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INSTABILITY OF A POSITIVE COLUMN IN A MAGNETIC FIELD*

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The particle and energy losses from a long positive column of a helium glow discharge were shown by Lehnert[1] to decrease with increasing longitudinal magnetic field as predicted by ambipolar diffusion theory, but only for fields less than a critical field, B_c. Above B_c the losses increased, leading to speculation about the possibility of "enhanced diffusion." Further measurements by Lehnert and Hoh,[2] and at this laboratory, extended Lehnert's measurements to other gases and discharge tube parameters; in particular, it was shown[3] that at B_c the discharges lose their azimuthal symmetry and appear as constricted, rotating, luminous helices (Fig. 1). In addition to the critical magnetic fields, the wavelengths and oscillation frequencies of such helices have now been measured for a number of discharge conditions. In this Letter the experimental results are compared with the two existing theories[4,5] and are shown to be in reasonable agreement with one of them.[5]

Recently, Hoh[4] and Kadomtsev and Nedospasov[5] have considered the origin of the instability from two different standpoints. Hoh made use of a criterion for the stability of a wall sheath and showed that it is no longer satisfied when the magnetic field exceeds a critical value. The manner in which the failure of the sheath-stability criterion affects the discharge was not discussed. Kadomtsev and Nedospasov, on the other hand, show that perturbations of the discharge column can grow when diffusion to the walls can no longer overcome the effects of $\vec{j}\times\vec{B}$ forces that tend to increase the perturbation. $\vec{j}$ cannot be zero, but the value of B_c obtained does not depend on the magnitude of the current density. In addition to predicting B_c, their theory predicts the frequency, wavelength, and growth rate of the oscillation.

In our experiment, discharge tubes 250 cm long were placed at the axis of a solenoid which could be adjusted in length up to 200 cm. As in references 1 and 2, the discharges were run at constant current, and the axial field was used as a measure of particle losses. These fields were obtained from a number of probes. The time and space variations of the visible light from the dis-

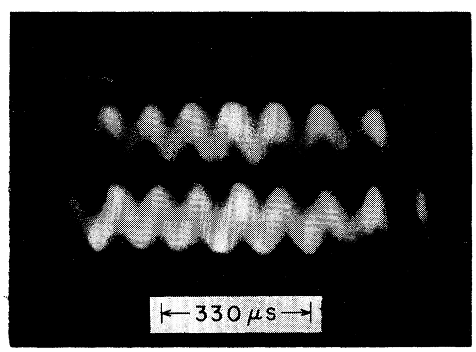

FIG. 1. 90-degree stereo streak photograph of helical constriction in helium. $R = 2.75$ cm, $p = 0.23$ mm Hg, $B = 710$ gauss. Traces show variation of light intensity with time and radius as seen through a slit perpendicular to axis of discharge tube. Note that this is not a photograph of the helix; the helicity has been established by wavelength measurements with photomultipliers.

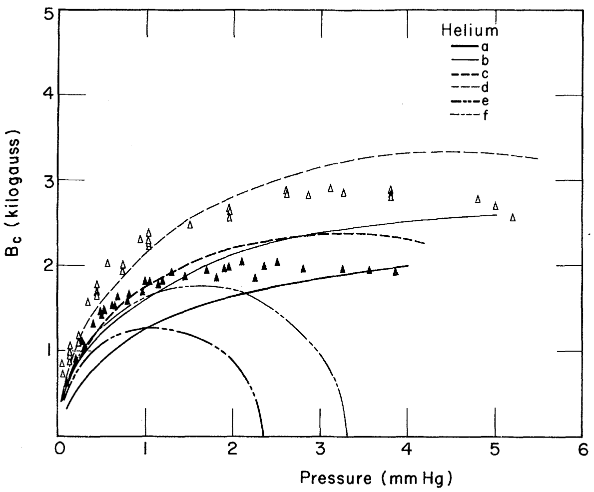

FIG. 2. The critical magnetic field, B_c, vs gas pressure. ▲: $R = 1.27$ cm, $I = 400$ ma; Δ: $R = 0.9$ cm, $I = 200$ ma. (a) $R = 1.27$ cm and (b) $R = 0.9$ cm, calculated from Kadomtsev's theory. (c) $R = 1.27$ cm and (d) $R = 0.9$ cm, calculated from Hoh's theory, assuming only He_2^+ ions present. (e) $R = 1.27$ cm and (f) $R = 0.9$ cm, from Hoh's theory, assuming only He^+ ions present.

charge were photographically recorded from a rotating mirror and a series of well-collimated photomultipliers. From the measurements it was possible to obtain data on the critical magnetic fields and the oscillation frequencies, wavelengths, and growth rates for H_2, D_2, He, Ne, and A in tubes with radii of 0.9, 0.95, 1.27, and 2.75 cm.

Luminous helices of the clarity shown in Fig. 1 were observed only for special combinations of gas, pressure, and tube radius. In general, the appearance was more chaotic, often appearing as moving striations superimposed on the spirals. The pitches of the spirals were such that $\vec{j}\times\vec{B}$ was directed toward tube walls.

Some of the experimental B_c vs p data are shown in Figs. 2 and 3, together with calculations from the formulas[6] of references 4 and 5. Numerical data were taken from the compilation by Brown.[7] Considerable uncertainty exists as to the actual conditions in a positive column, e.g., both atomic and molecular ions are known to be present. The predictions of the sheath theory are not particularly sensitive to temperature, but depend critically on the mobilities; we show calculated curves for two ionic species to illustrate the latter point. On the other hand, the results of the current-channel instability theory depend much more sensitively on temperature than on mobility, B_c varying roughly as T_e for a given pressure. The moving striations that were generally present in our discharges may appreciably affect the electron energy distribu-

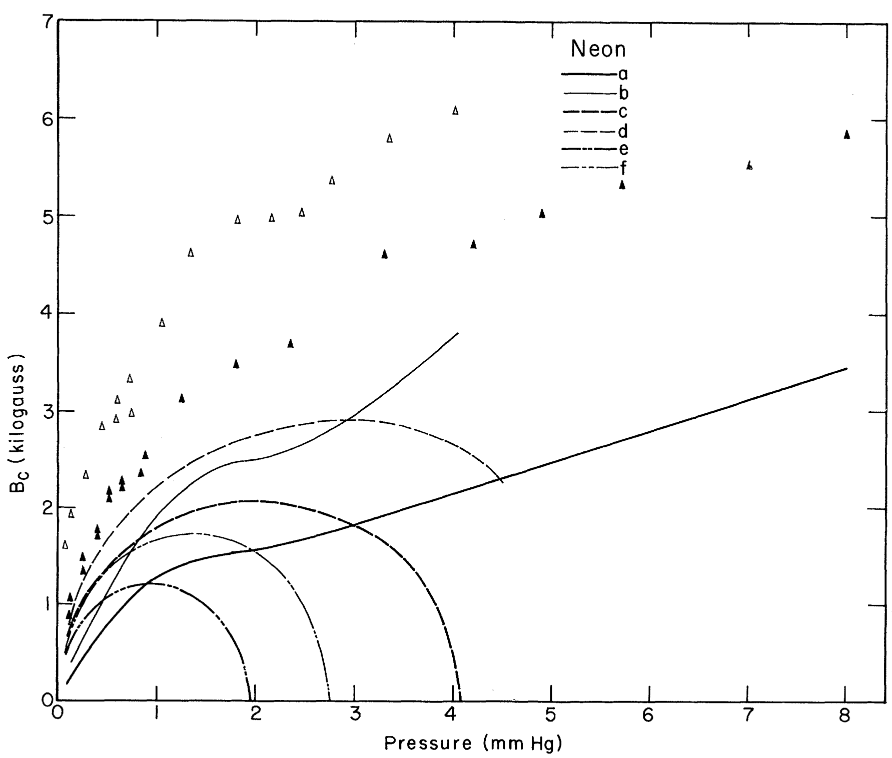

FIG. 3. The critical magnetic field, B_c, vs gas pressure. ▲: $R = 1.27$ cm, $I = 400$ ma; Δ: $R = 0.9$ cm, $I = 200$ ma. (a) $R = 1.27$ cm and (b) $R = 0.9$ cm, calculated from Kadomtsev's theory. (c) $R = 1.27$ cm and (d) $R = 0.9$ cm, calculated from Hoh's theory, assuming only Ne_2^+ ions present. (e) $R = 1.27$ cm and (f) $R = 0.9$ cm, from Hoh's theory, assuming only Ne^+ ions present.

tions, although the B_c vs p curves go smoothly through transitions from unstriated to striated regimes. Similar uncertainties exist for the H_2, D_2, and A discharges.

Where it was possible to obtain oscillation frequencies and wavelengths from the experimental results, these quantities were in good agreement with values obtained from the formulas in reference 5. For example, helium at 0.3 mm Hg pressure in a 2.75-cm radius tube gave an angular frequency of 5×10^4 radians/sec and a wavelength of 73 cm, the calculated values being 4.3×10^4 and 63, respectively.[8] In addition, the frequencies increased with decreasing radius and decreasing pressure, as predicted in Eq. (22) of reference 5. Growth rates were difficult to determine experimentally, but showed agreement in order of magnitude.

We conclude that with the proper choice of parameters either theory can approximate the experimental B_c vs p curves, although the shapes of the latter are perhaps most like the results of Kadomtsev and Nedospasov. Their theory also yields perturbation frequencies and wavelengths in good agreement with experiment.

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[1]B. Lehnert, Proceedings of the Second International Conference on the Peaceful Uses of Atomic Energy (United Nations, Geneva, 1958), Vol. 32, p. 349.

[2]B. Lehnert and F. C. Hoh, Phys. Fluids 3, 600 (1960).

[3]T. K. Allen, G. A. Paulikas, and R. V. Pyle, Bull. Am. Phys. Soc. 5, 314 (1960).

[4]F. C. Hoh, Phys. Rev. Letters 4, 559 (1960).

[5]B. B. Kadomtsev and A. V. Nedospasov, J. Nuclear Energy Part C 1, 230 (1960). We are grateful to the authors for a prepublication copy of this paper.

[6]Equation (4) of reference 4 has apparently been misprinted; the second quantity in brackets should read $(g'p_0\lambda/R - p_0^2)^{1/2}$.

[7]S. C. Brown, Basic Data of Plasma Physics (John Wiley & Sons, Inc., New York, 1959).

[8]The experimental frequencies and wavelengths were obtained from the fully developed, steady-state helix. It is not obvious that they should be the same as the quantities calculated from the small-amplitude theory. The amplitude dependence might be examined experimentally by using pulsed magnetic fields.

APPROACH TO EQUILIBRIUM IN QUANTUM SYSTEMS

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In recent years, Prigogine and his co-workers have developed a general theory of approach to equilibrium in classical mechanics, using a perturbation technique on the Liouville equation.[1-3] We want to indicate here how it is possible to get similar results in quantum systems, using a representation for the density matrix, introduced first by Prigogine and Ono,[4] which stresses the formal analogy between the von Neumann equation and the classical Liouville equation. This representation has already been used in the particular cases of weakly coupled systems[4,5] and of quantum plasmas.[6]

We shall consider the case of a Fermi gas, but the method is quite general and can be applied to boson-boson or boson-fermion interactions. The Hamiltonian of the system is

$$H = H_0 + \lambda V, \tag{1}$$

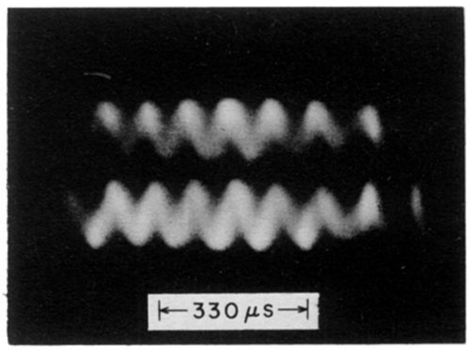
$$H_0 = \sum_k \epsilon_k a_k^\dagger a_k, \tag{2}$$

$$V = \frac{1}{2\Omega} \sum_{klpr} v(klpr) a_k^\dagger a_l^\dagger a_p a_r \delta(k+l-p-r), \tag{3}$$

$$[a_k, a_{k'}^\dagger]_+ = \delta_{kk'}; \quad [a_k, a_{k'}]_+ = [a_k^\dagger, a_{k'}^\dagger]_+ = 0. \tag{4}$$

In the representation of the occupation numbers $|n\rangle$:

$$H_0 |n\rangle = \sum_k \epsilon_k n_k |n\rangle \equiv \epsilon n |n\rangle, \tag{5}$$



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