PION PARAMETERS FROM HIGH-ENERGY INELASTIC INTERACTIONS*

S. D. Drell
Institute of Theoretical Physics, Department of Physics, Stanford University, Stanford, California
(Received August 25, 1960)

In an earlier Letter[1] we employed the physical principle that a transition amplitude has a pole for real one-particle intermediate states, to study the photoproduction of secondary beams of high-energy particles. We found there that photons are very effective in initiating collimated beams of some very high-energy strongly interacting particles. It is the object of this note to apply these "polology" considerations to further experiments in order to check their quantitative content as well as to determine parameters of interest in pion physics such as the strengths of the $(\pi\pi\pi\pi)$, $(\gamma\pi\pi\pi)$, and $(\pi\pi KK)$ interactions.

What we propose here is a straightforward extension to different inelastic processes of the work of Chew and Low.[2] Our main point is illustrated by comparing Figs. 1 and 2 for elastic and inelastic nucleon-nucleon scattering. For forward elastic scattering as shown in Fig. 1 we are near the pole for exchange of one real pion between (a) and (b). The pole is at $(p-q)^2 = \mu^2$ or, in terms of the scattering angle θ and the magnitude of the momentum of a nucleon, k, in the barycentric system, at $\cos\theta = 1 + \mu^2/2k^2$. Chew[3] proposed extrapolating to the pole by letting $\theta \to 0°$ at fixed k, the hope being that the contribution from Fig. 1 would dominate in the physical region near the pole. In practice[4] this has turned out to be a difficult procedure and the main reason is that the nucleon is being requested to emit and absorb a low-velocity pion at forward scattering angles, with the limit $v_\pi \to 0$ at $\theta \to 0°$. For pseudoscalar mesons the coupling to nucleons is proportional to v_π as $v_\pi \to 0$ and so the amplitude at the pion-nucleon vertex vanishes at $\theta = 0°$. Therefore the amplitude corresponding to Fig. 1 is reduced for small scattering angles, its contribution to the cross section being proportional to

$$\frac{4(p\cdot q - M^2)^2}{[(p-q)^2-\mu^2]^2} = \left(\frac{1-\cos\theta}{1-\cos\theta+\mu^2/2k^2}\right)^2. \qquad (1)$$

In this circumstance it is possible that contributions from heavier intermediate states connecting (a) and (b) in Fig. 1 may be relatively important and should be included in the analysis even though their singularities are more removed from the physical region.

For the inelastic scattering, Fig. 2, we are also near the pole for the one-pion exchange. However, two factors are operating here to give a large cross section for a high-energy nucleon to emerge in the forward direction at $\theta_l \sim 0°$ along with anything else (n). First of all, by opening up the vertex at (b) to all inelastic channels (n) we have replaced the amplitude for pion absorption which vanishes as $v_\pi \to 0$, by a total inelastic cross section which is ≈ 30 mb in the Bev range. This removes one of the vanishing factors in the numerator of Eq. (1). Secondly, the vertex at (a) does not vanish either since the incident nucleon transfers energy to the pion emitted there. The pion emerges with a finite velocity and therefore with a finite amplitude, proportional to

$$p\cdot q - M^2 \approx (M^2/2)(E_p - E_Q)^2/E_pE_Q$$

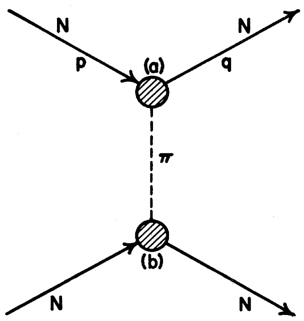

FIG. 1. Diagram for one-pion contribution to elastic nucleon-nucleon scattering.

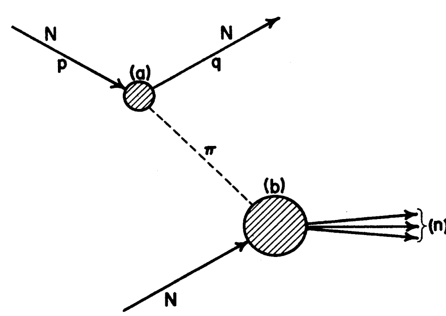

FIG. 2. Diagram for one-pion contribution to inelastic nucleon-nucleon scattering.

for relativistic nucleons and $\theta_l = 0°$; here E_p is the energy of the incident nucleon and E_Q and θ_l are the energy and angle of the observed final nucleon in the laboratory frame of reference. The parameters should be chosen so as to locate the pole in the pion propagator near to the physical region and therefore to make the extrapolation to it feasible. This will be the case for a small energy loss $\Delta \equiv E_p - E_Q \lesssim (\mu/M)E_p$, and small scattering angle $\theta_l \sim \mu/E_p$. Then Fig. 2 should dominate the inelastic cross section which, summed over all final states (n), is

$$d^2\sigma(E_p, E_Q, \theta_l) = \frac{1}{2\pi}\left(\frac{G^2}{4\pi}\right)\frac{p\cdot q - M^2}{(p\cdot q - M^2 + \frac{1}{2}\mu^2)^2} \times \frac{d\Omega_Q}{4\pi}\Delta\frac{E_Q}{E_p}dE_Q\sigma_{\pi+X,\,\mathrm{total}}(\Delta), \tag{2}$$

where $G^2/4\pi = (2M/\mu)^2 f^2$ with f^2 the pion-nucleon constant which, by pion-nucleon scattering, is ≈ 0.08. Simplifying for small angles and energy losses, Eq. (2) becomes

$$d^2\sigma(E_p, E_Q, \theta_l) \cong \frac{4}{\pi}f^2\left[\frac{(\theta_l E_p/\mu)^2 + (M\Delta/\mu E_p)^2}{[(\theta_l E_p/\mu)^2 + (M\Delta/\mu E_p)^2 + 1]^2}\right] \times \frac{d\Omega_Q}{4\pi}\frac{M^2\Delta dE_Q}{\mu^4}\sigma_{\pi+X,\,\mathrm{total}}(\Delta). \tag{3}$$

Equations (2) and (3) lead to an intense flux of inelastically scattered nucleons in the forward direction which should be readily measurable. For $E_p = 6$ Bev, $E_Q = 5$ Bev, and $\theta_l = \mu/E_p \approx 1\frac{1}{2}°$, the cross section is $35f^2\sigma_{\pi+X,\,\mathrm{total}}(1\text{ Bev})\ \mathrm{sr}^{-1}\ \mathrm{Bev}^{-1}$. Experimental test of (3) by measurements of the inelastic nucleon spectrum at fixed energies E_p and E_Q and decreasing θ_l is of interest for three reasons. First, it checks the mechanism underlying this formula and gives an independent determination of f^2 in terms of measured pion- and nucleon-initiated inelastic processes. We would like to know what, if any, is the region of high energy and small angles in which we can attach quantitative significance to (3). Second, if it is found that an accurate extrapolation is practical on the basis of (3), we may turn it around and use it as a means of measuring π^0-nucleon cross sections. Third, if the extrapolation is successful it opens the door to several further experiments which offer the hope of measuring other coupling constants of importance in pion physics.

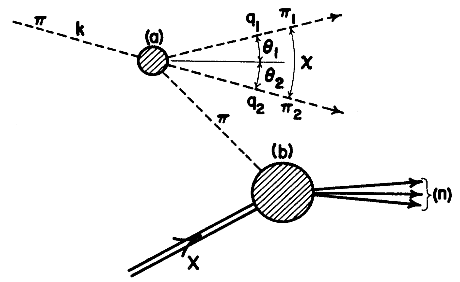

FIG. 3. Diagram for one-pion contribution to the process $\pi + X \rightarrow \pi_1 + \pi_2 + (n)$.

One of these is the π-π scattering length which is measured by the experiment[5] indicated in Fig. 3. In this process, a pion of four-momentum $k:(\omega, \vec{k})$ is incident on a nucleon target and two high-energy pions, $q_1:(\omega_1, \vec{q}_1)$ and $q_2:(\omega_2, \vec{q}_2)$ are observed in coincidence emerging in the forward cone of opening angle $\theta_{1/2} \sim \mu/k$. Summing again over inelastic channels (n), we obtain just as before (footnote 2 of reference 1) for the differential cross section to detect the two emerging pions

$$d^6\sigma_{\pi,\pi\pi}(k, q_1, q_2) = \frac{16}{\pi^2}\frac{|A(s_1, s_2, s_3)|^2}{(s_1+s_2+s_3-4\mu^2)^2}\frac{d\Omega_1}{4\pi}\frac{d\Omega_2}{4\pi} \times \frac{(\omega-\omega_1-\omega_2)\omega_1\omega_2 d\omega_1 d\omega_2}{\omega}\sigma_{\pi+X,\,\mathrm{total}}(\omega-\omega_1-\omega_2). \tag{4}$$

In (4) we use the following notation: $s_1 \equiv (k-q_1)^2$, $s_2 \equiv (k-q_2)^2$, $s_3 \equiv (q_1+q_2)^2$; $[s_1+s_2+s_3-4\mu^2]^{-1} = [(k-q_1-q_2)^2 - \mu^2]^{-1}$ is the pion propagator; $A(s_1, s_2, s_3)$ is the invariant amplitude for π-π scattering as defined by Chew and Mandelstam[6] and is the object of interest on the mass shell $s_1+s_2+s_3 = 4\mu^2$. The invariant momentum transfers s_i are all small, $\approx \mu^2 \ll \omega^2$ for forward scattering angles θ_1, θ_2, and χ as shown in Fig. 3:

$$s_1 \cong -\mu^2\left[\frac{(\omega-\omega_1)^2}{\omega\omega_1} + \frac{\omega\omega_1\theta_1^2}{\mu^2}\right];$$

$$s_2 \cong -\mu^2\left[\frac{(\omega-\omega_2)^2}{\omega\omega_2} + \frac{\omega\omega_2\theta_2^2}{\mu^2}\right];$$

$$s_3 \cong \mu^2\left[\frac{(\omega_1+\omega_2)^2}{\omega_1\omega_2} + \frac{\omega_1\omega_2\chi^2}{\mu^2}\right].$$

Therefore we try the scattering length approximation to A:

$$A_0^0 = -5\lambda, \quad A_1^1 = 0, \quad A_0^2 = -2\lambda, \tag{5}$$

where the lower and upper indices indicate the orbital and isotopic angular momentum channels, respectively. The scattering lengths are the extrapolation to the symmetry point $s_1 = s_2 = s_3 = (4/3)\mu^2$ of the π-π scattering amplitude, and λ is the coupling strength in terms of which Chew and Mandelstam[6] have formulated their solution.[7] P-wave contributions vary as $(s_1 - s_2)$ near the symmetry point and in principle may be studied along with effective-range corrections to (5). By carrying out the experiments for (4) with different combinations of pion charges, one can check predictions of (4) that are independent of the magnitude of λ. For example, choosing $s_1 = s_2$ to remove p-wave effects, the ratio of cross sections I (π^+ incident, $2\pi^+$'s detected) to II (π^- incident, $\pi^+\pi^-$ detected) to III (π^+ incident, $\pi^+\pi^-$ detected) is

$$\text{I:II:III} = |A_0^2|^2\sigma_{\pi^-X} : [\tfrac{2}{3}|A_0^0|^2 + \tfrac{1}{3}|A_0^2|^2]\sigma_{\pi^-X} : [\tfrac{2}{3}|A_0^0|^2 + \tfrac{1}{3}|A_0^2|^2]\sigma_{\pi^+X} \cong 2:9:9[\sigma_{\pi^+X}/\sigma_{\pi^-X}]. \tag{6}$$

Whether or not it is possible to make accurate determinations of scattering lengths and effective ranges in this way, it will be very valuable to see if the amplitude A is especially large or small.

Another experiment of this type corresponds to replacing the incident pion by a photon in Fig. 3 and measures the strength of the $\gamma \to 3\pi$ vertex which is of interest in connection with the understanding of the isotopic scalar part of the electromagnetic form factors of nucleons. Aside from reinterpretation of A as the $\gamma \to 3\pi$ coupling constant, it is only necessary to change $-4\mu^2$ to $-3\mu^2$ in the propagator in Eq. (4) for this application. We might similarly also get an idea of the strength of the $(\pi\pi K\bar{K})$ interaction by detecting a $K\bar{K}$ pair instead of a $\pi\pi$ pair emerging in Fig. 3 from an incident pion. The appropriate modification of Eq. (4) is $-4\mu^2 \to -2\mu^2 - 2\mu_K^2$ in the propagator.

The counting rates for these coincidence arrangements are "reasonable" according to D. M. Ritson, to whom the author is grateful for valuable discussion. It is also a pleasure to acknowledge a valuable discussion with J. D. Bjorken.

Note added in proof. At the Tenth International High-Energy Conference at Rochester, F. Salzman and G. Salzman reported related considerations for one-pion exchange processes in pion-nucleon collisions [Phys. Rev. (to be published)]. They observed that the π-π interaction is measured in the process π + nucleon $\to 3\pi$ + nucleon. In addition, it is interesting to note in connection with Eqs. (2) and (3) that the ratio of cross sections corresponding to Fig. 2 for incident antiprotons to the cross sections for incident protons measures the ratio of G' for coupling of pion to antinucleon to G for coupling of pion to nucleon.

*Supported in part by the U. S. Air Force through the Air Force Office of Scientific Research.

[1]S. D. Drell, Phys. Rev. Letters 5, 278 (1960).

[2]G. F. Chew and F. E. Low, Phys. Rev. 113, 1640 (1959).

[3]G. F. Chew, Phys. Rev. 112, 1380 (1958).

[4]P. Cziffra and M. J. Moravcsik, Phys. Rev. 116, 226 (1959). Extrapolations of recent measurements have indicated somewhat different and smaller coupling constants [G. F. Chew and W. R. Frazer (private communication)]. A similar difficulty faces the extrapolation in $\cos\theta$ for single pion photoproduction [J. G. Taylor, M. J. Moravcsik, and J. L. Uretsky, Phys. Rev. 113, 689 (1959)].

[5]Comparing with the Chew-Low proposal (reference 2) to measure the π-π scattering cross section in the reaction $\pi + N \to \pi + \pi + N$, this has two factors in its favor: Just as we saw in comparing elastic and inelastic nucleon-nucleon scattering (Figs. 1 and 2), we here take advantage of the large absorption amplitude at (b) in Fig. 3. In addition the present experiment calls for a double and not a triple coincidence arrangement.

[6]G. F. Chew and S. Mandelstam, Phys. Rev. 119, 467 (1960).

[7]G. F. Chew, S. Mandelstam, and P. Noyes, Phys. Rev. 119, 478 (1960). Values of λ are limited to the range $-0.46 < \lambda < +0.3$.