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## THEORY OF HOT-ELECTRON EFFECTS IN MANY-VALLEY SEMICONDUCTORS IN THE REGION OF HIGH ELECTRIC FIELD

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(Received September 26, 1960; revised manuscript received October 10, 1960)

The quantitative description of hot-electron effects in many-valley semiconductors is obtained by means of the solution of a properly formulated Boltzmann equation for high fields. A proper formulation implies that one takes into account the actual band structure and the actual scattering mechanisms (acoustical, optical, and intervalley scattering) in terms of the deformation potential theory.

To solve this equation, one defines for convenience the effective momenta $\vec{p}^*$ and the effective electric fields $\vec{F}^*$ for each valley by

$$p_x^* = p_x, \quad p_y^* = p_y, \quad p_z^* = (m_t/m_l)^{1/2} p_z;$$

$$F_x^* = F_x, \quad F_y^* = F_y; \quad F_z^* = (m_t/m_l)^{1/2} F_z,$$

where $z$ is the longitudinal axis of the valley under consideration.

A detailed analysis of the case of $n$-type Ge has shown that one may write the distribution function for electrons in the valley $i$ as

$$f^{(i)}(p^*, \theta) = f_0^{(i)}(\epsilon) + f_1^{(i)}(\epsilon) P_1(\cos\theta). \tag{1}$$

Here $\epsilon$ is the energy of the electron, and $\theta$ is the angle between the directions of the effective field and the effective momentum.

In this note we neglect intervalley scattering and electron-electron collisions. We restrict ourselves to a range of mean energies of the electrons so that the following conditions are met: The mean energies of the electrons are much larger than $\hbar\omega_{\text{opt}}$. Momentum relaxation of the electrons is due to acoustical and optical scattering; energy relaxation is due only to optical scattering. We then find the following results:

1. $f_0^{(i)}(\epsilon)$ is a Maxwellian distribution over most of the energy range with only slight deviations for $\epsilon < 2\hbar\omega_{\text{opt}}$ and $\epsilon > 100kT_{\text{latt}}$.[1] The electron temperature

$$T_e^{(i)} = (eh^3 \rho c_l F^{*(i)})^2 / (96\pi^4 k^2 m_t^3 m_l \Xi_{\text{av}}^2 D^2 T^*), \tag{2}$$

where

$$T^* = T_{\text{latt}} + [hc_l(2n_{q\,\text{opt}} + 1)D^2/(4\pi k\omega_{\text{opt}} \Xi_{\text{av}}^2)],$$

is in general different for different valleys. It is proportional to the energy supplied by the electric field $[\sim(F^{*(i)})^2]$ and inversely proportional to the losses in energy and momentum, which depend on the lattice temperature and the deformation potential constants.

2. The drift velocity is given by

$$v_d = hc_l DS/[\pi^{3/2}(3m_t kT^*)^{1/2} \Xi_{\text{av}}], \tag{3}$$

and is independent of the electric field. $S$ is a slowly varying function of the orientation of the sample. For a Sasaki type of experiment[2] [drift velocity in a (110) plane] $S$ varies between 0.83 and 0.76.

3. The optical deformation potential constant can be evaluated from measurements of the saturation drift velocity using (3). For the evaluation a mean value of $S = 0.80$ and $\Xi_{\text{av}} = 11$ ev (calculated with $\Xi_u = 17$ ev,[3] $\Xi_d = -5.8$ ev[4]) has been used.

The result is shown in Table I. (Koenig's values of $v_d$ are calculated from measurements of the saturation current density.)

The values of $D$ are consistent with the analysis of mobility data by Brooks.[5] They are of the same order of magnitude as a value given by Meyer,[6] $D = 1.15 \times 10^9$ ev/cm, which was extracted from experimental data on infrared ab-

Table I. Evaluation of the optical deformation potential constant $D$ from measurements of the saturation drift velocity $v_d$ at different lattice temperatures.

| Author | $T_{latt}$ | $v_d$ (cm/sec) | $D$ (ev/cm) |
|---|---|---|---|
| Ryder[a] | 77 | $0.9\times10^7$ | $0.37\times10^9$ |
| | 193 | $0.76\times10^7$ | $0.49\times10^9$ |
| | 298 | $0.62\times10^7$ | $0.49\times10^9$ |
| Gunn[b] | 300 | $0.55\times10^7$ | $0.43\times10^9$ |
| Koenig[c] | 80 | $1.08\times10^7$ | $0.47\times10^9$ |
| | 300 | $0.60\times10^7$ | $0.47\times10^9$ |
| Koenig et al.[d] $\langle100\rangle$ sample | 297 | $0.67\times10^7$ | $0.51\times10^9$ |

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sorption by free carriers obtained by Fan, Spitzer, and Collins.[7]

4. The calculated anisotropy of hot electrons in a Sasaki type of experiment is shown in Fig. 1.

The values of $\tan\psi$ are much higher than those calculated by Shibuya[8] but they are still too low compared with measurements of Koenig[9,10] and Schmidt-Tiedemann.[11] The deviations between this theory and the experiments can be accounted for by intervalley scattering (to be treated in a shortly forthcoming note).

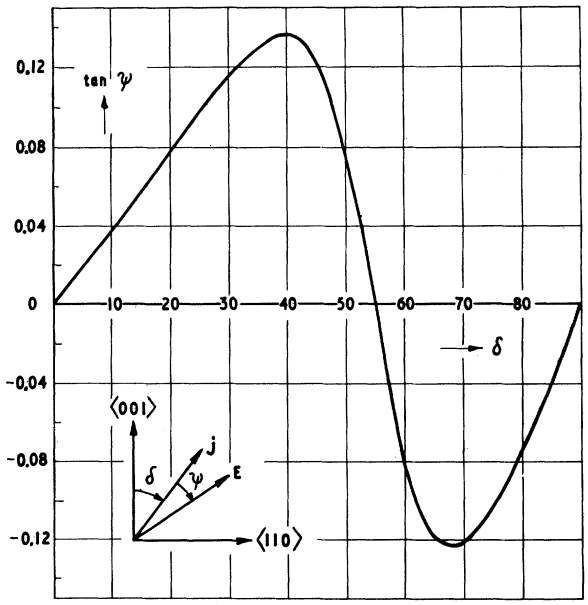


FIG. 1. Anisotropy of hot electrons.

5. In order to estimate the limits of applicability of the results, we shall now briefly outline our calculations. For mean energies of the electrons much larger than the energy of an optical phonon, we retain only optical terms in the energy relaxation and expand all optical scattering terms in the Boltzmann equation linearly in $\hbar\omega_{opt}/\epsilon$. $f_0^{(i)}$ and $f_1^{(i)}$ are then determined by the equations:

$$\epsilon df_1^{(i)}/d\epsilon + f_1^{(i)} = -\alpha_0^{(i)}(\epsilon df_0^{(i)}/d\epsilon + f_0^{(i)}), \qquad (4)$$

$$df_0^{(i)}/d\epsilon = \alpha_1^{(i)} f_1^{(i)}, \qquad (5)$$

$$\alpha_0^{(i)} = 6\pi m_t^{3/2} m_l^{1/2} D^2/(eh^2\rho F^{*(i)}),$$

$$\alpha_1^{(i)} = 16\pi^3 k m_t^{3/2} m_l^{1/2} \Xi_{av}^2 T^*/(eh^4\rho c_l^2 F^{*(i)}).$$

The usual elimination procedure leads to the Maxwellian type of $f_0^{(i)}$ with the electron temperature (2); use of (5) gives (3). The approximations made in deriving (4) and (5) are justified for electron temperatures of 1500°K and higher. The assumption that energy loss is due only to optical scattering is true for $T_{el} < 100 T_{latt}$.[1] Equation (2) gives thus a relation between effective fields and lattice temperatures, determining the range of experimental conditions for the applicability of the theory.

6. For electron temperatures below 1500°K one has to retain quadratic terms in $\hbar\omega_{opt}/\epsilon$ in the expansion of optical scattering terms in the Boltzmann equation. We then find a new Maxwellian distribution as solution for $f_0^{(i)}$ with $T_e^{(i)}$ replaced by $T_e^{(i)}[1+(\hbar\omega_{opt}/2kT_e^{(i)})(2n_{q\,opt}+1)]$.

The mean effective momentum $\bar{p}^{*(i)}$, which before was independent and equal to $\bar{p}_{sat}^{*(i)}$, now becomes field dependent:

$$\bar{p}^{*(i)} = \bar{p}_{sat}^{*(i)}\left(1 - \frac{\hbar\omega_{opt}}{4kT_e^{(i)}}(2n_{q\,opt}+1)\right).$$

The drift velocity also becomes field dependent and the anisotropy is lowered.

We thank Professor D. Polder, Dr. H. J. G. Meyer (Eindhoven), Professor W. Franz, and Dr. K. J. Schmidt-Tiedemann (Hamburg) for

helpful and stimulating discussions.

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## RESONANT SPIN-SPIN INTERACTION BETWEEN DONORS AND ACCEPTORS IN SILICON

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(Received September 27, 1960; revised manuscript received October 11, 1960)

A reduction of the direct relaxation time of donor electrons in silicon, believed to be due to a resonant spin-spin interaction[1] with a background acceptor resonance line, has been observed in compensated silicon containing[2] approximately $5\times10^{15}$ phosphorus donors/cm$^3$ and approximately $3\times10^{15}$ boron acceptors/cm$^3$. Observations were made in both strained and unstrained samples after populating all the donor and acceptor states with optical radiation at liquid helium temperature.[2,3] Two effects were observed: The first was a reduction, after irradiation, of the relative amplitude of one of the donor hyperfine components with respect to the other; the second was a reduction of relative amplitude of both donor hyperfine components with respect to the amplitude before irradiation. Results were obtained by studying the decay of magnetization at the resonant mixing field using the electron spin resonance signal as a monitor.

The procedure for each run was as follows: (1) With rf power turned off, magnetize the sample at a high field (9000 oersteds) long enough to come to thermal equilibrium at this field; (2) reduce the field to $H_{\mathrm{mix}}$ and leave it there for a specified length of time; (3) turn on rf power and sweep through resonance to check how much the magnetization had decayed while at the mixing field. Figure 1 shows recordings of the phosphorus resonance[4] with a calibrating marker of conduction electrons added (0.5 mg of silicon containing $\approx3\times10^{18}$ P/cm$^3$). *a*, *b*, and *c* constitute a series of runs taken before irradiation; *d*, *e*, and *f* are a series taken after irradiation. The top row is for no time in the mixing field and is a measure of the equilibrium magnetization at 9000 oersteds. The increased donor concentration after irradiation is to be noted.

Data taken before irradiation show no difference between the two hyperfine components of the phosphorus donor resonance, and the relaxation time indicated by successive traces is in substantial agreement with previously reported[5] work on donors in silicon without compensation (~500 seconds).

Data taken after irradiation with the sample unstrained (middle row of Fig. 1) indicate that the $m_I=-1/2$ component is relaxing about 50% faster than the $m_I=+1/2$ component. According to Feher's[1] description of the resonant spin-spin interaction, this indicates an overlap with a background resonance line at the mixing field ($\approx$80 oersteds for this case). Since this type of behavior does not occur before light populates the acceptor states, it is concluded that the acceptor resonance is responsible.

Comparison with data before irradiation also indicates that the $m_I=+1/2$ component is relaxing about twice as fast as it was before, showing that the background acceptor line is broad enough to overlap both hyperfine components; however, the $m_I=-1/2$ component is closer to the center. Since the sample was under no externally applied uniaxial strain, the width of the acceptor line[6] would be expected to be quite large. As pointed out by Feher,[1] the sensitivity of this technique makes it possible to observe relatively low concentrations of species with broad resonances. In this case the observation of a line perhaps several hundred oersteds wide