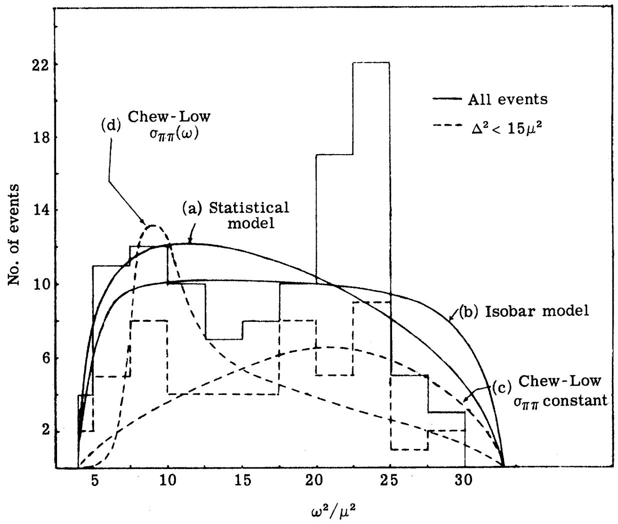

FIG. 2. Spectra of Cambridge events against ω^2/μ^2, where ω^2 is the square of the total energy of $(\pi^-\pi^0)$ in their barycentric system. The solid curves, derived from (a) statistical model and (b) isobar model, are normalized to the same area as the solid histogram which includes all events. Dashed curves were obtained by integrating the Chew-Low Eq. (1) over Δ^2 up to $\Delta^2 = 15\mu^2$, assuming (c) $\sigma_{\pi\pi}$ to be constant and (d) the Frazer-Fulco (reference 13) expression for $\sigma_{\pi\pi}(\omega)$, having $\omega_\gamma{}^2 = 8\mu^2$, $\Gamma = 0.4$; they are normalized to the area of the dashed histogram of events with $\Delta^2 < 15\mu^2$.

sults with the predictions[12] of the statistical model and of Chew and Low. The phase-space density for the $N\pi\pi$ system is

$$d\rho = \delta(p_2{}^2 + \mu^2)\delta(q_2{}^2 + \mu^2)\delta(k_2{}^2 + M^2) \times \delta(p_2 + q_2 + k_2 - p_1 - k_1)dk_2 dp_2 dq_2,$$

where k_1 and k_2 are four-momenta of initial and final nucleons, and q_1, p_2, q_2 of the initial pion and two final pions, respectively. On integrating over superfluous variables, this becomes

$$d\rho = \frac{\pi^2}{2Mq_{1L}}(1 - 4\mu^2/\omega^2)^{1/2} d\omega^2 d\Delta^2, \tag{3}$$

and therefore in the statistical model,

$$\frac{\partial\sigma}{\partial\Delta^2} = \text{const} \int_{\omega_{\text{min}}{}^2}^{\omega_{\text{max}}{}^2(\Delta^2)} (1 - 4\mu^2/\omega^2)^{1/2} d\omega^2. \tag{4}$$

The strong low-Δ^2 peaking of the curves in each region obtained from Eq. (1) when integrated between the corresponding ω^2 limits is characteristic of the $\Delta^2/(\Delta^2 + \mu^2)^2$ term which has a sharp maximum at $\Delta^2 = \mu^2$. This peaking is more marked than in the total spectrum against T

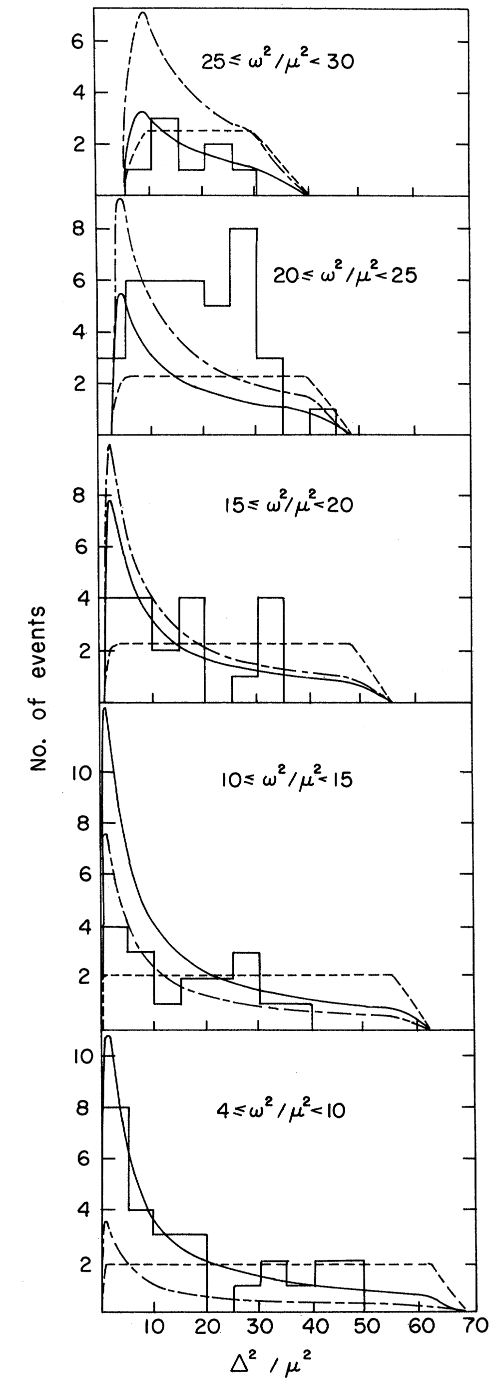

FIG. 3. Spectrum of Cambridge events divided into strips of ω^2/μ^2; the abscissae are expressed in units of Δ^2/μ^2. A solid curve was obtained by integrating the Chew-Low formula [Eq. (1)] between the limits of ω^2 shown, after inserting the Frazer-Fulco expression (reference 13) for $\sigma_{\pi\pi}(\omega)$, with $\omega_\gamma{}^2 = 8\mu^2$, $\Gamma = 0.4$. A dot-dashed curve is the same except that $\sigma_{\pi\pi}$ is assumed constant. A dashed curve gives the result of the statistical theory. Curves of each type are normalized to make the area under the total histogram of all events equal to the area under the curve obtained by integrating the corresponding expression over all values of ω^2.

(Fig. 1) and should be easier to recognize in practice. Recalling the possible peak at $\omega^2/\mu^2 \approx 22$ referred to above, there is no apparent resonance effect in the fourth ω^2 "strip" $(20 \leq \omega^2/\mu^2 < 25)$, the fairly even distribution in Δ^2 being suggestive only of a two-body breakup into nucleon and "di-pion," to which the special assumptions of the Chew and Low theory are not appropriate. The low-Δ^2 peak in the lowest "strip" of ω^2 $(4 \leq \omega^2/\mu^2 < 10)$ is perhaps significant, despite the meager statistics, and suggests the possibility of a resonance in or near this region of ω^2. Correspondingly a set of curves are shown incorporating into Eq. (1) the Frazer and Fulco[13] $t_{\pi\pi} = 1$ resonance expression, in this case with $\omega_r^2 = 8\mu^2$, $\Gamma = 0.4$.[14,15]

The effect of such a resonance on the spectrum of events against ω^2/μ^2 can be estimated by integrating Eq. (1) over Δ^2 up to $\Delta^2 = 15\mu^2 \approx 0.3M^2$ from the lower limit given by the allowed region in phase space, i.e.,

$$\frac{\partial\sigma}{\partial\omega^2} = \frac{f^2}{2\pi q_{1L}^{\ 2}}\,\sigma_{\pi\pi}(\omega)\,\frac{\omega^2}{2\mu^2}\,(1 - 4\mu^2/\omega^2)^{1/2} \times \int_{\Delta_{\min}^2(\omega^2)}^{\Delta_{\max}^2(\omega^2)} \frac{\Delta^2}{(\Delta^2+\mu^2)^2}\,d\Delta^2, \qquad (5)$$

and inserting the Frazer-Fulco $\sigma_{\pi\pi}(\omega)$. The resulting curve (d) can now be compared with the spectrum of events (broken histogram of Fig. 2) corresponding to a region in which the pion-pion interaction is known to be important from Fig. 1. A curve (c) obtained from Eq. (5) assuming $\sigma_{\pi\pi}$ to be a constant is also shown. The experimental evidence is inconclusive when looked at from this point of view.

It is difficult to see how a single Frazer-Fulco resonance of narrow width (i.e., $\Gamma = 0.4$) without a significant nonresonant contribution could fit all the above spectra, as the bulk of events would tend to be concentrated into a region of ω^2 near ω_r^2, as may be seen in Fig. 2, curve (d) and from the solid curves of Fig. 3. Final-state interactions would tend to mask this effect, however. A quantitative test of this point could only be made by performing an experiment designed to detect low-energy proton recoils, when, with large enough statistics, spectra of the type shown in Fig. 3 should reveal the presence of a pole operating in any accessible region of ω^2/μ^2.

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[6]I. Derado, Nuovo cimento 15, 853 (1960).

[7]G. F. Chew and F. E. Low, Phys. Rev. 113, 1640 (1959).

[8]T, the recoil nucleon laboratory kinetic energy, is related to Δ^2 by $\Delta^2 = 2MT$.

[9]The isobar model curve shown refers to the conventional isobar model [R. M. Sternheimer and S. J. Lindenbaum, Phys. Rev. 109, 1723 (1958)], calculated on the assumption that the isobar decays isotropically in the c.m. system. The "extended" isobar model (reference 4) includes an admixture of cases in which the nucleon has greater c.m. kinetic energy as it stems from the decay of an isobar of larger mass; this results in a smaller proportion of nucleons of low laboratory kinetic energy.

[10]R. F. Peierls, Phys. Rev. Letters 5, 166 (1960).

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[12]The corresponding curve predicted by the isobar model (reference 9) may be shown to resemble the statistical curve in this case.

[13]W. R. Frazer and J. R. Fulco, Phys. Rev. Letters 2, 365 (1959); Phys. Rev. 117, 1609 (1960).

[14]A π-π resonance of this shape has the effect of moving the peak of the Chew-Low curve of Fig. 1 to a smaller value of T, though the general shape is preserved.

[15]The only other direct evidence concerning a pion-pion resonance is from an experiment involving meson production in proton-deuteron collisions [A. Abashian, N. E. Booth, and K. M. Crowe, Phys. Rev. Letters 5, 258 (1960)] which favors $\omega_r^2 \approx 5\mu^5$, $\Gamma \sim 1.0 - 2.0$. A reasonable fit to nucleon charge structure is obtained with $\omega_r^2 = 11.5\mu^2$, and to the anomalous magnetic moment with $\Gamma = 0.4$, though these parameters are not determined very precisely.