$R(H \parallel c)$ is small because electrons with $k_\perp < \beta$ rotate in a sense opposite to that for electrons with $k_\perp > \beta$ and because ω_c tends to zero as $k_\perp$ approaches β.

For CdS Thomas et al.[5] have estimated an upper bound of the order of a few hundredths of an ev for the displacement of any energy minima below $E(0)$ in the conduction band. In the toroidal model this energy displacement equals $A\beta^2$. Setting $A\beta^2 = 0.04$ ev, one obtains $\Lambda \approx 6$ at $T = 77°\text{K}$. Thus, one might hope to observe a large anisotropy in the Hall effect, if the model is correct.

It should be emphasized that the model results from an approximation. Extension of the $\vec{k}\cdot\vec{p}$ perturbation calculation to higher orders will lower the symmetry of the surfaces to one of sixfold rather than continuous rotational invariance. Whether or not the corrections are important will depend upon the size of β and the proximity of other bands. For $A\beta^2 = 0.04$ ev and $A = (2m)^{-1}$, β is typically of the order of one-tenth the shortest distance in $\vec{k}$ space between the point $\vec{k} = 0$ and any of the reduced zone boundaries parallel to the k_z axis.

Added note. The toroidal model is a direct consequence of the spin-orbit interaction. The nature of $E(\vec{k})$ in the limit of zero spin-orbit coupling is described in the earlier work of Rashba,[11] who has recently given independent consideration to the effects of including the interaction.[12]

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NUCLEAR POLARIZATION IN He^3 GAS INDUCED BY OPTICAL PUMPING AND DIPOLAR EXCHANGE*

M. A. Bouchiat,† T. R. Carver,‡ and C. M. Varnum
Palmer Physical Laboratory, Princeton University, Princeton, New Jersey

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Although almost complete polarization of alkali metal nuclei can be produced by optical pumping utilizing a buffer gas and a filter to remove the D_2 resonance light,[1,2] the number of atoms polarized is necessarily very small and is limited by optical opacity of the sample vapor. The usual range of pressure is 10^{-7} to 10^{-3} mm for successful detection and production of optical polarization. The Overhauser nuclear polarization effect[3] involving dipolar interactions[4-8] between such an optically polarized atom and the nucleus of a suitable buffer gas should provide a mechanism for the transfer of polarization to a gas at considerably higher pressure than that of the alkali vapor. In other words, the Overhauser effect should work not by the dipole-coupled relaxation of a saturated paramagnetic impurity toward a polarized equilibrium, but by the relaxation of an optically polarized impurity toward a nearly depolarized equilibrium. We have observed this effect in He^3 gas used as the buffer gas for the optical pumping of natural rubidium vapor.

The nuclear magnetic relaxation time in a liquid or a monatomic gas depends inversely on both the square of the magnetic moment of the nucleus and the square of the moment of other spins causing the relaxation. If He^3 is contaminated with "dissolved" Rb atoms having a moment 10^3 times as great as the He^3, a concentration of more than 1 part in 10^6 would dominate the relaxation. The dipole interaction terms involving the spin-flip operators I_+S_+ and I_-S_- for the nuclear and electronic spins, respectively, are known to dominate the relaxation process,[8] so that as the Rb atoms relax from the optically polarized condition they flip He^3 nuclei to a polarized condition. In our experiment a partial pressure of 10^{-3} mm of Rb^{85+87} vapor should polarize the 2.8-atmosphere He^3 sample with a characteristic time of between 10^5 and 10^6 seconds, comparable to the He^3 relaxation time at that pressure. In actual fact the relaxation time was found to be shortened by what is almost certainly wall relaxation. This competing relaxation process reduces the amount of He^3 polarization below the approximately 10% polarization of the Rb atoms, but we have observed enhancement of the nuclear polarization by a factor of 10^4 above the initial Boltzmann distribution of 10^{-8}.

The experimental technique was as follows: A 30-cm^3 Pyrex bulb was outgassed and treated with G.E. Dri-Film (dimethyl-dichloro silane), a treatment[9] which has been found to be effective in preventing the recombination of atomic hydrogen,[10] and in increasing the relaxation time of optically pumped vapors.[11] The bulb was again baked out in vacuum, and natural Rb was distilled into a side tube connecting with the bulb through a small re-entrant orifice so that an excess of metallic Rb would not be present in the bulb itself. He^3 was admitted at 2.8 atmospheres STP after purification with hot calcium and a discharge with titanium electrodes to remove contaminant O_2 and N_2, and the bulb was sealed off with a breakoff to reclaim the He^3. The entire sample was placed within another Pyrex tube to conduct hot air and maintain the sample at 140°C, at which point the Rb pressure is 3×10^{-3} mm.

The Rb was polarized by irradiating only with the D_1 (7947A) line from a 4-cm diameter rf-excited Rb lamp passed through a thin film D_2 interference filter, and subsequently through plastic sheet type polarizer and quarter-wave plate designed for near infrared use. The optical pumping was monitored by light transmission through the cell and the observation of the $\Delta M = 1$, $\Delta F = 0$ transitions excited by a small rf loop around the sample.

In order to measure the degree of nuclear polarization, the nuclear resonance of He^3 was observed at 400 kc/sec in a field of 123.3 gauss produced by a large corrected solenoid capable of a homogeneity of a few milligauss over the sample volume. The detection system consisted of a crystal oscillator, twin-T bridge, preamp, i.f. strip, and a 400-kc/sec lock-in amplifier which directly phase-or amplitude-detects the signal. This apparatus had sufficient sensitivity to display the proton resonance in a 5-atmosphere propane sample with a single sweep equivalent to a 90° pulse. To make possible the detection of He^3 resonance for calibration and comparison with optical polarization, the He samples could be observed after prior polarization in a 10 000-gauss field. In the case of the optically polarized sample, the 400-kc/sec nuclear magnetic resonance (NMR) coil was wound directly on the outer Pyrex heating tube in a Helmholtz pair arrangement so that the optical pumping radiation could pass through the sample along the axis of symmetry of the solenoid. The NMR signals obtained were sufficiently intense ($S{:}N = 1000$) so that observation of the long relaxation time could be made with successive almost nondestructive fast-passage sweeps by reducing the incident 400-kc/sec power to a very low level. The time constants involved were 1800, 2450, 1200, and 2250 seconds for polarization by Rb at 140°C, depolarization at 140°C, polarization at 27°C by large external magnetic field, and depolarization at 30°C, respectively. The nuclear polarization could be inverted and observed in either emission or absorption by simply changing the circular polarizer from left to right.

There appears to be no reason in principle why this technique cannot be used to produce polarization comparable to that induced in the metal vapor. The polarization is reduced, however, by other relaxation processes. We have investigated the relaxation time in a number of samples which do not contain Rb and are not above room temperature. The results are not very reproducible, but we have generally found that the relaxation time is proportional to the density of the He^3, that the relaxation time doubles in going from 300°K to 77°K, and that deliberately dirtied walls or internal complexity of the sample shorten the relaxation time. This is not to be expected from relaxation in the bulk of the gas due to either He^3 or impurities, but is to be expected from wall relaxation. Although the justification and analysis cannot be presented here, the relaxation times are quantitatively consistent with the assumption that there exists a monolayer of He^3 on the walls in which the relaxation occurs,[12] and that each atom has probability of sticking per collision of greater than about 1/2. The efficacy of various wall coatings has not been completely determined but we have had good results with the treatment described, and we have found that bare Pyrex is certainly inferior. The sample described is obviously a superior one and exhibits not only longer nuclear relaxation time than others but also shows much better optical pumping signals than usual.

This technique should be applicable to any noble gas having a nuclear moment. It should not be applicable to any molecular gas since the nuclear relaxation time is greatly shortened by the interaction between the nuclear moments and the rotating charge distribution of the molecule. Although the amount of polarization (0.01%) is as yet not useful in nuclear physics experiments, we hope to improve it by better wall coatings, by the use of hotter walls to reduce the dwell time of the He^3, or possibly by the use of "mixing" frequencies[13] to increase the transition rate of the Rb-He flips.

It has recently been suggested[14] and discussed[15] that a possible study of the relaxation time in a monatomic gas might give information about the electric dipole moment of the nucleus through its interaction with the electric field produced during collisions. It is tempting to think that this method of study of the He^3 relaxation would be useful in this respect, but the actual frequency and density conditions of this experiment are still several orders of magnitude away from the optimum conditions for study of this question, in spite of the fact that our observed relaxation times are longer than a shortest estimate[15] for such a relaxation mechanism. The wall relaxation again appears to be a limiting factor.

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†Now at the Laboratoire de Physique, Ecole Normale Supérieure, Paris, France.

‡Alfred P. Sloan Foundation Fellow.

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MODIFICATION OF EFFECTIVE-RANGE THEORY IN THE PRESENCE OF A LONG-RANGE POTENTIAL*

Larry Spruch, Thomas F. O'Malley, and Leonard Rosenberg
Physics Department, Washington Square College, and Institute of Mathematical Sciences
New York University, New York, New York

(Received September 16, 1960)

The great success of effective-range theory (ERT) in the analysis and interpretation of low-energy nuclear two-body scattering data has stimulated its application to a wide variety of other fields, in some cases perhaps somewhat less critically than might be warranted. It should therefore be stressed that there are limitations to the applicability of ERT in its usual form. For long-range forces, for example, the very form of the expansion as a power series in k^2 is incorrect.

The simplest derivation[1] of ERT for zero angular momentum starts with the identity

$$k \cot\eta = -1/A + \tfrac{1}{2} r(0,k) k^2,$$

where

$$\tfrac{1}{2} r(0,k) \equiv \int_0^\infty (v v_0 - u u_0)\, dr.$$

$u(r)$ and $u_0(r)$ are the solutions of the Schrödinger equation with the potential $V(r)$ at energies E and