FREQUENCY DISTRIBUTION OF RESONANCE LINE VERSUS DELAY TIME

C. S. Wu, Y. K. Lee, N. Benczer-Koller, and P. Simms
Columbia University, New York, New York
(Received September 26, 1960)

In the quantum theory, the intensity distribution of a line emitted in a transition between two states with $\Delta E = \hbar\omega$ is given by

$$I(\omega)d\omega = \frac{\gamma}{2\pi} \frac{\hbar\omega d\omega}{(\omega - \omega_0)^2 + \frac{1}{4}\gamma^2},$$

when the period of observation T is long compared to the mean lifetime T_{av} of the excited state. The distribution has a maximum intensity for the frequency ω_0 and the breadth at half maximum is equal to γ, the initial transition probability per unit time or the reciprocal lifetime, $\gamma = 1/T_{av}$. This distribution is identical with the natural line shape for emission and absorption of the light in a classical treatment where $\omega/2\pi$ is the frequency of an oscillator and the natural breadth γ is due to the damping force of the emitted radiation on the oscillator. Therefore, the quantum analog of the classical natural linebreadth γ is the initial transition probability per unit time. However, when the time T is short or comparable to the mean lifetime T_{av} of the excited state, the frequency spectrum of the emitted line follows the function

$$\frac{1 + e^{-\gamma T} - 2e^{-\gamma T/2}\cos(\omega - \omega_0)T}{(\omega - \omega_0)^2 + \frac{1}{4}\gamma^2}.$$

The propagation of such a composite wave[1] through a dispersive medium filled with resonators like the resonance absorber depends on both the frequency distribution of the incident wave, which is governed by the duration of the wave train T, and the complex dielectric constant of the medium which can be affected by Doppler shift.

The unusual frequency-sensitivity of the Mössbauer effect[2] in Fe^{57} provides an excellent means to demonstrate such phenomena. In a recent paper Holland and his co-workers[3] reported their interesting results on the time spectra of filtered resonance radiation of Fe^{57}. The filtered resonance radiation displayed a nonexponential time decay as a consequence of the Fourier spectrum of the short wave train and the dispersive nature of the filtering medium.

Our approach to this investigation is to observe directly the resonance frequency spectrum of Fe^{57} at different time intervals T after the formation of the 14.4-kev excited state or classically the starting of the wave train. The detection of the preceding radiation (123-kev line) serves as the signal of the formation of the 14.4-kev excited state.

A theoretical treatment based on classical theory has been developed by Hamermesh[4] assuming a single emission and absorption line. (A quantum mechanical treatment[5] yields identical results.) For a Co^{57} source imbedded in a metallic iron environment, the 14.4-kev line of Fe^{57} has a hyperfine structure of six lines and so does the iron absorber. For very short delay times, each of the lines spreads to such an extent that the adjacent lines overlap each other. Under these circumstances, the simple assumption that each component interacts only with its own counterpart in the absorber is no longer valid. To overcome this complication a Co^{57} source electroplated onto a thin stainless steel foil (No. 302; 8.3 mg/cm^2) and a stainless steel absorber cut from the same foil were used. According to Wertheim's results[6] the spin correlation time in stainless steel is sufficiently short to exhibit an unsplit line. The single-peak resonance spectrum as shown in Fig. 1(c) confirms the absence of the satellites. The observed linewidth, after the correction for the absorber thickness, closely approaches the expected natural width, therefore dispelling the doubt that the spin correlation time may not be sufficiently short. Both source and absorber were annealed in hydrogen at approximately 1000°C for a period of several hours. The estimated fraction of nonrecoil emission or absorption (assuming $f = f'$) is nearly 60%.

A continuous range of Doppler shifts[7] are obtained by fixing the source to one end of a light metal tube which passes coaxially through two loud speakers that are rigidly bolted back to back. The paper cones of the speakers have been removed and the tube fastened to the two voice coils. The tube is otherwise unsupported so that it is free to follow the voice coil motion. The source is placed in motion by driving one of the voice coils with a sinusoidal oscillator (12-30 cps). The other coil follows the motion, and the induced sine wave is amplified and directly coupled into a multichannel analyzer which

serves as an instantaneous velocity indicator and recorder. Normalization is obtained by accumulating the velocity spectrum of the nonresonant line (123 kev) and the resonant line (14.4 kev) side by side on the multichannel analyzer. For the delayed coincidence investigation, the 14.4-kev resonance spectrum is gated by a fast-slow coincidence system operated by the 14.4-kev, and 123-kev line. The time-delay coincidence circuit is very similar to that of Green and Bell[8] with some minor improvements. The detectors used are thin NaI scintillation counters and the time resolution was determined from the prompt curve to be 25 mμsec. The source (5-mm diam) is placed 2 cm away from the absorber. The exposed area of the absorber is defined by a thick lead shield with an opening of 3-cm diameter. The 14.4-kev detector is 2 cm from the absorber and the 123-kev detector views the source sidewise at 3 cm.

Our results can be summarized in Figs. 1-3. In Fig. 1, three resonance spectra [1(a), 1(b), and 1(c)] are exhibited. They are Armco iron source (0.7 mil) with Armco iron absorber (0.3 mil), stainless steel source (No. 302; 8.3 mg/cm^2) with Armco iron absorber (0.7 mil), and stainless steel source (No. 302; 8.3 mg/cm^2) with stainless steel absorber (No. 302; 8.3 mg/cm^2), respectively. The separations between the central line and the first pair of satellites in 1(a) and between the first and second satellite lines in 1(b) should be equal to each other and equal to the energy separation between the sublevels of the upper state. This measurement provides a convenient velocity calibration of our system because this separation is known from earlier direct "lathe" method. Curve 1(c) shows only a single, narrow line; therefore it fulfills our requirements. The most pronounced prediction of the theory is that the apparent width of the main resonance line is initially very broad and then compresses as the duration of the de-

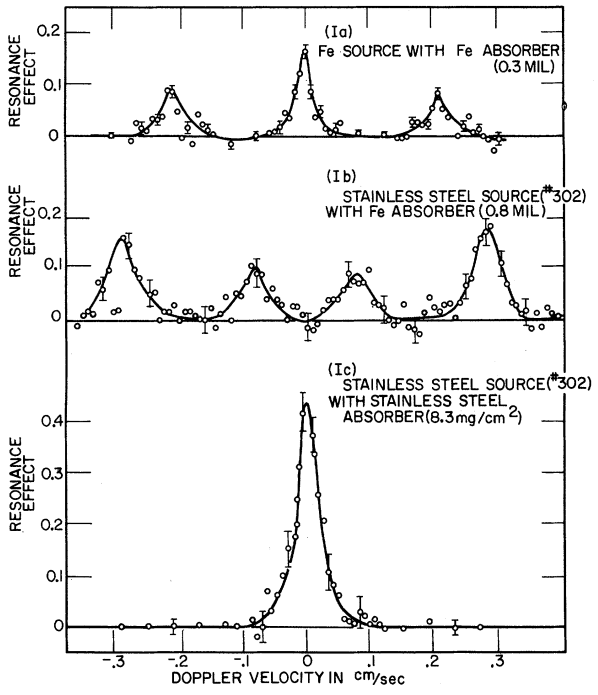

FIG. 1. Resonance effect of the 14.4-kev line from Fe^{57} imbedded in various source environments with various absorbers: (a) Co^{57} electroplated on Armco iron foil with Armco iron foil as absorber; (b) Co^{57} electroplated on stainless steel foil (No. 302) as source and Armco iron foil as absorber; (c) Same stainless steel foil (No. 302) used both as the backing material for the source and as absorber.

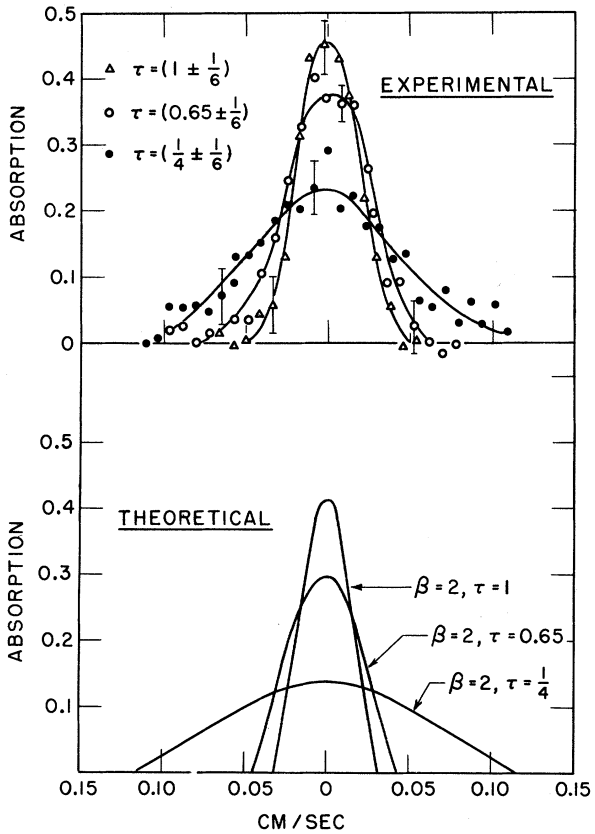

FIG. 2. The central resonance line versus delay time, where $\beta = N\sigma_0 f'$, and $\tau = T/T_{av}$. T is the delay time used. The upper portion shows the experimental results with delay times of $\tau = 1/4$, 0.65, and 1. The lower portion exhibits the theoretical curves calculated for the above delay times.

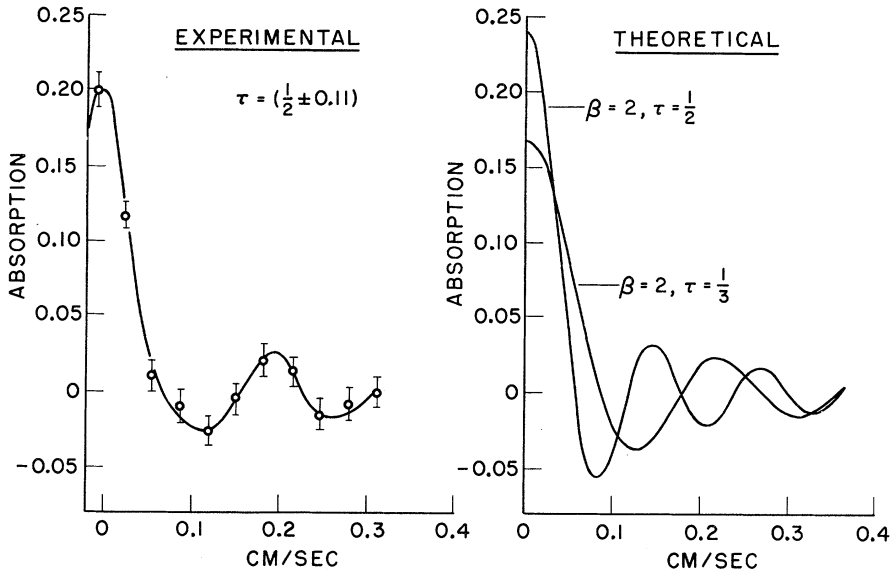

FIG. 3. The extended resonance spectrum for delayed coincidences at $\tau = T/T_{av} = (1/2 \pm 0.11)$ is shown on the left. The observed distribution falls somewhere between the two theoretical curves calculated for $\tau = 1/3$ as shown on the right.

layed time increases. To illustrate this point, the observed central resonance lines alone were plotted with various delayed time intervals, approximately 35 mμsec, 94 mμsec, and 150 mμsec as shown in Fig. 2. The theoretical curves for $\beta = 2$ and $T = \frac{1}{4}T_{av}$, $0.65T_{av}$, and T_{av} are plotted side by side for comparison.

The theoretical formula for the time dependence of the transmitted intensity derived by Hamermesh[4] involves the summation of Bessel functions:

$$|a'(\tau)|^2 = e^{-\tau}\left|\sum_{n=0}^{\infty}\left(i\frac{4}{\beta}\frac{\Delta\omega}{\gamma}\right)^n\left(\frac{\beta\tau}{4}\right)^{\frac{1}{2}n} J_n(\beta^{\frac{1}{2}}\tau^{\frac{1}{2}})\right|^2 .$$

For large values of $\Delta\omega$ the transmitted intensity is

$$|a'(\tau)|^2 = e^{-\tau}\left|\exp\left[i\left(\frac{\Delta\omega}{\gamma}\tau + \frac{\gamma}{\Delta\omega}\frac{\beta}{4}\right)\right] + \sum_{n=1}^{\infty}\left(i\frac{\beta}{4}\frac{\gamma}{\Delta\omega}\right)^n\left(\frac{\beta\tau}{4}\right)^{-\frac{1}{2}n} J_n(\beta^{\frac{1}{2}}\tau^{\frac{1}{2}})\right|^2 ,$$

where $\beta = n\sigma_0 f'$ is the absorber thickness in terms of number of mean free paths (n = number of Fe^{57} atoms/cm^2, σ_0 = cross section at resonance, f' = the recoilless fraction of absorption) and $\tau = \gamma T = T/T_{av}$. However, for a thin absorber of $\beta = 2$ a series expansion of the exponential term in the amplitude function retaining the first three terms is quite sufficient for most purposes. This gives[9]

$$|a'(\tau)|^2 = \left[1 - \frac{\gamma\beta}{2}\frac{\sin(\Delta\omega T)}{\Delta\omega} + \left(\frac{\gamma\beta}{4}\right)^2\left\{\frac{1-\cos(\Delta\omega T)}{(\Delta\omega)^2} + \frac{T\sin(\Delta\omega T)}{\Delta\omega}\right\}\right]e^{-\tau} .$$

In our stainless steel case, the value of β calculated for the absorber is between 1 and 2. However, one must also take into account the source thickness but this is rather difficult to estimate without knowledge of the environment of the source. Furthermore, for precise comparison, the finite time resolution and the observed linewidth must be folded into the formula. This is rather involved and was not attempted. The effect of the delayed time interval on the linewidth as well as the amount of resonance effect is most apparent in Fig. 2 and it illustrates the points predicted theoretically.

As was previously stated, the absorber behaves like a resonant filter. It reverberates in response to the incident damped oscillation. For a wave train of short duration, the theoretical resonance spectrum for a single line predicts many repeated waves oscillating up and down across the zero-absorption axis with diminishing intensities as the Doppler frequency increases. Those portions of the spectrum extending below the zero-absorption axis imply the existence of a negative absorption, which means more radiation transmitted with absorber than without.

This interesting phenomenon was actually observed and the results for a delayed time interval of 80 mμsec are shown in Fig. 3(a) which is to be compared with the theoretical curves 3(b) calculated for $\beta = 2$ and $\tau = T/T_{av} = 1/2$ and $\tau = 1/3$. In order to observe this effect, the time resolution was improved to 15 mμsec in order to reduce the smearing effect due to the uncertainty in time resolution. The agreement is very good.

The authors are deeply indebted to Professor N. Kroll and Dr. M. Morita who theoretically formulated the behavior of the resonance effect versus delay time for us before we were aware of Hamermesh's calculation. We are particularly grateful to Professor W. Lamb, Jr., Professor H. Frauenfelder, and Dr. E. L. Garwin for their stimulating discussions. To many of our colleagues, H. Fleischman, H. Goldberg, and H. Dowds who have contributed in the preparation of the experiment, we express our thanks.

[1]In actual calculations for the dispersive phenomena the amplitude of the incident wave is used.

[2]R. L. Mössbauer, Z. Physik 151, 125 (1959); Naturwissenschaften 45, 538 (1958).

[3]R. E. Holland, F. J. Lynch, G. J. Perlow, and S. S. Hanna, Phys. Rev. Letters 4, 181 (1960). We are also indebted to the authors for the preprint of a more detailed paper: F. J. Lynch, R. E. Holland, and M. Hamermesh, Phys. Rev. 120, 513 (1960).

[4]M. Hamermesh, Argonne National Laboratory Summary Report ANL-6111, February, 1960 (unpublished).

[5]S. Harris, reported at the Symposium on Mössbauer Effect, Allerton House, University of Illinois, June 6 and 7, 1960 (unpublished).

[6]G. K. Wertheim, Phys. Rev. Letters 4, 403 (1960).

[7]This idea was originally suggested by S. DeBenedetti.

[8]R. E. Green and R. E. Bell, Nuclear Instr. 3, 127 (1958).

[9]W. Lamb, Jr. (private communication).

DOUBLE PION PHOTOPRODUCTION*

M. Feldman,† V. Highland, J. W. DeWire,‡ and R. M. Littauer
Laboratory of Nuclear Studies, Cornell University, Ithaca, New York
(Received October 7, 1960)

The photoproduction of two pions from hydrogen is interesting not only from the viewpoint of the isobar model[1] and pion-nucleon forces but for the study of pion-pion forces.[2] The possible reactions are

$$\gamma + p \rightarrow p + \pi^0 + \pi^0, \quad (1)$$

$$\gamma + p \rightarrow p + \pi^+ + \pi^-, \quad (2)$$

$$\gamma + p \rightarrow n + \pi^+ + \pi^0. \quad (3)$$

Various workers[3] have looked at the π^- from (2). Chasan et al.[4] observed all the particles from (2) in a cloud chamber. Recently Silverman and Richert[5] observed the recoil proton and one of the π's from (1) and (2).

We have used the bremsstrahlung beam of the Cornell synchrotron to observe (1) and (3) by detecting the two π's in coincidence. A photon from the decay of a π^0 enters the totally absorbing lead glass Čerenkov counter C (Fig. 1), producing a pulse proportional to its energy. An anticoincidence counter, A, prevents charged particles from being counted. Because the π^0 decays into a spectrum of photons, there is no sharp energy threshold for its detection. With a 150-Mev bias on the photon energy, the efficiency for detecting

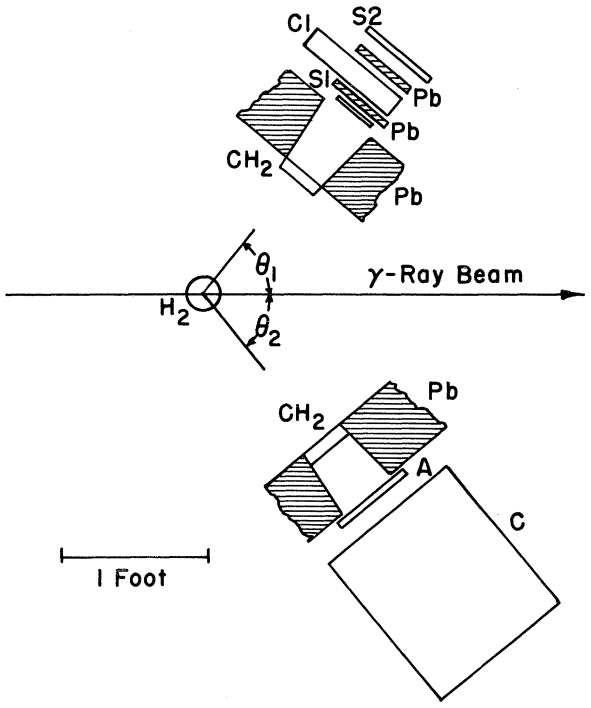

FIG. 1. Experimental arrangement.