RANGE OF PROTON-ANTIPROTON ANNIHILATION*

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Experimental data on elastic and inelastic scattering cross sections of antiprotons in hydrogen are now available up to energies of the order of several Bev.[1] Although the total p-$\bar{p}$ cross section is decreasing slowly when the energy increases, it still remains, at 2 Bev, of the order of twice the p-p cross section. Preliminary data[2] obtained at CERN tend to indicate that the limit of equality of the two cross sections which is predicted by the theorem of Pomeranchuk[3] is still not reached at 10.7 Bev. This can be taken as an indication that the pure annihilation process still remains fairly important at that high energy.[4]

Several theoretical investigations have been made on the p-$\bar{p}$ interaction, either phenomenologically[5] or on the basis of meson theory.[6-7] One of the most puzzling questions is the determination of the range of the annihilation region. Following familiar arguments of meson theory, this range is expected to be of the order of twice the nucleon Compton wavelength ($\sim 0.5\times10^{-13}$ cm, which is also the radius of the repulsive core in the nucleon-nucleon interaction) rather than of the order of the pion Compton wavelength. Since one cannot expect that the validity of a potential model or of the static meson field theory can be extended to the multi-Bev energy region we would like to present arguments based qualitatively on semiclassical concepts. In particular, a "maximum theorem" has been demonstrated[8] which relates the total and elastic cross sections to the maximum number of partial waves $(L+1)$ which contribute to the scattering process. The theorem can be expressed as follows:

$$(\sigma_{\rm tot})^2/\sigma_{\rm el} \leqslant 4\pi\lambda\!\!\!^{-}{}^2(L+1)^2; \tag{1}$$

it is obtained by making all the phase shifts purely imaginary:

$$\delta_l = i\beta_l, \quad (l \leqslant L) \tag{2}$$

and varying the real parameter β_l so that $(\sigma_{\rm tot})^2/\sigma_{\rm el}$ is maximum. In other words, the maximum theorem is obtained when the interaction takes place in a purely absorbing region, the elastic contribution coming solely from shadow scattering; $L+1$ is therefore the number of partial waves which contribute to the inelastic process.

Now, if we consider all the experimental data between 200 Mev and 2 Bev, we find that, within 20% (which is certainly of the order of the combined experimental errors), the ratio $(\sigma_{\rm tot})^2/\sigma_{\rm el}$ remains practically constant and equal approximately to 300 mb. On the other hand, $(L+1)\lambda\!\!\!^{-}$ is roughly the range R of the interaction region.[9] Inequality (1) can then be understood as meaning that the interaction region remains approximately constant in the above energy range, and that 300 mb $\leqslant 4\pi R^2$, or

$$R \geqslant 1.5\times10^{-13} \text{ cm}. \tag{3}$$

One could object to the above argument that, if there is an attractive real potential outside the annihilation region, particles of high angular momenta which strike the target well outside that region will be curved in and annihilated, so that the number given by Eq. (3) is more nearly equal to the range of the real scattering potential. This argument, however, does not hold if L is differently large, because the centrifugal barrier will prevent any such effect. The calculations of Ball and Chew,[6] for example, show that beyond $l=2$ the centrifugal barrier dominates the meson-theoretical scattering potential. Actually, if there is an attractive potential $V(r)$, classical arguments show that the minimum radius of interaction will be given by solving the equation:

$$R = \frac{(L+1)\lambda\!\!\!^{-}}{1+|V(R)|/E}. \tag{4}$$

However, if there is an attractive real potential, the maximum theorem (1) will be modified, since the scattering phase shifts can no longer be purely imaginary. But this is merely a second order effect, as can be seen by the following calculation. We suppose that the scattering phase shifts contain a real part α <u>which is independent</u> of l:

$$\delta_l = i\beta_l + \alpha, \tag{5}$$

and we vary β_l, keeping α fixed, so that $(\sigma_{\rm tot})^2/\sigma_{\rm el}$ is maximum. With the simplifying assumption on α, the partial wave contributions can again be summed, and we obtain the following result:

$$\frac{(\sigma_{\text{tot}})^2}{\sigma_{\text{el}}} \leq \frac{4\pi\lambda\!\!\!^{-2}(L+1)^2 \cos^2\alpha}{1+\frac{4\pi\lambda\!\!\!^{-2}(L+1)^2}{\sigma_{\text{tot}}}\sin^2\alpha\left[\frac{4\pi\lambda\!\!\!^{-2}(L+1)^2}{4\sigma_{\text{tot}}}-1\right]}, \qquad (6)$$

which is a generalization of Eq. (1) to the case where phase shifts are not purely imaginary. In the limit of small α, this gives

$$(\sigma_{\text{tot}})^2/\sigma_{\text{el}} \leq 4\pi\lambda\!\!\!^{-2}(L+1)^2\left[1-\left(1-\frac{2}{t}\right)^2\alpha^2\right], \qquad (7)$$

where we have put $t=\sigma_{\text{tot}}/\pi\lambda\!\!\!^{-2}(L+1)^2$. [Note that if one allows α to vary, the maximum of the right-hand side of Eq. (6) is obtained by putting $\sin^2\alpha=0$, which gives back Eq. (1).]

Now, if we assume that $V(r)$ is approximately constant for $r \geq R$ and if we replace the inequality of Eq. (1) by an approximate equality, we obtain, by combining Eqs. (1) and (4),

$$(\sigma_{\text{tot}})^2/\sigma_{\text{el}} \simeq 4\pi R^2\left(1+\frac{|V|}{E}\right),$$

valid to the first order in V. This linear dependence in $1/E$ agrees better with experiment than our previous assumption of a constant value of $(\sigma_{\text{tot}})^2/\sigma_{\text{el}}$. One finds that the experimental data can be fitted quite well by taking $R \simeq 1.43\times10^{-13}$ cm and $|V| \simeq 38$ Mev, which are rather reasonable values.

In order to understand the meaning of this result, one has to keep in mind that there can be two different contributions to the inelastic cross section: either the truly inelastic scattering with multiple production of pions, or the pure p-$\bar{p}$ annihilation. If the first process is dominant, then it is not surprising that the range which we have obtained should be of the order of the pion Compton wavelength. If, however, the main contribution comes from the second process, then we are facing again the puzzling result which we have indicated previously, namely that the radius of the annihilation region is of the order of $\hbar/m_\pi c$. There are, at present, some indications, which are however not at all conclusive, that it is the latter hypothesis which seems to be verified.

Preliminary data obtained at CERN on the total p-$\bar{p}$ cross section[2] give a value of 55 ± 2 mb at 10.7 Bev. If our arguments were to remain valid up to that energy, one should expect then an elastic scattering cross section of the order of 12 ± 4 mb. This appears reasonable, since one expects that the p-$\bar{p}$ and p-p elastic scattering cross sections should be approximately equal at high energy, because they both reduce to diffraction scattering. If one assumes also that the pure annihilation cross section at very high energy is roughly equal to the difference between the p-$\bar{p}$ and p-p total cross sections, one obtains for it about 15 mb. There remain 30 mb for the inelastic p-$\bar{p}$ scattering, so that the ratio of this process to pure annihilation should be approximately 2 to 1 at that energy.

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[4]See a discussion of this effect in M. Lévy, Report to the Tenth Annual Rochester Conference on High-Energy Nuclear Physics (to be published).

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[7]M. Lévy, Nuovo cimento 8, 92 (1958); see also J. Mendelbrojt (to be published).

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[9]The well-known classical arguments show that particles of angular momentum L strike the target in a region contained between two circles of radii $L\lambda\!\!\!^{-}$ and $(L+1)\lambda\!\!\!^{-}$. In the absence of an external attractive potential, quantum mechanics predicts that the minimum radius, due to the centrifugal force, is equal to $[L(L+1)]^{1/2}\lambda\!\!\!^{-}$. All these arguments should be understood as mainly valid when L is large.