is completely in the shadow of the fiber. Nor can they be attributed to a return field H_z parallel to the whisker in the region a' outside it, due to the flux emerging from the sides and ends, for two reasons. An estimate of the magnitude of this field shows that it might be strong enough to displace the pattern by perhaps one fringe width, but not more, and observation confirms that the displacement of the envelope is very small; secondly, we have seen experimentally that an extended field H_z in fact produces a completely different effect (Fig. 2). Thus the patterns of Fig. 3 might be taken to demonstrate the existence of the predicted quantum shift. Indeed they do; nevertheless the tilt of the fringes can be attributed to a leakage field, as Pryce has pointed out to me, and it is illuminating to consider this.[10] Immediately outside a tapering whisker, the leakage field is in fact primarily radial and is given by $H_r = (d\Phi/dz)/2\pi r$. This field exerts a force on the electron and gives it a momentum $p_z = \pm\frac{1}{2}(e/c)d\Phi/dz$, the different signs applying to paths on either side of the whisker. The two beams which converge to interfere at o are thus tilted one above and one below the plane of Fig. 1, thus skewing the interference fringes. There is a progressive change in the phase difference between the two beams as one moves in the z direction. This is easily calculated from p_z by de Broglie's relation, and amounts to a phase-difference gradient of $(e/\hbar c)d\Phi/dz$. This is precisely the rate of change of the "quantum" phase difference $e\Phi/\hbar c$ calculated by Aharonov and Bohm. One thus sees fairly intuitively how the "quantum" phase difference is progressively built up from the free end of the whisker, where it is zero, to any section where the interference is being observed. It remains true, however, that the total displacement of a given fringe is a direct measure not of the leakage field from that section but of the flux enclosed within it, and that a displacement will occur even in a parallel-sided region of the whisker where the radial leakage field is zero.

I am indebted to Mr. Aharonov and Dr. Bohm for telling me of their work before publication, and to them and to Professor Pryce for many discussions.

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[10]The following analysis is due to Professor Pryce.

ACOUSTICALLY MODULATED γ RAYS FROM Fe^{57}

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The relationship between the emission of γ rays by nuclei bound in a crystal and the creation (or destruction) of phonons has been discussed by Visscher,[1] and suggests that a careful study of the "off-resonance" line shape in a Mössbauer-type experiment may be used to observe the frequency distribution of lattice vibrations in the crystal. Unfortunately, a direct attempt at such a study seems difficult since it requires the measurement of nuclear γ-ray absorption cross sections much smaller than the photoelectric cross sections for the same atom. In an attempt to investigate the interactions between phonons and emitting nuclei, therefore, it was decided to generate low-energy phonons acoustically, and to study their effect on the γ-ray spectrum.

Source and absorber were one-mil thick 321 stainless steel (18% chromium, 8% nickel) foils. The source, into which had been diffused Co^{57}, could be driven by either or both of two methods: (1) a low-frequency (15 cps) drive utilizing a loud speaker, and (2) a piezoelectric quartz crystal drive mounted on the rear of the source foil. The quartz crystal is driven by a radiofrequency oscillator whose frequency and amplitude are continuously adjustable. The counting rate for the 14.4-kev γ ray, as a function of loudspeaker velocity, is measured by using the output of a single-channel analyzer to "command" a multichannel analyzer to measure the velocity at a particular instant. This is accomplished by feeding the amplified output of the velocity pickup coil (rigidly attached to the source) to the appropriate place in the analog-to-digital converter section of the analyzer.

The experiment was planned on the assumption that the density of ultrasonic phonons in a narrow frequency band could be markedly increased over that corresponding to 300°K, and that this should lead to pairs of satellite peaks, symmetrically located relative to the main Mössbauer peak, with a spacing in energy units of $\Delta E = \hbar q$ or, in velocity units, $v_s = (c/E_0)\Delta E$. ($q/2\pi$ = ultrasonic frequency, $E_0 = 14.4$ kev.) This corresponds to the creation or destruction of acoustic phonons with the emission of the γ ray. A similar discrete Doppler effect has been observed in optical light diffracted by acoustic "gratings." The optical effect is extremely small, of the order of 10^{-4} A for an optical wavelength of 5460 A and an ultrasonic frequency of 10 Mc/sec. The effect has been observed for both traveling and stationary sound waves.[2] A theory of this optical effect has been given by Raman and Nagendra Nath.[3]

Since the source foil is thin (approximately one-tenth the wavelength of sound at 20 Mc/sec), one can alternatively consider the quartz transducer as simply vibrating the foil with a sinusoidal velocity, $v_s = v_m \cos qt$. This corresponds merely to a sinusoidal motion of the center of mass of the foil. The "instantaneous" frequency of the 14.4-kev γ ray may therefore be expressed as

$$\nu = \nu_0 + \Delta\nu \sin qt, \qquad (1)$$

where

$$\nu_0 = E_0/h = 3.48 \times 10^{18} \text{ cycles/sec},$$

and the maximum frequency deviation is $\Delta\nu = \nu_0(v_m/c)$. Expressed in the language of frequency modulation, this corresponds to a carrier of frequency ν_0, modulated sinusoidally at a frequency $q/2\pi$. The modulation index is $m = (v_m/c) \times (2\pi\nu_0/q)$. The frequency spectrum can be shown[4] to consist of the carrier and an infinite set of side-bands, with the nth side frequency separated from the $(n+1)$th side frequency by the modulating frequency, $q/2\pi$. The maximum amplitude of the nth side frequency is given by $J_n^2(m)$, where $J_n(m)$ is the Bessel function of the first kind of order n.

In Fig. 1 the solid lines in curves a-e show the result of calculations for five values of m, with $q/2\pi = 20$ Mc/sec. The vertical scale of the drawing is based on curve a. Also shown in the figure are the experimental results, using a 20-Mc/sec

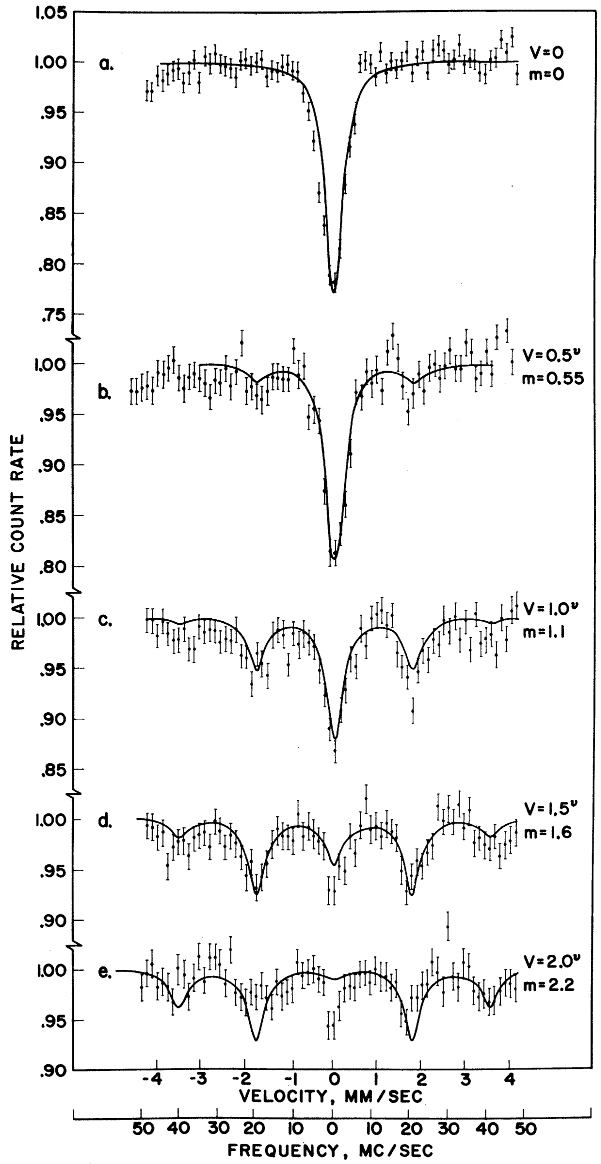

FIG. 1. Mössbauer pattern for Fe^{57} γ ray emitted by a stainless steel source driven by a 20-Mc/sec x-cut quartz transducer. The experimental points are shown in a-e for values of the driving voltage, V, from 0 to 2.0 volts rms. The solid curves are calculated on the basis of FM theory, using a single proportionality constant between m and V_{rms} which best fits the data.

x-cut quartz transducer, taken at five different transducer driving voltages. The maximum velocity of the iron atoms resulting from the ultrasonic vibration has not been measured directly, but is expected to be proportional to the driving voltage; the proportionality constant has been chosen so as to fit the solid curves as well as possible to the experimental points. Using $m = 1.1\,V_{\mathrm{rms}}$, one finds $v_m = 0.29$ cm/sec for $1.5\,V_{\mathrm{rms}}$ across the transducer. This value for v_m is consistent with the value calculated from the piezoelectric properties of the quartz transducer. The velocity at 20 Mc/sec corresponds to the rather small maximum displacement of approximately 2×10^{-9} cm.

The progressive disagreement between the calculated and experimental curves with higher drive voltage, especially near the carrier frequency, suggests that all of the iron atoms did not have the same velocities. A new source foil was then prepared, care being taken to preserve flatness of the foil and uniformity of the acoustic bond. In Fig. 2, a plot of the amplitude of the carrier (unshifted γ ray) vs the 20-Mc/sec driving voltage is given, together with a plot of $[1 - 0.24J_0^2(m)]$ with $m = 0.6\,V_{\mathrm{rms}}$. The calculated curve assumes that all the Fe^{57} atoms have the same maximum velocity v_m. The pattern using the new foil, however, still suggests a continuous range of velocities with perhaps 50% of the Fe^{57} nuclei moving considerably more slowly than the remainder. Such an effect could be caused by bonding defects, such as air bubbles trapped in the cement between foil and quartz. Velocity blurring also results from the fact that the thickness of the foil is not negligible compared to the wavelength of the sound waves.

Since the energy shift of the γ ray is determined solely by the frequency of the ultrasonic drive, this discrete Doppler technique offers a precise method for adding or subtracting known quantities of energy to the γ ray. This may be useful in providing a monochromatic calibration of energy or velocity in the measurement of line splittings (such as Fe^{57} in iron) or line shifting

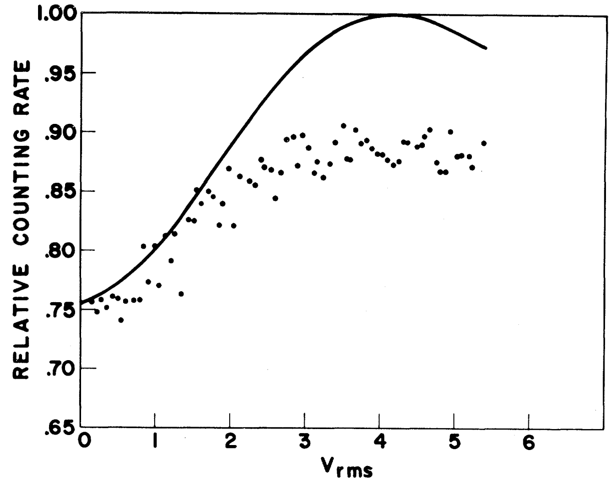

FIG. 2. Relative intensity of carrier (unshifted γ ray) vs voltage on quartz transducer. The solid line represents the theoretically predicted function $[1 - 0.24J_0^2(m)]$, where $m = 0.6\,V_{\mathrm{rms}}$ for this case. This prediction assumes that all the Fe^{57} atoms have the same maximum velocity v_m.

(such as temperature shifts of the Mössbauer peak due to zero-point vibration). This method for Doppler shifting may also be applicable at low temperatures when more conventional drives are inconvenient to use. For this purpose, broadbanding of the transducer frequency response will be desirable.

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