EFFECT OF NUCLEAR ROTATION ON THE PAIRING CORRELATION

Ben R. Mottelson and J. G. Valatin*
NORDITA and The Institute for Theoretical Physics, University of Copenhagen, Copenhagen, Denmark
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The empirically observed energy gap, the reduction in the effective moment of inertia below that of a rigid rotator, and the transition to spherical shape with the approach to closed shells all testify to the important role of the pairing correlation in the low excited states of deformed nuclei. The total binding energy associated with the pairing correlations is, however, quite small,[1] and thus a rather slight disturbance of the system may be sufficient to destroy the pairing and cause a transition to a less correlated particle motion.

An analogy with the correlations in superconductors has proved to be rather fruitful in interpreting the nuclear features mentioned above.[2] Pursuing this analogy, the close formal correspondence of the equations of motion in a constant magnetic field and in a rotating reference system suggests that the critical magnetic field phenomena of superconductors should also have their counterpart in the rotational spectra of nuclei. Indeed, the pairing force couples two particles in time-reversed single-particle states, and a rotation has an opposite effect on the particles forming the pair. The Coriolis forces act in opposite directions and tend to decouple the pairing correlations. This is in analogy with the effect of the magnetic forces in a metal, which act with a different sign on electrons moving in opposite directions and destroy the pairing when the field reaches a critical value. The observation or nonobservation of this effect in the rotational spectra of nuclei could serve as a test of the description of the pairing correlations in nuclei.

A suitable approximation scheme to investigate the quantum mechanics of this effect is provided by the generalized Hartree-Fock method[3] worked out in connection with the theory of superconductivity. The pairing forces are represented in this scheme by a self-consistent pairing potential, the strength of which is proportional to a gap parameter Δ. A reference system rotating with angular frequency Ω introduces a coupling term in the self-consistent energy which is Ω times the angular momentum operator l_x. The competing effect of the rotational motion and of the pairing finds its mathematical expression in the fact that Ωl_x does not commute with the pairing potential. Simple solvable models are being examined to investigate the detailed nature of the two simultaneous effects.

The main results of such a treatment can be seen from rather simple arguments. Since the Coriolis force counteracts the pairing correlations, the energy-gap parameter Δ will decrease with increasing Ω. This is reflected in an increase in the effective moment of inertia $\mathscr{I}$, and for sufficiently small Ω the effect may be described by a term in the rotational energy proportional to $I^2(I+1)^2$, where I is the rotational angular momentum. For larger values of the rotational frequency Ω, when the rotational energy is of the order of the binding energy due to the pairing correlation, the Coriolis force becomes comparable to the pairing force and gives rise to major modifications in the correlations of the particles. For Ω greater than a critical value Ω_c, the energy-gap parameter Δ vanishes and the particles perform an approximately independent motion in the average potential.

Though in the region $0<\Omega<\Omega_c$ one has a more involved intermediate coupling, the value Ω_c can independently be obtained from the simpler gap equation which results with a trial ground-state vector of the Bardeen, Cooper, Schrieffer type formed with eigenstates of the single-particle energy in the rotating system. In such a representation, the relevant matrix elements of the pairing interaction energy can be approximated by a constant which appears as an effective coupling constant so that, to second order[4] in Ω, the coupling constant G of the pairing interaction is replaced by

$$\bar{G}=G\left\{1-4\Omega^2\left\langle\sum_{k'}\left(\frac{\langle k'|l_x|k\rangle}{\epsilon_{k'}-\epsilon_k}\right)^2\right\rangle_{\mathrm{av}}\right\}. \tag{1}$$

The energies ϵ_k and matrix elements $\langle k'|l_x|k\rangle$ refer to the one-particle states k of the nonrotating system, and the averaging is over those states k which take part in the pairing interaction. The effect of Ω is to reduce the value $\bar{G}$ of the effective coupling constant, and Ω_c is the minimum value of Ω for which the gap equation with this effective interaction leads to a vanishing gap. This happens already for some finite value of $\bar{G}$. For $\Omega = 0$, the level spacing d is of the same order as the gap, and in the actual calculations carried out for the deformed nuclei[2] a reduction of G by about 30% is sufficient to cause Δ to vanish.

One can obtain an estimate of Ω_c by evaluating the averaged sum in (1), employing harmonic oscillator wave functions. For $\Delta = 0$, $\mathcal{I} = \mathcal{I}_{\text{rigid}}$, and so the critical angular momentum I_c is given by $I_c = \Omega_c \mathcal{I}_{\text{rigid}}$. One finds[5] in this way $I_c \sim 12$ for $A \sim 180$, and $I_c \sim 18$ for $A \sim 238$. These values are only slightly higher than the highest rotational states so far observed in these nuclei[6] ($I = 8$ for $A = 180$ and $I = 12$ for $A = 238$), and thus further study of high-angular-momentum states in these bands would be very interesting. One expects that in multiple Coulomb excitation studies, the rotational sequence based on the ground state will effectively terminate[7] for $I \sim I_c$. States with $I > I_c$ will have an intrinsic structure that is practically orthogonal to the ground state. One would further expect that in nuclear reaction processes populating states with $I > I_c$, the density of states as a function of I will be governed by a Boltzmann factor in which the rotational energy is given approximately by $(\hbar^2/2\mathcal{I}_{\text{rigid}})I(I+1)$ + constant, as is characteristic of a free Fermi gas.

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*On leave from the Department of Mathematical Physics, University of Birmingham, Birmingham, England.

[1]An estimate of this energy may be obtained from the model employed in references 2 and 3. Thus, for a nucleus with an average level spacing d between the unperturbed (twofold degenerate) single-particle orbitals, one has for the total binding energy associated with the pairing correlation, including both neutrons and protons, $W = -\Delta^2/d$, where 2Δ is the magnitude of the energy gap. Inserting the empirically observed values of Δ and d gives $W \sim -1.3$ Mev for deformed nuclei, approximately independent of A.

[2]A. Bohr, B. R. Mottelson, and D. Pines, Phys. Rev. 110, 936 (1958); S. T. Belyaev, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. 31, No. 11 (1959); A. B. Migdal, Nuclear Phys. 13, 655 (1959); Soviet Phys.—JETP 37(10), 176 (1960); J. J. Griffin and M. Rich, Phys. Rev. 118, 850 (1960); S. G. Nilsson and O. Prior, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. (to be published); V. G. Soloviev, Doklady Akad. Nauk (U.S.S.R.) 133, 325 (1960); and J. Exptl. Theoret. Phys. (U.S.S.R.) (to be published).

[3]J. Bardeen, L. N. Cooper, and J. R. Schrieffer, Phys. Rev. 108, 1175 (1957); J. G. Valatin, Nuovo cimento 7, 843 (1958), and a paper to be published which discusses the more general equations relevant in the case of the noncommuting coupling terms which appear in the present problem. N. N. Bogolyubov, Nuovo cimento 7, 794 (1958); Soviet Phys.—Uspekhi 67(2), 236 (1959).

[4]The use of a perturbation estimate at this point is justified since $\hbar\Omega_c \ll \Delta\epsilon$, where $\Delta\epsilon$ is a typical energy denominator in (1), i.e., the rotational motion implies only a small modification in the one-particle motion although it implies a major modification in the pairing correlation. Compare also with footnote 7.

[5]One obtains essentially the same estimate for Ω_c from the condition that the difference between the rotational energies of the correlated and uncorrelated states should be equal to the energy gain $-W$ due to the correlation.

[6]For the rotational band of Hf^{178} and Hf^{180}, see, e.g., D. Strominger, J. M. Hollander, and G. T. Seaborg, Revs. Modern Phys. 30, 585 (1958); for U^{238} see F. Stephens, R. Diamond, and I. Perlman, Phys. Rev. Letters 3, 435 (1959).

[7]It should be emphasized that the effect considered here is quite distinct from the modifications in the rotational motion that occur when the rotational frequency becomes of the order of the intrinsic particle frequencies. These latter modifications effectively terminate the rotational bands when $I \sim I_m$, where I_m is the maximum angular momentum that can be constructed out of the available single-particle configurations [see J. P. Elliott, Proc. Roy. Soc. (London) A245, 128 (1958)]. For the heavy nuclei which exhibit well-defined rotational band structure, $I_m \sim 40$.