penetration factor, for the scattering of charged particles in the presence of another long-range potential. This would, for example, be the case for the scattering of protons by a nucleus, though the coefficients of these new terms might be quite small since they are proportional to the polarizability of the nucleus. New terms will also appear in nucleon-nucleon scattering due to the long-range magnetic interactions; for $n$-$p$ or $p$-$p$ forces it is clear from our previous discussion that there will also be additional (though generally very small) terms due to the nucleon polarizabilities.

A detailed report is in preparation. This report will also include some applications, with emphasis on electron scattering by hydrogen.

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[4]T. Ohmura, Y. Hara, and T. Yamanouchi, Progr. Theoret. Phys. (Kyoto) 22, 152 (1959); 20, 80 (1958); T. Ohmura and H. Ohmura, Phys. Rev. 118, 154 (1960).
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[6]T. Ohmura and H. Ohmura, Phys. Rev. (to be published).
[7]R. M. Thaler, Phys. Rev. 114, 827 (1959).
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# SINGLE-VIRTUAL-BOSON-EXCHANGE INTERACTION IN HIGH-ENERGY COLLISIONS*

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Recent experimental evidence points towards the importance of a long-range interaction, i.e., $\sim m_\pi^{-1}$, in high-energy collisions of $\pi$ mesons with nucleons and of nucleons with nucleons.[1] A natural model to investigate is that in which the two incident particles collide by exchanging a single virtual $\pi$ meson. In this note it is shown that if these graphs dominate in collisions in which the square of the virtual pion's four-momentum is very small, then a simple field-theoretical model of such collisions results.

Because the kinematics and field theory discussed in this note are not restricted only to processes involving $\pi$ mesons and nucleons, we consider a general inelastic two-particle collision as shown in Fig. 1.[2] Particles $A$ and $A'$, with four-momenta $p_i$ and $p_i'$, exchange a boson $B$ with four-momentum $\Delta_i$, and two groups of particles emerge: $C$, with total four-momentum $P$, and $C'$, with total four-momentum $P'$. $A$ and $A'$ have rest masses $w$ and $w'$,

$$p_i^2 = -w^2 \text{ and } p_i'^2 = -w'^2,$$

and the "rest masses" $W$ and $W'$ of $C$ and $C'$ are defined by

$$P^2 = -W^2 \text{ and } P'^2 = -W'^2.$$

The over-all barycentric coordinate system ($U$) is defined by

$$\vec{p}_{iU} + \vec{p}_{iU}' = 0,$$

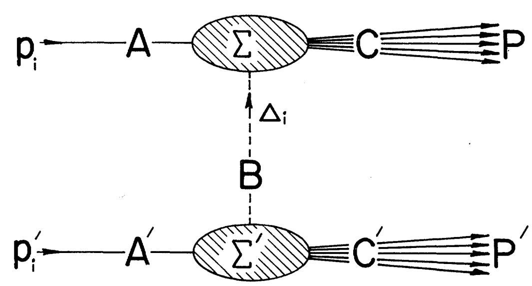


FIG. 1. A general binary inelastic collision in which particles $A$ and $A'$, with four-momenta $p_i$ and $p_i'$, interact by the exchange of a single boson $B$, with four-momentum $\Delta_i$, leading to two groups of final state particles: $C$, with total four-momentum $P$, and $C'$, with total four-momentum $P'$.

and the "scattering angle" $\theta$ in $(U)$ is defined by

$$\vec{p}_{iU}\cdot\vec{P}_U = p_{iU}P_U \cos\theta.$$

From conservation of four-momentum at each of the two "vertices," $\Sigma$ and $\Sigma'$, one obtains

$$\Delta^2 \equiv \Delta_i^{\,2} = [\Delta^2(U, w, w', W, W')]_{\min} + 4p_{iU}P_U \sin^2(\theta/2), \quad (1)$$

where $U$, the total energy in the system $(U)$, is defined by

$$(p_i + p_i')^2 = -U^2.$$

This is but a slight generalization of the corresponding equation for elastic scattering, for which case $W = w$, $W' = w'$, and $(\Delta^2)_{\min} = 0$. For the case that $w' = w$, e.g., proton-proton collisions, one obtains

$$(\Delta^2)_{\min} U^2 \approx (W^2 - w^2)(W'^2 - w^2), \quad (2)$$

provided that each of $W^2$, $W'^2$, $w^2$, and $(\Delta^2)_{\min}$ is much less than $U^2$. One finds:

(1) $\Delta^2$ may be small in collisions in which a number of additional particles are produced. If, for example, $A$ is a nucleon with laboratory energy $10^4$ Bev and $A'$ a nucleon at rest, so that $U \approx 140$ Bev, then $\Delta^2 = m_\pi^{\,2}$ can occur with $W = W' \approx 4.5$ Bev for $\theta = 0$.

(2) $\Delta^2$ is small only for small "scattering angles." In the example just mentioned, as $\theta$ increases from 0° to 1°, $\Delta^2$ increases from $m_\pi^{\,2}$ to about 80 $m_\pi^{\,2}$.

The main field-theoretical simplification results from the fact, which may at first appear surprising, that for small values of $\Delta^2$ the virtual boson behaves kinematically as an incoming, almost real, particle, both in its "collision" with $A$ as seen in the barycentric system $(W)$ of the group of particles $C$, and in its "collision" with $A'$ as seen in the barycentric system $(W')$ of the group of particles $C'$.[2] It is incoming in the sense that it carries positive energy into the vertex. This may be seen by evaluating the two equations,

$$(P - \Delta_i)^2 = p_i^{\,2} \quad \text{and} \quad (P' + \Delta_i)^2 = p_i'^2,$$

in the $(W)$ and $(W')$ coordinate systems, respectively. One obtains for the energy components of $\Delta_i$ in the $(W)$ and $(W')$ systems,

$$\omega_{\Delta W} = (W^2 - w^2 - \Delta^2)/(2W),$$

and

$$\omega_{\Delta W'} = -(W'^2 - w'^2 - \Delta^2)/(2W'). \quad (3)$$

It may be shown that if $W > w$ and $W' > w'$, then $\Delta^2 > 0$. Thus, for the case of interest, the boson $B$ carries positive energy $\omega_{\Delta W}$ into the vertex $\Sigma$, as seen in $(W)$, for $\Delta^2 < W^2 - w^2$, and carries negative energy $\omega_{\Delta W'}$ away from the vertex $\Sigma'$, as seen in $(W')$, for $\Delta^2 < W'^2 - w'^2$. This result is possible because the four-vector $\Delta_i$ is spacelike and the sign of its energy component can change under the Lorentz transformation that connects the $(W)$ and $(W')$ coordinate systems.

In order to help visualize this result it is useful to consider a Minkowski diagram for energy $E$ and momentum $\vec{p}$, for the case $\theta = 0$, as is shown in Fig. 2, where $z$ is taken to be the collision axis. Shown as dashed curves are the mass hyperboloids for real bosons, $\Delta^2 = -m_B^{\,2}$; for real photons, $\Delta^2 = 0$; and for virtual bosons with $\Delta^2 = m_B^{\,2}$. Shown as solid-line vectors are possible components of $\Delta_i$ in $(W)$ and in $(W')$, and the components of the four-momentum of a real boson with almost the same energy and momentum, in $(W)$, as $\Delta_i$.[3] From Fig. 2 it is clear that the effect of the sequence of Lorentz transformations that carries $(W)$ into $(W')$ is quite different in the case of a virtual particle than in the case of a real particle.[4] The four-momentum of the real

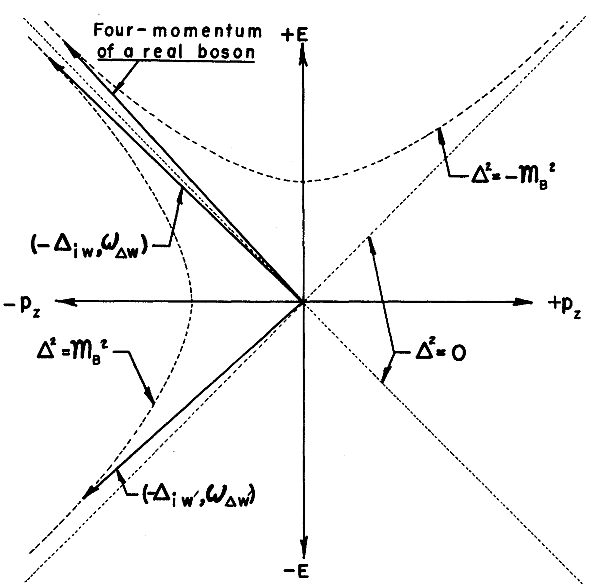


FIG. 2. Minkowski energy-momentum diagram illustrating the kinematical differences between real and virtual particles.

boson remains always on the hyperboloid $\Delta^2 = -m_B^2$, so that the sign of its energy component remains positive, while its three-momentum may change sign. For a virtual particle the sense of the three-momentum is invariant, but its energy may change sign.

If $\omega_{\Delta W}^2$ is large compared to $m_B^2$ and to $\Delta^2$, then there is a real boson four-momentum with energy and three-momentum components in $(W)$ very close to those of $\Delta_i$, as is shown in Fig. 2. It is in this sense that the virtual boson behaves kinematically as an "almost real" particle at the $\Sigma$ vertex in the system $(W)$. Of course, the corresponding result is obtained at the $\Sigma'$ vertex in the system $(W')$. The cross section for the process $A + A' \to C + C'$, as given by the graph of Fig. 1, will contain, according to the usual Feynman rules, the Lorentz-invariant vertex functions

$$\sum_f \bar{\sum_i} |\langle C|\Sigma|\Delta_i, p_i\rangle|^2,$$

and

$$\sum_f{}' \bar{\sum_i}{}' |\langle C'|\Sigma'|-\Delta_i, p_i'\rangle|^2,$$

where $\sum_f$ and $\bar{\sum_i}$ are the appropriate sums and averages over spins, and the Lorentz-invariant phase space factors $d\Omega_C$ and $d\Omega_{C'}$, where

$$d\Omega_C = (2\pi)^{-3(m+n)} \prod_{l=1}^{m} m_l \frac{d^3p_l}{E_l} \prod_{j=1}^{n} \frac{d^3k_j}{2\omega_j}$$

for the case in which $C$ consists of $m$ fermions and antifermions and $n$ bosons, and similarly for $d\Omega_{C'}$. Because the virtual boson is an incoming, almost real particle in the system $(W)$, it is reasonable to define

$$\sigma_{A+B\to C}(\Delta^2, W) \equiv \frac{(2\pi)^4 w}{2p_W W} \int d\Omega_C\, \delta^4(p_i + \Delta_i - P) \sum_f \bar{\sum_i} |\langle C|\Sigma|\Delta_i, p_i\rangle|^2, \tag{4}$$

where $p_W$ is given by

$$W = (p_W^2 + w^2)^{1/2} + (p_W^2 + m_B^2)^{1/2}, \tag{5}$$

and similarly for $\sigma_{A'+\bar{B}\to C'}(\Delta^2, W')$. These are the "scattering" cross sections for the collisions of an almost real boson $B$, $(\bar{B})$ with the real particles $A$, $(A')$. This is the simplest definition consistent with the requirement that

$$\lim_{\Delta^2 \to -m_B^2} \sigma_{A+B\to C}(\Delta^2, W) = \sigma_{A+B\to C}(W),$$

the physical cross section. With these definitions one obtains for the differential cross section,

$$\frac{d\sigma_{A+A'\to C+C'}(\Delta^2, U)}{d(\Delta^2)} = \frac{2}{(2\pi)^3 p_{iU}^2 U^2} [D_{m_B^2}^C(\Delta^2)]^2 \int dW dW' p_W W^2 \sigma_{A+B\to C}(\Delta^2, W) p_{W'}' W'^2 \sigma_{A'+\bar{B}\to C'}(\Delta^2, W'), \tag{6}$$

where $D_{m_B^2}^C(\Delta^2)$ is the boson propagator which for the case of a $\pi$ meson is $(\Delta^2 + m_\pi^2)^{-1}$, and the upper limits of integration depend upon $\Delta^2$. This formula is expected to be valid for all processes in which large-impact-parameter collisions are dominant.

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[1]See for example the Proceedings of the Tenth Annual International Conference on High-Energy Nuclear Physics at Rochester, August, 1960 (to be published).

[2]This note is a generalization of the paper on double pion production in pion-nucleon collisions, given by one of us at the Washington Meeting of the American Physical Society [F. Salzman, Bull. Am. Phys. Soc. 5, 236 (1960)]. A detailed discussion, including justification for the neglect of final-state interactions and of symmetrization, is contained in an Article by F. Salzman and G. Salzman, Phys. Rev. 120, 599 (1960).

[3]All signs are here chosen in accordance with the picture that $A$ is incident from the left (in the laboratory), $A'$ is at rest, and $C$ emerges towards the right, with smaller three-momentum than that with which $A$ entered. With the sense of $\Delta_i$ as shown in Fig. 1, the sign of the $z$ component of $\vec{\Delta}_{iW}$ is thus negative.

[4]We are indebted to Professor P. Morrison for a stimulating discussion on this point.