carried out on the IBM 704 in the U. S. Atomic Energy Commission Computation Center at New York University.

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POSSIBLE EXISTENCE OF $\Delta Q = -\Delta S$ DECAYS

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The possible existence of isotopic spin $I=3/2$ strangeness-changing currents, which lead to decays that satisfy $|\Delta S| \leq 1$ and to the $|\Delta I| = \frac{1}{2}$ rule for processes involving only strongly interacting particles, has been discussed recently.[1-3] In particular, it has been shown[1] that the quasi-conservation of the vector part of the $I=3/2$ currents in the presence of both the strong π and K interactions of the Yukawa type requires a lower symmetry of the strong couplings (the 14-parameter group $G2$) than the $I=\frac{1}{2}$ currents (the seven-dimensional rotation group). For this and other theoretical reasons,[2,3] it is worthwhile to examine in more detail the experimental implications of such currents.

As has been noted,[1] the existence of $I=3/2$ currents of strongly interacting particles which are coupled to the leptonic currents permits, but does not demand, the presence of leptonic modes of decay with $\Delta Q = -\Delta S$ (while still preserving the rule $|\Delta S| \leq 1$ for all decays). On the other hand, for pure $I=\frac{1}{2}$ currents, these decays are forbidden.

It follows from the existence of the K_{l2} decay that $I=\frac{1}{2}$ currents must be present in the space-time vector and/or axial vector part. For example, if the K were pseudoscalar (scalar), K_{l2} could occur through the axial vector (vector) part. However, this still allows one of the space-time parts to be composed of dominant, or even possibly "pure" $I=3/2$ currents through which the K_{l3} modes of decay could proceed. As the predictions of pure $I=\frac{1}{2}$ currents on the K_{l3} decays have been already given,[4] we will now examine the other "pure" case; namely, the possibility that the space-time part governing these decays consists of pure or dominant $I=3/2$ currents. It is the purpose of this note to discuss the implications of such an assumption with regard to the $\Delta Q = -\Delta S$ modes of decay. We shall also assume time-reversal invariance.

Let us consider the K_{l3} decays. The first interesting consequence is that we can obtain exact lower and upper bounds for the $\Delta Q = -\Delta S$ decays $K^0 \to \pi^+ + l^- + \nu$ in terms of observed lifetimes. For "pure" $I=3/2$ currents, neglecting electromagnetic effects, we obtain

$$\sqrt{2}M(K^0 \to \pi^- + l^+ + \nu) = -M(K^+ \to \pi^0 + l^+ + \nu). \quad (1)$$

By using Eq. (1) and the relation $K_2{}^0 = (K^0 - \bar{K}^0)/\sqrt{2}$, we get the following triangular inequalities for the total rates R:

$$R^{1/2}(\bar{K}^0 \to \pi^- + l^+ + \nu) + \frac{1}{\sqrt{2}}R^{1/2}(K^+ \to \pi^0 + l^+ + \nu) \geq \sqrt{2}R^{1/2}(K_2{}^0 \to \pi^- + l^+ + \nu), \quad (2)$$

and cyclically. These inequalities hold exactly for the e^+ and μ^+ modes and for the total rates for positive leptons, electron plus muon. The TCP theorem tells us that $R(\bar{K}^0 \to \pi^- + l^+ + \nu) = R(K^0 \to \pi^+ + l^- + \nu)$ and CP invariance implies that the l^+ and l^- modes of $(K_2{}^0)_{l3}$ decay occur at the same total rate. Inserting the existing experimental data,[5,6] we obtain for the total decay rates for the leptonic modes, e plus μ:

$$8.5^{+2.3}_{-1.6} \geq \frac{R(K^0 \to \pi^+ + \text{leptons})}{R(K^0 \to \pi^- + \text{leptons})} \geq 0.85^{+0.81}_{-0.45}. \qquad (3)$$

In Eq. (3), we have neglected the error in the $(K^+)_{l3}$ decays and have not corrected for possible τ^0 decays (which in this experiment[5] occur with a frequency $\lesssim 15\%$). Although the experimental errors are large, we see that the results (3) favor the existence of $\Delta Q = -\Delta S$ decays at an appreciable rate, if the assumption that these decays are governed by dominant $I = 3/2$ currents is correct.

In order to obtain more detailed predictions for the $K_1{}^0$ decays, we express K^0 in Eq. (1) in terms of $K_1{}^0$ and $K_2{}^0$ and observe that T invariance implies that the phases for the K_{l3} decays may be so chosen that all the matrix elements are real (assuming, of course, that the spin of the K is zero). If we limit ourselves to the electron modes and neglect terms proportional to m_e, each decay amplitude consists of one structure function multiplying a lepton matrix element. By taking this fact into account, the following rigorous equality follows from Eq. (1):

$$\sigma^{1/2}(K_1{}^0 \to \pi^- + e^+ + \nu) = |\sigma^{1/2}(K_2{}^0 \to \pi^- + e^+ + \nu) \pm \sigma^{1/2}(K^+ \to \pi^0 + e^+ + \nu)|, \qquad (4)$$

where σ is the differential decay rate at fixed pion energy E_π. [Equation (4) does not follow necessarily for the μ modes unless one adds the dynamical assumption that the ratio of the two structure functions describing those processes is the same for the three decays.] To obtain equalities for the total rates of the electron modes, it is necessary to add the dynamical assumption that the form factor for the $(K_2{}^0)_{e3}$ and $(K^+)_{e3}$ decays have approximately the same dependence on E_π. This is not unreasonable since most calculations suggest that the form factor is, anyway, a slowly varying function of E_π. If this is true, Eq. (4) is also valid for the total rates and the following relations are readily obtained:

$$r \equiv R(K_1{}^0 \to \pi^\mp + e^\pm + \nu)/R(K_2{}^0 \to \pi^\mp + e^\pm + \nu) = (1+c)^2, \qquad (5a)$$

$$s \equiv R(K^0 \to \pi^+ + e^- + \bar{\nu})/R(K^0 \to \pi^- + e^+ + \nu) = (1 + 2/c)^2, \qquad (5b)$$

$$R(K^+ \to \pi^0 + e^+ + \nu)/R(K_2{}^0 \to \pi^- + e^+ + \nu) = c^2. \qquad (5c)$$

As the experimental data include both leptonic decay modes of $(K_2{}^0)_{l3}$, we cannot use them directly in Eqs. (5). However, if it turns out that both modes of $(K_2{}^0)_{l3}$ occur at approximately the same rate (as is the case for K^+), the present experimental value for c^2 would be $1.08^{+0.43}_{-0.32}$. From this value, one obtains the predictions

$$r_+ = 4.2^{+0.8}_{-0.7}, \quad r_- = 0.00^{+0.05}, \qquad (6a)$$

$$s_+ = 8.5^{+2.3}_{-1.6}, \quad s_- = 0.85^{+0.81}_{-0.45}, \qquad (6b)$$

where the plus and minus subscripts refer to the two possible signs of c. This is to be compared with $r = 1$ and $s = 0$ when the $\Delta Q = -\Delta S$ decays are not present. The value r_+ is in close agreement with existing data,[7] but the experimental errors are too large at present to draw any definite conclusions. It should be noted that for pure $I = \frac{1}{2}$ currents, one has the prediction $c^2 = 1$. This result is also compatible with pure $I = 3/2$ currents if the $\Delta Q = -\Delta S$ decays occur at the rate predicted by Eqs. (5a) and (5b). Thus, an experimental determination of the ratio (5c) is not sufficient to distinguish between the $I = 1/2$ and 3/2 currents.

It might be interesting to consider the predictions which arise in the case for which the space-time vector part of the currents is predominantly $I = 3/2$ and is quasi-conserved in the presence of both π and K interactions of the Yukawa type.[1] This requires that the K be pseudoscalar with respect to ΣN and $\Xi\Sigma$. In this case, besides the predictions of Eqs. (6), it is possible to evaluate approximately the vector contribution to $\Sigma^+ \to n + e^+ + \nu$ from the known rates of K^+ and $K_2{}^0$, if one neglects renormalization effects originating from the mass differences. The vector contribution provides a lower bound for the total rate when the electron mass is neglected. In this way, one finds that the branching ratio for $\Sigma^+ \to n + e^+ + \nu$ must be greater than $(7.3^{+2.0}_{-1.4}) \times 10^{-4}$ or $(0.73^{+0.70}_{-0.39}) \times 10^{-4}$, corresponding, respectively, to the predictions r_+ and r_- of Eqs. (6).

From the above considerations, it is clear that the search for the $\Delta Q = -\Delta S$ decays, as well as the precise determination of the $K_2{}^0$ leptonic rates, are essential in order to establish the isotopic spin structure of the strangeness nonconserving currents. If the $\Delta Q = -\Delta S$ decays are found, it would be interesting to test the numerical predictions given above to determine whether the assumption of a dominant $I = 3/2$ current responsible for the K_{l3} decays is tenable. If this were the case, the possibility that the vector current is quasi-conserved in the presence of both π and K interactions would be particularly appealing. On the other hand, if the $\Delta Q = -\Delta S$ decays do not exist and the other predictions of the pure $I=\frac{1}{2}$ hypothesis are established, it might be interesting to consider the possibility that the vector part of such currents is quasi-conserved only in the presence of the π-baryon interactions of the Yukawa type. In fact, such a restricted conservation requirement implies the absence of $I=3/2$ currents.[1]

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REACTION $\mu + N \to e + N'$: INTERMEDIATE BOSON THEORY*

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A μ^- bound in a 1S state around a nucleus may decay via the process $\mu \to e + \nu + \bar{\nu}$ or may be absorbed by one of the nuclear protons to form a neutron plus a neutrino. However, if there exists a mechanism giving rise to a nonvanishing μ-e-γ matrix element, then one would expect a third process $\mu + N \to e + N'$ to occur. The μ^- may absorb a virtual photon from the Coulomb field of the original nucleus N, producing an electron and a recoil nucleus N' having equal and opposite momenta of magnitude 106 Mev/c. Because the photon involved is virtual rather than real, the experimental absence of $\mu \to e + \gamma$ decay does not necessarily preclude the existence of the process $\mu + N \to e + N'$. In fact, it is possible that instances of such a reaction have been observed by Sard et al., who set an upper limit 4×10^{-6} upon the branching ratio $\omega(\mu + N \to e + N')/\omega_{\text{abs}}$, although Conversi et al. have established a comparable upper limit 4.3×10^{-6} upon the branching ratio without having seen any $\mu + N \to e + N'$ events.[1] Because simple arguments allow one to suppose that the branching ratio predicted by an intermediate Boson theory of weak interactions will be close to the upper limit established by these experiments, it is appropriate at this time to investigate in detail the dependence of the predicted branching ratio upon the value of the high-momentum cutoff and the Boson mass. Furthermore, the calculation will be of value in predicting the rates of other processes like $\mu \to 3e$, which can also proceed through a μ-e-γ vertex.

A nonvanishing μ-e-γ matrix element could arise via a charged vector Boson of reasonably large mass M coupled to Fermion pairs with a coupling constant g.[2] The induced four-Fermion interaction would have the right strength, $G \approx 10^{-5}/M_{\text{nucleon}}^2$, providing g is chosen so that

$$(8)^{1/2} G = (g/M)^2. \tag{1}$$

The three Feynman graphs which contribute to the μ-e-γ matrix element in the lowest order of perturbation theory are shown in Fig. 1. Denoting the muon momentum by $p_{\mu\lambda}$, the electron momentum by $p_{e\lambda}$, and the momentum transfer by

$$q_\lambda = p_{e\lambda} - p_{\mu\lambda}, \tag{2}$$

the three contributions to the matrix element may be written as follows:

$$\langle e | J_\lambda{}^1 | \mu \rangle = -(2\pi)^{-7} e g^2 \bar{u}_e \tfrac{1}{2}(1-\gamma_5) \int d^4k [\delta_{\alpha\beta}(p_e + p_\mu - 2k)_\lambda - (p_\mu - k)_\alpha \delta_{\beta\lambda} - (p_e - k)_\beta \delta_{\alpha\lambda} - (1+\mu)(q_\alpha \delta_{\beta\lambda} - q_\beta \delta_{\alpha\lambda})]$$
$$\times \left(\frac{\gamma_\beta + M^{-2}(p_e - k)_\beta (\gamma \cdot p_e - \gamma \cdot k)}{(p_e - k)^2 + M^2} \right) \frac{\gamma \cdot k}{k^2} \left(\frac{\gamma_\alpha + M^{-2}(\gamma \cdot p_\mu - \gamma \cdot k)(p_\mu - k)_\alpha}{(p_\mu - k)^2 + M^2} \right) u_\mu, \tag{3}$$