ficacy of various wall coatings has not been completely determined but we have had good results with the treatment described, and we have found that bare Pyrex is certainly inferior. The sample described is obviously a superior one and exhibits not only longer nuclear relaxation time than others but also shows much better optical pumping signals than usual.

This technique should be applicable to any noble gas having a nuclear moment. It should not be applicable to any molecular gas since the nuclear relaxation time is greatly shortened by the interaction between the nuclear moments and the rotating charge distribution of the molecule. Although the amount of polarization (0.01%) is as yet not useful in nuclear physics experiments, we hope to improve it by better wall coatings, by the use of hotter walls to reduce the dwell time of the He^3, or possibly by the use of "mixing" frequencies[13] to increase the transition rate of the Rb-He flips.

It has recently been suggested[14] and discussed[15] that a possible study of the relaxation time in a monatomic gas might give information about the electric dipole moment of the nucleus through its interaction with the electric field produced during collisions. It is tempting to think that this method of study of the He^3 relaxation would be useful in this respect, but the actual frequency and density conditions of this experiment are still several orders of magnitude away from the optimum conditions for study of this question, in spite of the fact that our observed relaxation times are longer than a shortest estimate[15] for such a relaxation mechanism. The wall relaxation again appears to be a limiting factor.

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MODIFICATION OF EFFECTIVE-RANGE THEORY IN THE PRESENCE OF A LONG-RANGE POTENTIAL*

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The great success of effective-range theory (ERT) in the analysis and interpretation of low-energy nuclear two-body scattering data has stimulated its application to a wide variety of other fields, in some cases perhaps somewhat less critically than might be warranted. It should therefore be stressed that there are limitations to the applicability of ERT in its usual form. For long-range forces, for example, the very form of the expansion as a power series in k^2 is incorrect.

The simplest derivation[1] of ERT for zero angular momentum starts with the identity

$$k \cot\eta = -1/A + \tfrac{1}{2} r(0,k) k^2,$$

where

$$\tfrac{1}{2} r(0,k) \equiv \int_0^\infty (vv_0 - uu_0)\, dr.$$

$u(r)$ and $u_0(r)$ are the solutions of the Schrödinger equation with the potential $V(r)$ at energies E and

0, respectively, which satisfy the boundary conditions

$$u(0)=0, \quad u(r)\to \sin(kr+\eta)/\sin\eta \equiv v(r) \quad \text{for} \quad r\to\infty,$$

$$u_0(0)=0, \quad u_0(r)\to 1-r/A \equiv v_0(r) \quad \text{for} \quad r\to\infty.$$

The original derivations were concerned primarily with strong, short-ranged (range $\approx R$) potentials. It was argued that (1) the limits of integration could effectively be replaced by 0 and R, since $u(r)\approx v(r)$ and $u_0(r)\approx v_0(r)$ for $r>R$, and (2) in the region 0 to R, for E small compared to some average of $V(r)$ in that region, the replacement of $u(r)$ by $u_0(r)$ and of $v(r)$ by $v_0(r)$ introduces an error of order k^2.

One then obtains the shape-independent approximation,

$$k \cot\eta = -1/A + \tfrac{1}{2} r_0 k^2 + O(k^4),$$

where the energy-independent effective range, r_0, is defined by

$$\tfrac{1}{2} r_0 = \int_0^\infty (v_0^2 - u_0^2)\, dr.$$

This form of expansion cannot however be correct, for example, for a potential which behaves asymptotically as $(2\mu/\hbar^2)V(r) \sim -\beta^2/r^4$. (Effective potentials of this form arise, in the adiabatic approximation, in the scattering of charged particles by a neutral polarizable system.) For this potential, one finds that for $r\to\infty$

$$u_0(r) \sim \frac{r}{\beta}\sin\frac{\beta}{r} - \frac{r}{A}\cos\frac{\beta}{r}$$

$$\equiv v_{0p}(r) = \left(-\frac{r}{A}+1\right) + \frac{\beta^2}{2Ar} - \frac{\beta^2}{6r^2} + \ldots,$$

from which it follows that $r_0=\infty$. The approximations noted above are then clearly invalid for this potential. The point is that if $V(r)$ approaches zero faster than any power of $1/r$ the same will be true for $u_0(r)-v_0(r)$, whereas for the present potential $u_0(r)-v_0(r)$ approaches zero so slowly that the contribution to $r(0,k)$ from large values of r is anything but negligible.

The solution of the differential equation with a potential proportional to $1/r^4$ can be expressed in terms of Mathieu functions.[2] This enables one to derive the modified shape-independent approximation

$$k \cot\eta = -\frac{1}{A} + \frac{\pi\beta^2}{3A^2}k + \frac{4\beta^2}{3A}k^2 \ln\left(\frac{\beta k}{4}\right)$$

$$+\left[\tfrac{1}{2}r_{0p} + \frac{\pi\beta}{3} + \frac{20\beta^2}{9A} - \frac{8\beta^2}{3A}\Psi(\tfrac{3}{2}) - \frac{\pi\beta^3}{3A^2} - \frac{\pi^2\beta^4}{9A^2}\right]k^2 + \ldots, \quad (1)$$

where

$$\tfrac{1}{2}r_{0p} \equiv \int_0^\infty [v_{0p}^2(r) - u_0^2(r)]\,dr,$$

and

$$\Psi(\tfrac{3}{2}) = \Gamma'(\tfrac{3}{2})/\Gamma(\tfrac{3}{2}) = 0.0365.$$

Equation (1) is valid for k real and non-negative (and small). For k real and nonpositive, a different asymptotic expansion of the Mathieu function must be used, and one finds that k must be replaced by $-k$ on the right-hand side of Eq. (1). $k \cot\eta$ is therefore even in k for real k, as it must be.

To the extent that the adiabatic approximation is valid for the low-energy scattering of ions[3] and even of electrons by neutral atoms or molecules, as seems to be the case, Eq. (1) as appropriately modified to include symmetry effects is applicable to these situations. In this connection, we note the recent application[4] of ERT in its usual form to the singlet scattering of low-energy electrons by hydrogen atoms. The authors of that paper, however, expanded about the energy of binding of the H^- ion and it so happens, for reasons which we will go into at a later time, that this is a reasonably accurate approach. This explains the good agreement between their numerical result for the singlet scattering length and that deduced through the use of a rigorous minimum principle.[5] We also note that the effects of the term linear in k on continuous absorption due to free-free transitions in hydrogen are not negligible.[6]

Another application of Eq. (1) is to the scattering of neutrons by charged systems.[7] Due to the small polarizability of the neutron, the β^2/r^4 term can be treated as a perturbation. [No such approximation was necessary for the derivation of Eq. (1).] Doing so, Thaler found the term linear in k.

For $L>0$ and $V(r)\propto 1/r^4$, even the leading term of ERT is not of the correct form. Thus, it is $k^2 \cot\eta$ which approaches a constant as $k\to 0$, rather than $k^{2L+1}\cot\eta$ as is the case[8] for short-range potentials.

One cannot expand $k^{2L+1}\cot\eta$ as a power series in k^2 for any long-range potential $[V(r)\propto 1/r^n]$ though the point at which new types of terms appear depends upon the potential. For Van der Waals scattering at $L=0$, for example, new terms will appear between k^2 and k^4. New terms will also appear, taking $L=0$ again, in the expansion of $C^2 k \cot\eta$, where C^2 is the Coulomb penetration factor, for the scattering of charged particles in the presence of another long-range potential. This would, for example, be the case for the scattering of protons by a nucleus, though the coefficients of these new terms might be quite small since they are proportional to the polarizability of the nucleus. New terms will also appear in nucleon-nucleon scattering due to the long-range magnetic interactions; for *n-p* or *p-p* forces it is clear from our previous discussion that there will also be additional (though generally very small) terms due to the nucleon polarizabilities.

A detailed report is in preparation. This report will also include some applications, with emphasis on electron scattering by hydrogen.

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SINGLE-VIRTUAL-BOSON-EXCHANGE INTERACTION IN HIGH-ENERGY COLLISIONS*

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Recent experimental evidence points towards the importance of a long-range interaction, i.e., $\sim m_\pi^{-1}$, in high-energy collisions of π mesons with nucleons and of nucleons with nucleons.[1] A natural model to investigate is that in which the two incident particles collide by exchanging a single virtual π meson. In this note it is shown that if these graphs dominate in collisions in which the square of the virtual pion's four-momentum is very small, then a simple field-theoretical model of such collisions results.

Because the kinematics and field theory discussed in this note are not restricted only to processes involving π mesons and nucleons, we consider a general inelastic two-particle collision as shown in Fig. 1.[2] Particles A and A', with four-momenta p_i and p_i', exchange a boson B with four-momentum Δ_i, and two groups of particles emerge: C, with total four-momentum P, and C', with total four-momentum P'. A and A' have rest masses w and w',

$$p_i^2 = -w^2 \quad \text{and} \quad p_i'^2 = -w'^2,$$

and the "rest masses" W and W' of C and C' are defined by

$$P^2 = -W^2 \quad \text{and} \quad P'^2 = -W'^2.$$

The over-all barycentric coordinate system (U) is defined by

$$\vec{p}_{iU} + \vec{p}_{iU}' = 0,$$

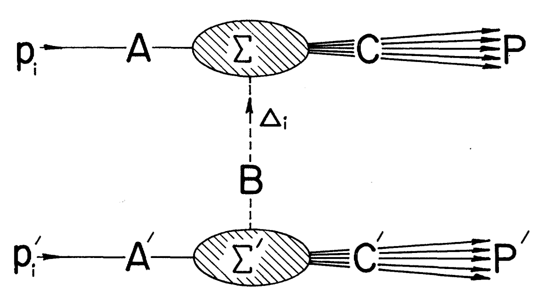

FIG. 1. A general binary inelastic collision in which particles A and A', with four-momenta p_i and p_i', interact by the exchange of a single boson B, with four-momentum Δ_i, leading to two groups of final state particles: C, with total four-momentum P, and C', with total four-momentum P'.