PARTIALLY CONSERVED CURRENTS AND THE K' MESON*

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Very recently some theorists[1,2] have suggested that there may exist a strong S-wave $T=1/2$ K-π resonance, which can be thought of as a "particle" (called K' by Gell-Mann) similar to the K meson. Of course, there is no evidence for a second long-lived K meson, so K' must decay by strong coupling; hence $m_{K'} > m_K + m_\pi$. We are free to define the K-meson parity as odd; then the K' parity is even. There are three main reasons for suspecting the existence of such an object.

(1) It is needed to construct a highly symmetric theory[1] of the $\Delta S \neq 0$ weak interaction currents $\mathcal{J}_\lambda{}^V$, $\mathcal{J}_\lambda{}^A$.

(2) The second reason for proposing a K' again comes from the theory of the weak interactions,[3] and is related to the first. We sketch it very briefly. In our present account of the weak couplings the leptonic decays are described by at least four currents[4]; vector and axial vector with $\Delta S = 0$ ($J_\lambda{}^V, J_\lambda{}^A$) and with $\Delta S=1$ ($\mathcal{J}_\lambda{}^V$, $\mathcal{J}_\lambda{}^A$). Hopefully, $J_\lambda{}^V$ is conserved. The divergences of $J_\lambda{}^A$ and $\mathcal{J}_\lambda{}^A$ are operators which transform like the π- and K-meson fields, respectively, and in fact one may exhibit theories in which these divergences are actually proportional to the π and K fields.[5] All this suggests that these divergences have certain "gentleness" properties,[5] properties which allow a convincing derivation[6] of the successful Goldberger-Treiman formula[7] for pion decay and of a similar formula for $K \to \mu + \nu$.

One is naturally then led to suppose also that the divergence of the $\Delta S \neq 0$ current $\mathcal{J}_\lambda{}^V$ is related to the field of a particle with the supposed properties of the K'.

(3) The well-known sharp backward peaking of the Λ^0 in the reaction $\pi^- + p \to \Lambda^0 + K^0$ (in the center-of-mass system) finds an explanation if one imagines that the pole at the K' mass in the diagram of Fig. 1 dominates the associated production process.

This is in fact an old story. In the days of the parity doublet K-particle theory, several physicists[8] (e.g., Goldhaber, Schwinger) proposed a strong $KK\pi$ coupling. One of the consequences of this coupling, noted then, was just the backward peaking of Λ^0's. If there is a scalar K particle, the K', then one may have a $KK'\pi$ coupling which is perfectly parity conserving. On the basis of such a coupling, Tiomno[8] has estimated that a reasonable mass to explain the Λ^0 data would be $m_{K'} \simeq m_K + m_\pi$. Quite clearly, the K and K' are not to be taken as parity doublets in the old-fashioned sense.

A $KK'\pi$ vertex does not give rise to any distinctive new features of elastic K-nucleon scattering, but just gives additional contributions to the two-pion exchange potential, etc. Thus its effect on the KN scattering cross sections is hard to estimate, but there is no reason to suppose that it is in contradiction to anything known about them. Of course, the K' would show itself in inelastic K-nucleon scattering, through processes like $K^+ + p \to K'^+ + p$, $K'^+ \to K^+ + \pi^0$, and this is very likely a good way to look for the K'.[1]

In this note we will consider how the K' particle may show up in the $\Delta S=1$ leptonic decay processes, assuming the divergence $\partial^\lambda \mathcal{J}_\lambda{}^V$ to be a "gentle" operator. In this case, the Goldberger-Treiman formula which comes out connects essentially unmeasurable quantities, but, as we shall see below, the argument can be turned around to yield a rather clear prediction of all features of $K_{\mu 3}$ and K_{e3} decay, except the total rate of $K_{\mu 3} + K_{e3}$.

All properties of $K_{\mu 3}$ and K_{e3} decay are determined by two scalar form factors $f_V(q^2)$ and $g_V(q^2)$, defined by the relation[7]:

$$\langle \pi | \mathcal{J}_\lambda(0) | K \rangle = \frac{i(2\pi)^{-3}}{(4E_\pi E_K)^{1/2}} \{ f_V(q^2) P_{K\lambda} + g_V(q^2) q_\lambda \}, \quad (1)$$

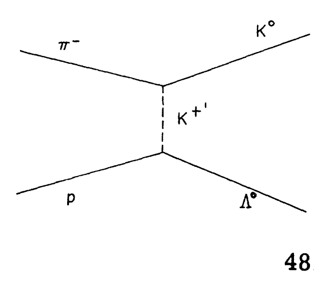

FIG. 1. Associated-production diagram with an intermediate K' particle.

where $q_\lambda \equiv (P_K - P_\pi)_\lambda$, $q^2 \equiv (\vec{P}_K - \vec{P}_\pi)^2 - (E_K - E_\pi)^2$. For example, the distribution in the pion momentum P and the angle θ between pion and neutrino is given in a system in which $P_K = (0, m_K)$, $P_\pi = (\vec{P}, E)$ by[9]

$$W(P,\theta)dPd\theta \sim (1-x^2-y^2)^2(m_K - E)^2 P^2 E^{-1} \times (1 + x\cos\theta)^{-4} x^2 |f_V(q^2)|^2 \times U(P,\theta)dPd\theta, \tag{2}$$

where

$$U(P,\theta) = \sin^2\theta + \frac{y^2}{x^2}\left|1 + \frac{m_K - E}{m_K}\frac{g_V(q^2)}{f_V(q^2)}(1 + x\cos\theta)\right|^2,$$

$x = P/(m_K - E)$, $y = m_\mu/(m_K - E)$ or $m_e/(m_K - E)$,

and in this frame of reference,

$$q^2 = -m_K^2 - m_\pi^2 + 2m_K E.$$

It is easy to see that the K' "pole" can appear only in $g_V(q^2)$, and not in $f_V(q^2)$. We have no idea whether this resonance gives the dominant contribution for the range of q^2 in K_{e3} and $K_{\mu 3}$ decay $[-(m_K - m_\pi)^2 \leq q^2 \leq -m_\mu^2$ or $-m_e^2]$, but to be constructive we will assume that it does. However, this is not enough to calculate g_V and f_V; we also need to know something about their behavior at high energies. We might guess, for instance, that they approach constants as $|q^2| \to \infty$, and would then need to use once-subtracted dispersion relations for them, of the form

$$f_V(q^2) = a + \int_{\mu_0^2}^{\infty} \frac{\rho(\mu^2)}{\mu^2 + q^2} d\mu^2,$$

$$g_V(q^2) = b + \int_{\mu_0^2}^{\infty} \frac{\sigma(\mu^2)}{\mu^2 + q^2} d\mu^2, \tag{3}$$

where $\mu_0 = m_K + m_\pi$. The assumption that only a sharp K-π S-wave resonance contributes to the spectral functions ρ and σ would then give

$$\rho(\mu^2) \simeq 0, \quad \sigma(\mu^2) \simeq c\delta(\mu^2 - m_{K'}^2), \tag{4}$$

so that

$$f_V(q^2) \simeq a, \quad g_V(q^2) \simeq b + \frac{c}{q^2 + m_{K'}^2}, \tag{5}$$

where c is a constant proportional to the unknown strong $KK'\pi$ coupling constant and to the amplitude for the unobservable processes $K' \to \mu + \nu$ and $K' \to e + \nu$.

So far, this is hardly a very useful set of formulas. But if we now make use of the idea that $\partial^\lambda \mathcal{J}_\lambda^V$ is a "gentle" operator, we can calculate not only b, but (almost paradoxically) the constant a as well. We do this by following along the lines of one of the derivations of the Goldberger-Treiman formula,[6] and assuming that the amplitude $h(q^2)$ related to $\partial^\lambda \mathcal{J}_\lambda^V$ by

$$\langle \pi | \partial^\lambda \mathcal{J}_\lambda^V(0) | K \rangle = (2\pi)^{-3}(4E_\pi E_K)^{-1/2} h(q^2) \tag{6}$$

must vanish as $|q^2| \to \infty$. Since we have

$$h(q^2) = P_K \cdot q f_V(q^2) + q^2 g_V(q^2) = q^2[\tfrac{1}{2} f_V(q^2) + g_V(q^2)] - \tfrac{1}{2}(m_K^2 - m_\pi^2) f_V(q^2), \tag{7}$$

the vanishing of $h(\infty)$ means that

$$-b = \tfrac{1}{2}a = \frac{1}{m_K^2 - m_\pi^2}\int_{\mu_0^2}^{\infty} [\tfrac{1}{2}\rho(\mu^2) + \sigma(\mu^2)] d\mu^2. \tag{8}$$

Assuming the dominance of the K' "pole," we have now

$$f_V(q^2) \simeq \frac{2c}{m_K^2 - m_\pi^2},$$

$$g_V(q^2) \simeq c\left[\frac{1}{q^2 + m_{K'}^2} - \frac{1}{m_K^2 - m_\pi^2}\right]. \tag{9}$$

The constancy of $f_V(q^2)$ determines the spectrum as well as the angular correlations in K_{e3} decay (since $m_e \simeq 0$, $U \simeq \sin^2\theta$) and implies that all features of $K_{\mu 3}$ decay, including the $K_{\mu 3}/K_{e3}$ branching ratio, are determined by the ratio

$$g_V(q^2)/f_V(q^2) \simeq \tfrac{1}{2}\left[\frac{m_K^2 - m_\pi^2}{q^2 + m_{K'}^2} - 1\right]. \tag{10}$$

This is a slowly decreasing function of E and of $m_{K'}$, which stays between the limits -0.04 and -0.2 in $K_{\mu 3}$ decay with the K' mass $m_{K'} \simeq m_K + m_\pi$.

It is instructive to compare our result with that obtained by Weinberg et al.,[9] who assumed that $\mathcal{J}_\lambda^V$ was actually conserved. In that paper

$$g_V/f_V = \tfrac{1}{2}(m_K^2 - m_\pi^2 - q^2)/q^2 \tag{11}$$

has an apparent pole at $q^2 = 0$. This is somewhat unphysical and reflects on the conserved-current hypothesis made there.[10] Their result is the same as ours would be for $m_{K'} = 0$, but in our case their unphysical pole at $q^2 = 0$ has migrated to $q^2 = -m_{K'}^2$, and represents a physical intermediate state.

We will defer a detailed analysis of the consequences of Eq. (9). It should be kept in mind that we have been very optimistic in assuming that the K' resonance dominates all dispersion integrals, and that the K' lifetime (which must be of order 10^{-23} sec) is long enough for this resonance to be represented by a pole; if our optimism is justified the experiments now being planned or completed[11] on the $K_{\mu 3}$ decay should show it up. Of course, our approach in deriving Eq. (8) does not rest on any approximation but on a fundamental assumption, and so it may have a wider utility than Eq. (9).

A similar approach can be applied to hyperon beta decay, but the results obtained are less complete and very difficult to check experimentally.

If the $K'YN$ relative parity is even, then

$$\langle N | \mathcal{g}_\lambda^{V} | Y \rangle \sim \gamma_\lambda f_V(q^2) + \sigma_{\lambda\eta} q^\eta g_V(q^2) + i q_\lambda h_V(q^2), \qquad (12)$$

and our analysis gives

$$f_V(q^2) \simeq c_Y/(m_Y - m_N), \quad h_V(q^2) \simeq c_Y/(q^2 + m_{K'}^2). \qquad (13)$$

If the $K'YN$ relative parity is odd, then

$$\langle N | \mathcal{g}_\lambda^{A} | Y \rangle \sim i\gamma_5 \gamma_\lambda f_A(q^2) + q_\lambda \gamma_5 g_A(q^2) + i\sigma_{\lambda\eta}\gamma_5 q^\eta h_A(q^2), \qquad (14)$$

and our analysis gives just the Gamow-Teller formulas

$$f_A(q^2) \simeq c_Y/(m_Y + m_N), \quad g_A(q^2) \simeq c_Y/(q^2 + m_{K'}^2). \qquad (15)$$

(Here $q \equiv P_Y - P_N$, and c_Y is proportional to the $K'YN$ coupling constant and to the amplitude for $K' \to \mu + \nu$.)

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[1]M. Gell-Mann, Proceedings of the Tenth Annual Rochester Conference on High-Energy Nuclear Physics, 1960 (to be published).

[2]J. Tiomno, Proceedings of the Tenth Annual Rochester Conference on High-Energy Nuclear Physics, 1960 (to be published).

[3]J. Bernstein and M. Gell-Mann (unpublished).

[4]For an alternate scheme see A. Pais, CERN preprint, 1960, and further unpublished work which was kindly explained to us by Professor Pais.

[5]M. Gell-Mann and M. Lévy, Nuovo cimento 16, 705 (1960).

[6]Y. Nambu, Phys. Rev. Letters 4, 380 (1960); J. Bernstein, S. Fubini, M. Gell-Mann, and W. Thirring (to be published). See also Chou Kuang-chao, Dubna report, 1960 (unpublished).

[7]M. L. Goldberger and S. B. Treiman, Phys. Rev. 110, 1178 (1958).

[8]See, e.g., J. Steinberger, Proceedings of the Second United Nations International Conference on the Peaceful Uses of Atomic Energy, Geneva, 1958 (United Nations, Geneva, 1958). A similar mechanism operates in the reaction $\pi^- + p \to K^0 + \Sigma^0$, in which the Σ^0 is also observed to go backwards, and in the reaction $\pi^+ + p \to \Sigma^+ + K^+$ where the backwards peaking has not been observed, although the data here are especially meager. It is easy to see that there is no K' mechanism available to the reaction $\pi^- + p \to \Sigma^- + K^+$ since the intermediate K' would have to have an impossible charge. For this case the experiments show isotropy at lower pion energies and some forward peaking at the high energies. J. G. Taylor, Nuclear Phys. 9, 357 (1959), has noted that in the usual theories a firm establishment of this pole would very likely indicate that K^+ and K^0 have opposite parities. For the early theoretical papers see M. Goldhaber, Phys. Rev. 101, 433 (1956), and J. Schwinger, Phys. Rev. 104, 1164 (1956).

[9]See A. Pais and S. B. Treiman, Phys. Rev. 105, 1616 (1957), and S. Weinberg, R. E. Marshak, S. Okubo, E. C. G. Sudarshan, and W. B. Teutsch, Phys. Rev. Letters 1, 25 (1958). Our notation is essentially that of the latter reference. Equation (7) of this reference should have $1 - x^2$ replaced by $x^2 - 1$.

[10]One of us (J.B.) has profited from several discussions with D. A. Geffen and M. Gell-Mann on this point and others related to the K' particle.

[11]D. Glaser and D. Ritson (private communications).