CONSISTENCY OF THE $K^+ \to \pi^+ + \pi^0 + \gamma$ RATE WITH THE $\Delta T = \frac{1}{2}$ RULE

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It was proposed by Gell-Mann and Pais[1] that in the decays of hyperons and K mesons into strongly interacting particles the isotopic spin change is given by $\Delta T = \frac{1}{2}$ when electromagnetic interactions are neglected. The subsequent detailed investigation of the implications of the rule[2] led to predictions which seem now to be satisfied by the most recent data.[3]

For $K^+ \to \pi^+ + \pi^0$ the $\Delta T = \frac{1}{2}$ rule predicts no rate at all in the absence of electromagnetic corrections. Experimentally the $K^+ \to \pi^+ + \pi^0$ rate is found to be almost two hundred times smaller than the rate for $K^0 \to 2\pi$. This result is in the direction of the prediction from the $\Delta T = \frac{1}{2}$ rule. The question then arises of discovering the mechanism which makes the amplitude for $K^+ \to \pi^+ + \pi^0$, arising from electromagnetic corrections of order e^2, large enough to account for the observed rate. Independently of this question it has been pointed out that the amplitude for $K^+ \to \pi^+ + \pi^0 + \gamma$ would be of the order e and might thus compete favorably with the amplitude for $K^+ \to \pi^+ + \pi^0$.[4] Such a possibility would strongly contradict present experimental evidence based on a few $K^+ \to \pi^+ + \pi^0 + \gamma$ events which can be accounted for in terms of the internal bremsstrahlung process accompanying the $K^+ \to \pi^+ + \pi^0$ decay.

In this note we show that there is no contradiction between the observed low frequency of $K^+ \to \pi^+ + \pi^0 + \gamma$ events and the $\Delta T = \frac{1}{2}$ rule. Specifically we show that the total $K^+ \to \pi^+ + \pi^0 + \gamma$ amplitude is not expected to be much larger than the internal bremsstrahlung contribution. This contribution would be of the order e^3, according to the exact $\Delta T = \frac{1}{2}$ rule, but it may thus compete favorably with the direct amplitude of order e. Such a rather unique situation would make very interesting a measurement of the π^+ spectrum of $K^+ \to \pi^+ + \pi^0 + \gamma$.

We shall construct an amplitude for $K^+ \to \pi^+ + \pi^0 + \gamma$ satisfying the requirements of gauge invariance. The total amplitude will consist of an internal bremsstrahlung amplitude (of order e^3 if $\Delta T = \frac{1}{2}$ holds exactly) and of a direct amplitude (of order e). The (necessarily off-shell) amplitude for $K^+ \to \pi^+ + \pi^0$ satisfying $\Delta T = \frac{1}{2}$ can be approximated by retaining only those terms with the lowest dependence on momentum transfer. Such an approximation should be valid if high-mass intermediate states contribute to the decay, as is expected. The form of the amplitude is then uniquely fixed by $\Delta T = \frac{1}{2}$ and corresponds to an effective Lagrangian,[5]

$$\mathcal{L}' = \sqrt{2} f M \frac{\partial \Phi_+}{\partial x_\nu} \left[\varphi_+^* \left(\frac{\overrightarrow{\partial}}{\partial x_\nu} - \frac{\overleftarrow{\partial}}{\partial x_\nu} \right) \varphi_0 \right] + \text{H.c.}, \tag{1}$$

where f is a constant, M is the K mass, Φ_+ is the K^+ field, and φ is the charged π field. The amplitude for $K^+ \to \pi^+ + \pi^0$ on the mass shell (which necessarily violates $\Delta T = \frac{1}{2}$) can be derived from the simplest Lagrangian,

$$\mathcal{L}'' = g M^3 \Phi_+ \varphi_+^* \varphi_0 + \text{H.c.}, \tag{2}$$

on the basis that any structure effect from this vertex would certainly be negligible considering the smallness of the K^+ decay constant g. The current producing the radiative decay $K^+ \to \pi^+ + \pi^0 + \gamma$ includes contributions from the π and K currents and from the transition current derived from (1) through the substitution $\partial/\partial x_\nu \to \partial/\partial x_\nu - ieA_\nu$. The total amplitude is given by

$$\frac{e}{(4\pi)^2} \frac{M}{(ME_0 E_+ E_\gamma)^{1/2}} \epsilon_\mu j_\mu, \tag{3}$$

where

$$j_\mu = \left(\frac{p_\mu^{(+)}}{p_\nu^{(+)} \cdot q_\nu} - \frac{K_\mu}{K_\nu \cdot q_\nu} \right) \times [gM^2 - \sqrt{2} f q_\lambda \cdot (p_\lambda^{(+)} - p_\lambda^{(0)})]. \tag{4}$$

In (3) and (4), E_0, E_+, and E_γ are the energies of the final particles; ϵ_μ is the polarization vector; $p_\mu^{(0)}$, $p_\mu^{(+)}$, K_μ, and q_μ are the momenta of the final pions, of the K meson, and of the photon, respectively. It may be noted that the current (4) does not lead to any magnetic dipole emission; this is a consequence of our approximation which neglects higher structure terms. The pion spectrum from (3) and (4) is given by

$$\frac{1}{w_0} \frac{dw}{d\omega} = \left(\frac{e^2}{4\pi} \right) \frac{2qp}{\pi E \beta} [\mathcal{B} + x\mathcal{g} + x^2 \mathcal{D}], \tag{5}$$

with

$$\mathfrak{B}=\frac{2}{q^2}\left[\frac{\omega}{p}\ln\frac{\omega+p}{\mu}-1\right],$$

$$\mathfrak{I}=\frac{\sqrt{2}}{M^2}\frac{E}{q}\left[6-\frac{\mu^2+2M\omega}{Mp}\ln\frac{M(\omega+p)-\mu^2}{M(\omega-p)-\mu^2}\right.$$
$$\left.-\frac{M-4\omega}{p}\ln\frac{M-\omega+p}{M-\omega-p}\right],$$

$$\mathfrak{D}=\frac{E^2}{M^4}\left[2\frac{\mu^2+M\omega}{Mp}\ln\frac{M(\omega+p)-\mu^2}{M(\omega-p)-\mu^2}\right.$$
$$\left.+4\frac{M-2\omega}{p}\ln\frac{M-\omega+p}{M-\omega-p}-12\right],$$

in the following notation: q is the photon momentum in the (γ,π^0) center-of-mass system, ω and p are the energy and momentum of π^+ in the K rest system, E is the total energy of (γ,π^0) in their center-of-mass system, β is the velocity of π^+ in the nonradiative decay, $x=f/g$, μ is the π mass, and w_0 is the rate for nonradiative decay. The expressions for E, q, and β are

$$E^2=M^2+\mu^2-2M\omega,$$
$$2Eq=M^2-2\omega M,$$
$$M^2\beta^2=M^2-4\mu^2.$$

In (5), $\mathfrak{B}$ is the bremsstrahlung term arising from the small amplitude (2) for $K^+\to\pi^++\pi^0$, $\mathfrak{D}$ arises entirely from the off-mass-shell amplitude (1), and $\mathfrak{I}$ is an interference term. The bremsstrahlung term coincides with that given by Good,[6] but the form of the other terms differs from estimates contained in reference 6 on the basis of dimensional and invariance considerations. In Fig. 1 we report $A\mathfrak{B}$, $Ax\mathfrak{I}$, and $Ax^2\mathfrak{D}$ with $A=qp/E\beta$, as given by (5), and for $x=15$. In the ratio x, the quantity g can be derived from the $K^+\to\pi^++\pi^0$ rate. A simplest model that gives a unique value for f is the following. The $K\to 2\pi$ decays are supposed to proceed through virtual dissociation of the K meson into a nucleon and an antilambda (for both K^+ and K^0); the Λ-decay interaction (subject to $\Delta T=\frac{1}{2}$) contains the derivative of the pion field. It then follows that $K^0\to 2\pi$ comes from an effective Lagrangian

$$fM\frac{\partial\Phi_0}{\partial x_\nu}\left(\vec{\varphi}\cdot\frac{\partial\vec{\varphi}}{\partial x_\nu}\right), \qquad (6)$$

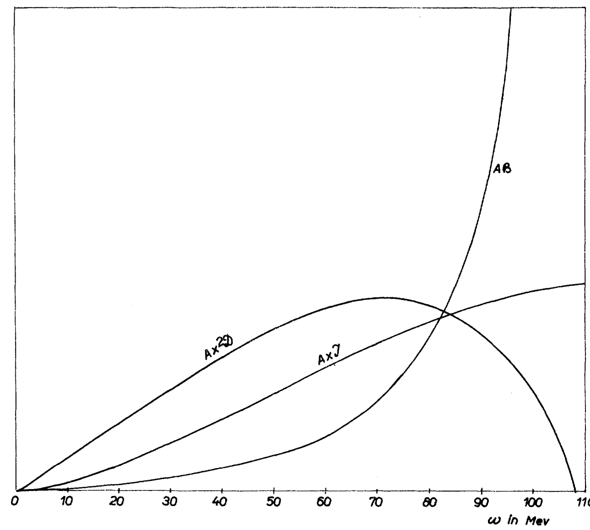

FIG. 1. The different contributions to the π^+ spectrum in $K\to\pi+\pi^0+\gamma$. $A\mathfrak{B}$ is the bremsstrahlung term, $Ax^2\mathfrak{D}$ is the direct term, and $Ax\mathfrak{I}$ is the interference between the two. The value of the parameter x is here chosen to be 15.

with the same f as in (1). The value of f can then be obtained by comparing the expression for the total $K^0\to 2\pi$ rate derived from (6) with its experimental value. In this way one obtains $x=\pm 15$. According to the choice of the sign, one finds that the number of $K^+\to\pi^++\pi^0+\gamma$ events in the best explored region of 55-75 Mev is bigger by a factor 1.6 (for negative interference term) or 5 (for positive interference) than the number predicted on the basis of internal bremsstrahlung alone (i.e., 1 event for the 8653 decays examined, among which two anomalous decays were reported). We consider our conclusion to be in agreement with experiment also, in view of our ambiguous determination of f. For instance, if our model to determine f is extended to include Σ particles, then one can derive in perturbation theory, assuming special forms for the hyperon decay Lagrangians,[7] cutoff-independent relations between f and the K^{0}-decay constant of (6). Such relations do not give unique results, however, because of the possibility of varying some signs in the hyperon decay Lagrangians.[8]

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[1]M. Gell-Mann and A. Pais, Proceedings of the 1954 Glasgow Conference on Nuclear and Meson Physics (Pergamon Press, New York, 1955).

[2]R. Gatto, Nuovo cimento 3, 318 (1956); G. Wentzel, Phys. Rev. 101, 1214 (1956).

[3]See F. S. Crawford et al., Phys. Rev. Letters 2, 266 (1959).

[4]M. Gell-Mann, Nuovo cimento 5, 758 (1957).

[5]We report briefly the argument. The rule $\Delta T=\frac{1}{2}$ implies that the final pions appear in the combinations

$$\epsilon_{ikl}\varphi^{(k)}\varphi^{(l)}, \quad \epsilon_{ikl}\frac{\partial\varphi^{(k)}}{\partial x_\nu}\frac{\partial\varphi^{(l)}}{\partial x_\nu},$$

$$\epsilon_{ikl}\frac{\partial\varphi^{(k)}}{\partial x_\nu}\varphi^{(l)}, \text{ or } \epsilon_{ikl}\varphi^{(k)}\frac{\partial\varphi^{(l)}}{\partial x_\nu}.$$

Of such expressions only the last two differ from zero and they are equal apart from sign.

[6]J. D. Good, Phys. Rev. 113, 352 (1959).

[7]O. Hori, Nuclear Phys. 17, 227 (1960).

[8]Particular models including Σ particles are being investigated by Dr. Bassetti, whom we wish to thank for useful discussions.

ESTIMATE OF THE NEUTRAL π-MESON LIFETIME

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The lifetime of the π^0 meson may in principle be determined from an analysis of K^+-meson decays at rest in nuclear emulsion. Events are selected in which the $K_{\pi 2}$ mode, $K^+\rightarrow\pi^+ +\pi^0$, is followed by the decay of the π^0 meson to give a direct electron pair,[1] $\pi^0\rightarrow e^+ +e^- +\gamma$. Both the velocity and the direction of flight of the π^0 mesons are determined by the kinematics of the K-meson decay and the presence of the direct pair allows the point of decay of the π^0 to be found.

Harris, Orear, and Taylor[2] first proposed this method of estimating the π^0 lifetime, and using 12 events found in Ilford $G5$ nuclear emulsion stacks were able to set an upper limit to the lifetime at $\sim 7\times10^{-16}$ second. In the present experiment Ilford $L4$ emulsions of smaller grain size (~ 0.3 μ rather than the 0.6 μ typical of $G5$) have been used in an attempt to increase the spatial resolution of the method. The preliminary results, based on measurement of 26 events, suggest a finite mean lifetime $\sim 3\times10^{-16}$ second.

Thirty examples of K^+-meson decay accompanied by a direct electron pair have been found in a total of 27 000 K^+-meson endings examined. Measurements of the ionization of the secondaries have shown 2 events to be τ' or $K_{\mu 3}$ decays; there remains a possible 12 % contamination due to decay by the $K_{\mu 3}$ and K_{e3} modes but the effect on the result is small.

The measurements have been performed using a projection apparatus to form an image of the event on a screen at a magnification ~ 7500. Each grain in the tracks of the π^+ meson, the electron pair, and the K-meson ending was then traced. The length of track used for the meson and electrons was ~ 30 μ. From the drawing, the coordinates of the center points of each grain were taken and this information was used to compute best-fit lines to the π^+ and electron tracks by minimizing $\sum h_i^2$, where h_i is the perpendicular distance from a grain center to the fitted line. This calculation was performed on the Oxford University Ferranti "Mercury" digital computer. At the same time the program computed the distance d between A, the foot of the perpendicular from the center of the last K-track grain to the π^+ line (the estimated origin of the π^0), and the point B, the intersection of the π^+ and one of the electron lines (the estimated point of decay); this calculation is illustrated by Fig. 1.

If the π^+-meson track has a dip angle α, the π^0-decay path is then $x=d\sec\alpha$. In those cases where both electron tracks of the pair were used, the estimate of d was obtained by taking an average using $1/\csc^2\theta$ as a weighting factor. Three drawings were made for each event and an average value used for the result.

This method of determining the mean decay

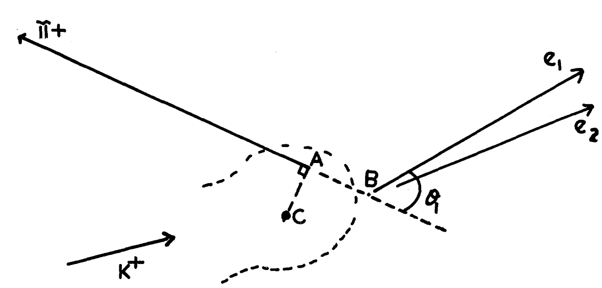

FIG. 1. Diagram showing the procedure for estimating the projected flight path of the π^0 meson. C is the center of the last grain in the K-meson track which is indicated by an irregular broken line.