sponsible for the K_{l3} decays is tenable. If this were the case, the possibility that the vector current is quasi-conserved in the presence of both π and K interactions would be particularly appealing. On the other hand, if the $\Delta Q = -\Delta S$ decays do not exist and the other predictions of the pure $I = \frac{1}{2}$ hypothesis are established, it might be interesting to consider the possibility that the vector part of such currents is quasi-conserved only in the presence of the π-baryon interactions of the Yukawa type. In fact, such a restricted conservation requirement implies the absence of $I = 3/2$ currents.[1]

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REACTION $\mu + N \rightarrow e + N'$: INTERMEDIATE BOSON THEORY*

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A μ^- bound in a 1S state around a nucleus may decay via the process $\mu \rightarrow e + \nu + \bar{\nu}$ or may be absorbed by one of the nuclear protons to form a neutron plus a neutrino. However, if there exists a mechanism giving rise to a nonvanishing μ-e-γ matrix element, then one would expect a third process $\mu + N \rightarrow e + N'$ to occur. The μ^- may absorb a virtual photon from the Coulomb field of the original nucleus N, producing an electron and a recoil nucleus N' having equal and opposite momenta of magnitude 106 Mev/c. Because the photon involved is virtual rather than real, the experimental absence of $\mu \rightarrow e + \gamma$ decay does not necessarily preclude the existence of the process $\mu + N \rightarrow e + N'$. In fact, it is possible that instances of such a reaction have been observed by Sard et al., who set an upper limit 4×10^{-6} upon the branching ratio $\omega(\mu + N \rightarrow e + N')/\omega_{\text{abs}}$, although Conversi et al. have established a comparable upper limit 4.3×10^{-6} upon the branching ratio without having seen any $\mu + N \rightarrow e + N'$ events.[1] Because simple arguments allow one to suppose that the branching ratio predicted by an intermediate Boson theory of weak interactions will be close to the upper limit established by these experiments, it is appropriate at this time to investigate in detail the dependence of the predicted branching ratio upon the value of the high-momentum cutoff and the Boson mass. Furthermore, the calculation will be of value in predicting the rates of other processes like $\mu \rightarrow 3e$, which can also proceed through a μ-e-γ vertex.

A nonvanishing μ-e-γ matrix element could arise via a charged vector Boson of reasonably large mass M coupled to Fermion pairs with a coupling constant g.[2] The induced four-Fermion interaction would have the right strength, $G \approx 10^{-5}/M_{\text{nucleon}}^2$, providing g is chosen so that

$$(8)^{1/2} G = (g/M)^2. \tag{1}$$

The three Feynman graphs which contribute to the μ-e-γ matrix element in the lowest order of perturbation theory are shown in Fig. 1. Denoting the muon momentum by $p_{\mu\lambda}$, the electron momentum by $p_{e\lambda}$, and the momentum transfer by

$$q_\lambda = p_{e\lambda} - p_{\mu\lambda}, \tag{2}$$

the three contributions to the matrix element may be written as follows:

$$\langle e | J_\lambda{}^1 | \mu \rangle = -(2\pi)^{-7} e g^2 \bar{u}_e \tfrac{1}{2}(1-\gamma_5) \int d^4k [\delta_{\alpha\beta}(p_e + p_\mu - 2k)_\lambda - (p_\mu - k)_\alpha \delta_{\beta\lambda} - (p_e - k)_\beta \delta_{\alpha\lambda} - (1+\mu)(q_\alpha \delta_{\beta\lambda} - q_\beta \delta_{\alpha\lambda})]$$
$$\times \left(\frac{\gamma_\beta + M^{-2}(p_e - k)_\beta (\gamma \cdot p_e - \gamma \cdot k)}{(p_e - k)^2 + M^2} \right) \frac{\gamma \cdot k}{k^2} \left(\frac{\gamma_\alpha + M^{-2}(\gamma \cdot p_\mu - \gamma \cdot k)(p_\mu - k)_\alpha}{(p_\mu - k)^2 + M^2} \right) u_\mu, \tag{3}$$

$$\langle e|J_\lambda{}^2|\mu\rangle=(2\pi)^{-7}ieg^2\bar{u}_e\tfrac{1}{2}(1-\gamma_5)K(p_e)\,\frac{i\gamma\cdot p_e-m_\mu}{p_e{}^2+m_\mu{}^2}\gamma_\lambda u_\mu,$$

$$\langle e|J_\lambda{}^3|\mu\rangle=(2\pi)^{-7}ieg^2\bar{u}_e\gamma_\lambda\,\frac{i\gamma\cdot p_\mu-m_e}{p_\mu{}^2+m_e{}^2}K(p_\mu)\tfrac{1}{2}(1+\gamma_5)u_\mu. \qquad (3)$$

Here μ is the anomalous magnetic moment of the Boson in Boson magnetons, and

$$K(p)=\int d^4k\,\frac{-2\gamma\cdot k+M^{-2}(\gamma\cdot p-\gamma\cdot k)\gamma\cdot k(\gamma\cdot p-\gamma\cdot k)}{[(p-k)^2+M^2]k^2}.$$

Neglecting the electron mass m_e and assuming that $(m_\mu/M)^2\ll 1$ and that $-q^2/M^2\ll 1$, the above integrals may be evaluated by expanding the denominators in powers of p_μ and p_e. Although individually the contributions are not gauge invariant, the sum $\langle e|J_\lambda|\mu\rangle$ is, and it may be written in the form

$$\langle e|J_\lambda|\mu\rangle$$
$$=-ie(2\pi)^{-3}\bar{u}_e[f_0(q^2)(1-\gamma_5)(\gamma_\lambda+iq_\lambda m_\mu/q^2)$$
$$+f_1(q^2)(1-\gamma_5)\sigma_{\lambda\eta}q^\eta/m]u_\mu, \qquad (4)$$

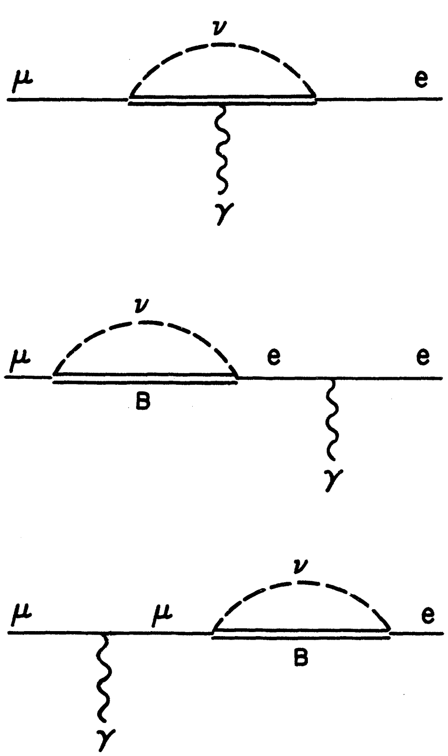

FIG. 1. Feynman graphs which contribute to the μ-e-γ matrix element. The intermediate vector Boson is designated by a double line.

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where $f_0(q^2)$ and $f_1(q^2)$ are form factors, which for $-q^2\ll M^2$ are given by

$$f_0(q^2)=-2^{-9/2}\pi^{-2}Gq^2N',$$
$$f_1(q^2)=-2^{-9/2}\pi^{-2}Gm_\mu{}^2N. \qquad (5)$$

The dependence of the dimensionless constant N upon the ratio of cutoff to Boson mass Λ/M and upon the Boson anomalous moment μ was investigated by Ebel and Ernst.[3] To study processes in which virtual photons take part it will be necessary to include N' as well as N. For these constants we find

$$N=(\mu-1)I_0+(2\mu+1)I_1-3I_2,$$
$$N'=-\tfrac{1}{2}\mu I_{-1}-(\tfrac{3}{2}\mu+\tfrac{5}{3})I_0+(2\mu+\tfrac{2}{3})I_1+I_2, \qquad (6)$$

where

$$I_n=\frac{M^{2n}}{\pi^2 i}\int\frac{d^4p}{(p^2+M^2)^{2+n}}. \qquad (7)$$

Unless $\mu=0$ it is necessary to cope with a quadratically divergent integral I_{-1} as well as the logarithmically divergent integral I_0 studied by Ebel and Ernst. Thus, for $\mu=0$ it suffices to introduce a cutoff factor such as $\Lambda^2/(p^2+\Lambda^2)$ in the definition of the integrals I_n, while for nonzero Boson anomalous moment it is necessary to use a stronger cutoff factor such as $\Lambda^4/(p^2+\Lambda^2)^2$. In Fig. 2 are given N and N' as functions of the ratio Λ/M for two values of anomalous moment, $\mu=0$ and 0.7 Boson magneton. The latter choice of magnetic moment was shown in reference 1 to lead to a marked suppression of the muon decay mode $\mu\to e+\gamma$, i.e., $N\approx 0$. The same conclusion is reached in the present case with the stronger cutoff factor. For any cutoff less than four or five times the Boson mass the rate for $\mu\to e+\gamma$ is compatible with the experimental branching ratio $\omega(\mu\to e+\gamma)/\omega(\mu\to e+\nu+\bar{\nu})<2\times 10^{-6}$.

For a wide range of elements from copper to lead, Weinberg and Feinberg have estimated the branching ratio for the reaction $\mu+N\to e+N'$

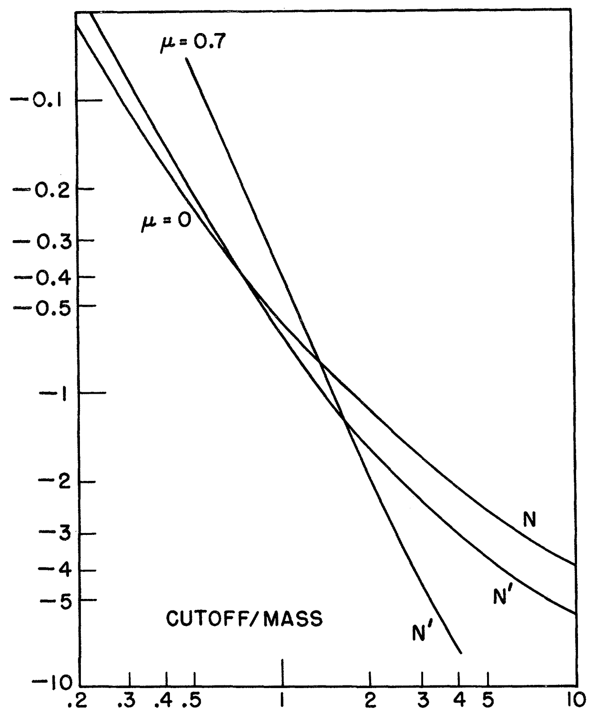

FIG. 2. Cutoff dependence of the dimensionless constants N and N' for Boson anomalous moment $\mu = 0$ and 0.7 Boson magneton. $N \approx 0$ for $\mu = 0.7$.

compared with μ absorption to be approximately[4]

$$\omega(\mu + N \to e + N')/\omega_{\text{abs}} \approx \xi_0^2/4G^2 m_\mu^4,$$

$$\xi_0^2 = |f_0(-m_\mu^2) + f_1(-m_\mu^2)|^2. \quad (8)$$

Therefore, the branching ratio according to the intermediate Boson theory is given by

$$\omega(\mu + N \to e + N')/\omega_{\text{abs}} \approx 2^{-11}\pi^{-4}(N - N')^2, \quad (9)$$

which has been tabulated for $\mu = 0.7$ (see Table I). In particular, for a cutoff equal to the Boson mass, we have

$$\omega(\mu + N \to e + N')/\omega_{\text{abs}} \approx 0.8 \times 10^{-6}. \quad [\mu = 0.7] \quad (10)$$

On the other hand, it is to be noted that for $\mu = 0$, $N \approx N'$, and hence there is a marked suppression of the reaction $\mu + N \to e + N'$ compared with the value given in Eq. (10). Of course, the choice $\mu = 0$ gives much too large a probability for $\mu \to e + \gamma$. Therefore, if one insists that $\mu = 0$, it is necessary to impose the condition $N = 0$ and employ the intermediate Boson theory to calculate N' only. If this is done the branching ratio for $\mu = 0$ becomes (at $\Lambda = M$)

$$\omega(\mu + N \to e + N')/\omega_{\text{abs}} \approx 2 \times 10^{-6}. \quad [\mu = 0] \quad (11)$$

In conclusion, insofar as it is possible to adjust the intermediate Boson theory to account for the absence of $\mu \to e + \gamma$, this theory predicts a branching ratio $\omega(\mu + N \to e + N')/\omega_{\text{abs}}$ of order 10^{-6}, although the result is very much dependent upon the choice of cutoff.

Finally, one may use Eqs. (4) and (5) in conjunction with the graphs in Fig. 2 to evaluate the rate for any other process which goes via the $\mu - e - \gamma$ interaction. For example, Bander and Feinberg[5] have estimated the branching ratio for $\mu \to 3e$ compared with $\mu \to e + \nu + \bar{\nu}$ to be

$$\omega(\mu \to 3e)/\omega(\mu \to e + \nu + \bar{\nu}) \approx 6 \times 10^{-3} \xi_0^2/G^2 m_\mu^4, \quad (12)$$

where ξ_0 has been defined in connection with Eq. (8). For an anomalous moment of 0.7 Boson magneton and for a cutoff equal to the Boson mass, we have

$$\omega(\mu \to 3e)/\omega(\mu \to e + \nu + \bar{\nu}) \approx 2 \times 10^{-8}. \quad (13)$$

The author would like to thank Dr. G. Feinberg for helpful discussions concerning the reaction $\mu + N \to e + N'$ and $\mu \to 3e$ decay.

Table I. The branching ratio for reaction $\mu + N \to e + N'$ for various choices of cutoff Λ if the Boson anomalous moment μ is 0.7 Boson magneton. M is the Boson mass.

Λ/M	$\omega(\mu + N \to e + N')/\omega_{\text{abs}}$
0.5	$.4.1 \times 10^{-8}$
1	0.85×10^{-6}
2	2.0×10^{-5}
4	2.9×10^{-4}

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