

laxed and the dependence of the amplitude of the oscillations on  $H$  will not be simple.

Some remarks can be made on the absence of a second component in the oscillations contributed by Landau levels in the valence band. Because of instrumental limitations, fields of 15 kgauss or more are required for the observation of the electron Landau levels. Hence, those for the heavy holes would require considerably higher fields. Another factor tending to obscure the Landau levels on the  $p$  side will be the greater scattering frequency due to the relatively greater impurity concentration. On the other hand, it might be possible, in principle, to detect oscillations in the tunnel current of electrons from well below the Fermi level in the conduction band to the Fermi surface in the light-hole band. However, these have not been observed, the probable reason being that at the fields at which oscillations can just be detected ( $\sim 15$  kgauss), the maximum energy of the light-hole band is already lower than the Fermi energy in the valence band.

In the case of  $E \parallel H$ , the infinite degeneracy of Landau states is responsible for the sharp peaks in the density of states. In the  $E \perp H$  geometry, this degeneracy is broken since the energy of a

Landau level now depends upon the position of the electron in the plane normal to  $H$ . Thus the density of states will vary with  $H$  in the perpendicular geometry, so accounting for the absence of oscillations.

The fact that, to within experimental error, the Landau spacings at the Fermi level in the conduction band determine the current oscillations at relatively high biases implies that the tunnel current is still dominated by transitions to or from states close to the Fermi level on the  $n$  side.

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## LOW-TEMPERATURE TRANSPORT IN "SPLIT $p$ -GERMANIUM"

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By measuring conductivity and Hall mobility of  $p$ -Ge at low temperature under large strains ( $> 10^{-3}$ ), we have been able to determine the sign, the ratio, and the approximate magnitude of the two deformation potentials that describe the change, under shear, of the valence band. In addition we have been able to verify that the velocity dependence of the hole-acceptor recombination cross section is independent of the acceptor ground-state energy over a limited range, in agreement with the prediction of the trap mechanism proposed by Lax.<sup>1</sup>

When germanium is subjected to a shear stress the valence band, normally 4-fold degenerate (including spin) at the Brillouin zone center, splits into two doubly degenerate bands separated by an energy  $\delta\mathcal{E}$ .<sup>2-5</sup> In a neighborhood within an energy  $\ll \delta\mathcal{E}$  of each (new) band edge, the surfaces of constant energy are ellipsoids of revo-

lution about the stress direction, and the effective masses are functions of the geometry of the strain, and not its magnitude.<sup>2,3,5</sup>

For a uniaxial stress  $\chi$  in either the  $\langle 100 \rangle$  or  $\langle 111 \rangle$  directions, the splitting is given by

$$\begin{aligned} \delta\mathcal{E}_{100} &= 2\Delta_{100} = 2S_{11}(1+\lambda)|b\chi| = 4S_{11}(1+\lambda)|D_u\chi|/3 \\ &= 2|b|\epsilon_{100} = 2.44 \times 10^{-12} |\chi b| \text{ ergs,} \end{aligned} \quad (1a)$$

$$\begin{aligned} \delta\mathcal{E}_{111} &= 2\Delta_{111} = \sqrt{3}S_{44}|d\chi|/3 = 2S_{44}|D_u'\chi|/3 \\ &= \sqrt{3}|d|\epsilon_{111} S_{44}/(S_{11} + 2S_{12} + S_{44}) \\ &= 0.838 \times 10^{-12} |\chi d| \text{ ergs,} \end{aligned} \quad (1b)$$

and  $b = -2D_u/3$ ;  $d = -2D_u'/\sqrt{3}$ . Here  $b$  is the deformation potential constant (for hole energies) appropriate to a strain of tetragonal symmetry (with respect to the conventional cubic basis) as

defined by Pikus and Bir<sup>3</sup> (PB);  $d$  is the similar constant for a deformation of rhombohedral symmetry.  $D_u$  and  $D_u'$  are similar constants defined (for electron energies) by Kleiner and Roth<sup>4</sup> (KR). The definitions used are those in Eq. (2) and Table I of KR, rather than their verbal description of  $D_u$  and  $D_u'$ , which is apparently inconsistent with these.  $S_{11}$ ,  $S_{12}$ ,  $S_{44}$  are the elastic compliance constants, and  $\lambda$  the Poisson ratio.  $\epsilon_{100}$  and  $\epsilon_{111}$  are the respective longitudinal components of strain that result from the stress  $\chi$ .<sup>6</sup> The relation to the notation of other authors is given in Table I of Price and Kao.<sup>7</sup>

Sufficiently large stresses may be applied to germanium so that at low temperatures ( $\approx 10^\circ\text{K}$ ),  $\delta\epsilon \gg kT$ . In this limit only one band is appreciably populated; the Hall mobility then becomes independent of strain. Figure 1 shows the strain dependence of the Hall mobility for two samples of 20 ohm cm  $p$ -type Ge cut from the same crystal, one  $\langle 111 \rangle$  oriented, the other  $\langle 100 \rangle$ . The abscissa scale was chosen so that if  $b=d$ , it would be proportional to  $\delta\epsilon$  for both samples. The saturation of the mobility is clearly indicated. From these data, a ratio  $|d/b| = 2.1 \pm 15\%$  or  $|D_u'/D_u| = 3.6$  is obtained. This result is a factor of  $\sim 2$  less than the estimates made by PB<sup>3</sup> and KR<sup>4</sup> from piezoresistance data. It is difficult to obtain any precise value for  $d$  from the present data. However, if one assumes that when the mobility is at 90% saturation,  $\delta\epsilon \sim 3kT$ , then  $d \approx 1$  ev.

According as the sign of  $b$  and  $d$ , the relevant effective-mass ellipsoid is either oblate or pro-

late.<sup>3</sup> To determine the signs we have crudely measured the transverse conductivity under saturation strain for both samples by making a two-terminal measurement of the resistance between the Hall contacts. For the  $\langle 100 \rangle$  case,  $\mu_{\parallel}/\mu_{\perp} > 1$  corresponding<sup>3</sup> to  $b < 0$  or  $D_u > 0$ , ignoring the (unlikely) possibility of an extremely large scattering anisotropy. Similarly, for the  $\langle 111 \rangle$  case,  $\mu_{\parallel}/\mu_{\perp} < 1$  corresponding to  $d < 0$  or  $D_u' > 0$ . These results agree with the signs of the estimates made by KR<sup>4</sup> and those which may be deduced from PB.<sup>3</sup> It is interesting to note, however, that the work of Hensel and Feher<sup>5</sup> yields the opposite signs for the corresponding valence band deformation potentials in silicon.<sup>8</sup> (Actually only the signs of the products  $dD$  and  $bB$  are here determined experimentally, where  $D$  and  $B$  are the conventional matrix elements of the spin-orbit interaction; but the signs of  $D$  and  $B$  are presumed known.<sup>8,5</sup>)

If  $\delta\epsilon$  can be made sufficiently large, the acceptor ground state will be formed from Bloch states near the edge of the upper of the two split bands. The effective-mass approximation as extended to the prolate case by Keyes,<sup>9</sup> using the masses calculated by Pikus and Bir,<sup>3</sup> and assuming that the dielectric constant remains isotropic, gives for the saturation activation energy  $\epsilon$  the value  $4.25 \times 10^{-3}$  ev for a  $\langle 100 \rangle$  stress.

Figure 2 shows a plot of reciprocal Hall constant vs strain for the  $\langle 100 \rangle$  orientation at  $T \approx 6^\circ\text{K}$ . The maximum stress on the  $\langle 100 \rangle$  sample, which corresponds to a strain of  $0.7 \times 10^{-2}$ , was limited by the available apparatus. (This corresponds to

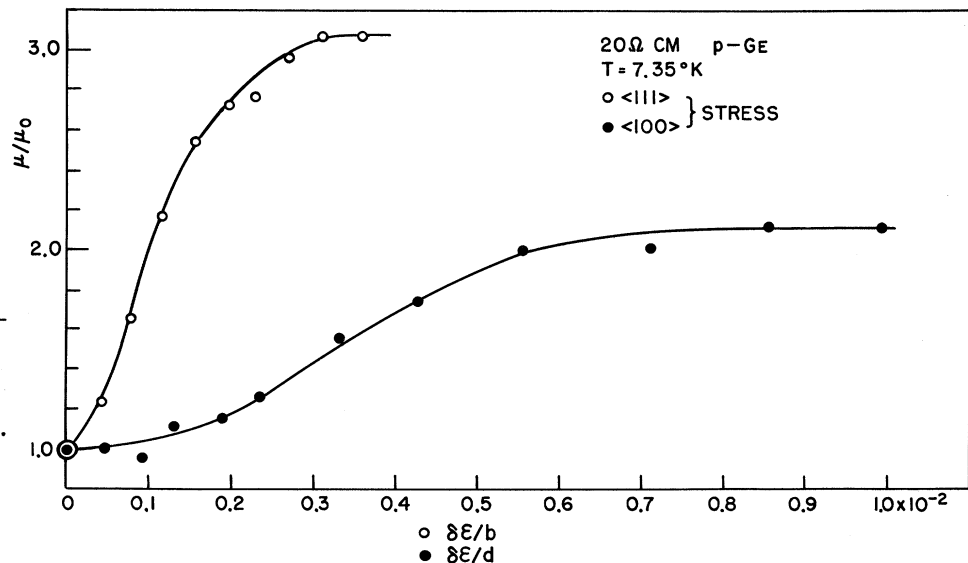


FIG. 1. Hall mobility (normalized to zero strain value) vs strain for a  $\langle 100 \rangle$  and  $\langle 111 \rangle$  oriented sample of  $p$ -Ge. The abscissa scale is in units of the strain-dependent splitting of the valence band at the zone center divided by the appropriate deformation potential [see Eq. (1)]. The strains at saturation are  $> 10^{-3}$ .

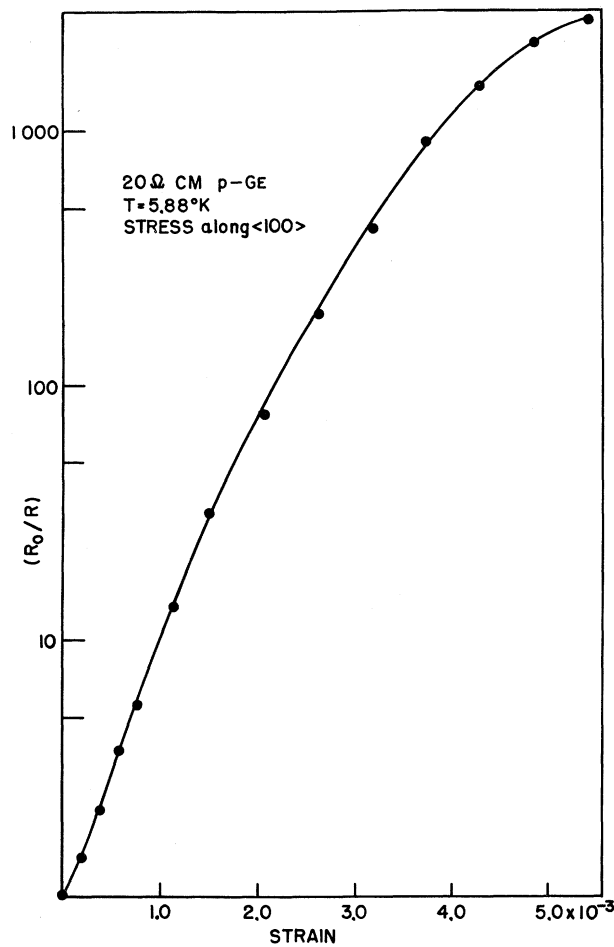


FIG. 2. Reciprocal Hall constant vs strain. The variation is due to a change in impurity activation energy which, in terms of the effective-mass approximation, is caused by a large reduction in at least one component of the mass tensor.

$\delta\epsilon \sim 7 \times 10^{-3}$  ev.) It would appear from the data that the condition  $\epsilon \ll \delta\epsilon$  has almost been reached. The activation energy measured at this strain is  $6.8 \times 10^{-3}$  ev. Whether there is indeed a discrepancy with the predictions of the effective-mass approximation remains to be determined. For a <111> stress, the reciprocal mass tensor is oblate; the calculated  $\epsilon$  in this case is  $5.0 \times 10^{-3}$  ev.<sup>10</sup>

This "new" semiconductor, "split *p*-germanium," has a unique application in hot electron problems for values of strain and electric field such that  $\delta\epsilon \gg kT_e$  where  $T_e$  is the electron "temperature." The nonlinear current-voltage relation<sup>11</sup> should then be independent of strain (except for a strain-dependent scale factor in the current caused by the varying activation energy) until impact ionization of neutral impurities makes a significant contribution to the current density. The electric field at which this occurs has been a matter of conjecture for some time.<sup>12</sup> However, it is now possible, by varying the strain, to vary continuously the impurity activation energy, without changing any transport parameters. Figure 3 shows recorder traces of

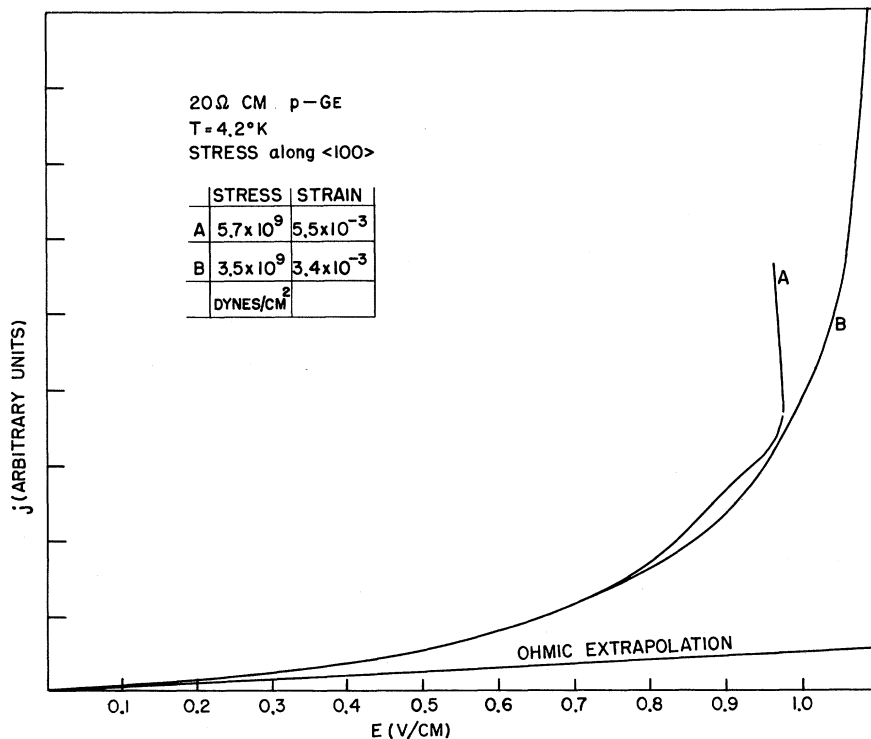


FIG. 3. Variation of current with electric field for two values of strain sufficiently large so that the transport parameters are strain independent but the activation energy not. The two curves diverge when impact ionization of neutral impurities begins to contribute appreciably to the carrier density. (The current scales have been adjusted to compensate for the activation energy change by making the curves overlap in the Ohmic range.)

current density vs electric field for two large values of strain; the current sensitivity has been adjusted to compensate for the activation energy change (~20%). There is overlap of the two curves for a considerable range of electric field where the nonlinearity in conductivity is due to the velocity dependence of the recombination coefficient  $\sigma$ .<sup>12</sup> It is seen that  $\sigma$  varies by a factor of 3.5 over this range. The (velocity-dependent) recombination cross section is expected to be dependent, not on the acceptor ground-state energy, but rather on a continuum of states  $\sim kT$  above the valence band edge,<sup>1</sup> which depends in this case on the parameters of the upper band. This expectation is here confirmed experimentally, and is additional strong confirmation that the cascade recombination mechanism of Lax is operative.

We are indebted to Rodney Brown for computational assistance and to Peter J. Price for discussions.

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## POLARIZATION OF THE CONDUCTION ELECTRONS IN THE FERROMAGNETIC METALS\*

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It has recently been proposed<sup>1</sup> that the polarization of the 4s conduction electrons in Fe, Co, and Ni is antiparallel to that of the electrons in the unfilled 3d shell; this is the reverse of the usual assumption. There are few ways in which this polarization is manifested. One, however, is in the effective magnetic field at a nucleus, which acts through the contact interaction with the 4s electrons. The magnitude of this field has been given by Marshall<sup>2</sup> as

$$H_c = (8\pi/3)\xi_s |\psi(0)|_A^2 \mu n p, \quad (1)$$

where  $\mu$  is the Bohr magneton,  $\xi_s |\psi(0)|_A^2$  is the average probability density of a 4s conduction electron evaluated at the nucleus [ $|\psi(0)|_A^2$  being the free atom value],  $n$  is the number of conduction electrons per atom, and  $p$  is their polarization. The sign of the field is positive, i.e., parallel to the direction of magnetization, if the polarization of the 4s electrons is parallel to the 3d polarization.

Hanna *et al.*<sup>3,4</sup> have measured the magnitude and sign of the field at the Fe nucleus in iron by observing the Zeeman splitting of the 14-kev transition in Fe<sup>57</sup> and obtain the value  $-3 \times 10^5$  koe. In this case, however, there are other contributions to the field beside  $H_c$ ; mainly those due to the electrons of the same atom, i.e., the polarized 3d electrons. These other contributions almost certainly outweigh the effect of the field  $H_c$  alone ( $H_c$  is probably of the order of 50 to 100 koe), and it is therefore difficult to draw any conclusion concerning its sign.

In an attempt to measure  $H_c$  directly, dilute (1%) solid solutions of Sn in the ferromagnetic metals (Fe, Co, and Ni) have been prepared. Since Sn is basically diamagnetic, we expect that there will be no contribution to the field at the Sn nucleus from its own inner electrons and that the field will be given simply by

$$H_s = (4\pi/3)M + H_c, \quad (2)$$