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TOTAL CROSS SECTION FOR $\gamma+n\to\pi^-+p$ NEAR THRESHOLD BY CHEW-LOW EXTRAPOLATION

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The basic reaction

$$\gamma+n\to\pi^-+p \tag{1}$$

has previously been studied[1] near threshold by measuring the ratio

$$R_d=\sigma(\gamma+d\to\pi^-+2p)/\sigma(\gamma+d\to\pi^++2n),$$

and inferring the parameters of interest from R_d and the measured cross section for the reaction $\gamma+p\to\pi^++n$. Two separate experiments are required, and Coulomb corrections to R_d near threshold are large and not well understood.

A new approach to the study of Reaction (1) is now possible through the recently developed Chew-Low technique[2] of extrapolating certain recoil-nucleon momenta into the nonphysical region. The method is both powerful and quite general. When the method is specialized to treat the reaction

$$\gamma+d\to\pi^-+2p \tag{2}$$

near threshold, the data are extrapolated to a pole in the transition amplitude in the nonphysical region of negative kinetic energy of the recoiling spectator proton [of Reaction (2)]. The residue of this pole is proportional to the T matrix for Reaction (1). At the pole, the "spectator" proton has a negative kinetic energy in the final state just equal to its share of the (negative) deuteron binding energy in the initial state. The recoil proton is really a noninteracting spectator at the pole in the nonphysical region. The total cross section near threshold for Reaction (1) on free stationary neutrons, deduced directly from the extrapolation, involves no uncertainties arising from the presence of the second nucleon in the target deuteron. The Chew-Low method requires somewhat greater numbers of events than usual for a given statistical error in the result, but this difficulty is compensated by the new ability to use unstable particles as free stationary targets.

Reaction (2) was observed in a deuterium bubble chamber,[1] with complete final-state kinematics known for each event. These data were extracted from the observations discussed in the authors' previous Letter.[1]

The free-nucleon cross sections $\sigma(\gamma+n\to\pi^-+p)$ have been obtained at five effective photon energies by this method, which requires that one carry out the following limiting procedure:

$$\sigma(w^2)=\lim_{p^2\to-\alpha^2}\frac{4\pi k^2}{\Gamma^2}\,\frac{M_d}{M_p}\,\frac{(p^2+\alpha^2)^2}{(w^2-M_n^{\;2})}\,\frac{\partial^2\sigma}{\partial p^2\partial w^2}.$$

Here k = laboratory-system photon energy, M_d = deuteron mass, M_p = proton mass, M_n = neutron mass, p = momentum (lab) of recoil proton (lower-energy proton), α^2 = (deuteron B. E.)M_p, w = internal total energy of the (π^-+p) system, $\Gamma^2=(4/M_p)\times\alpha/(1-\alpha r_0)$, $r_0=n$-p triplet effective range, and $\hbar=c=\mu=1$.

In compiling the data, plots of the events, such as Fig. 1, were used. The curves in p^2, w^2 designate the boundaries for kinematically possible values of p^2 and w^2 for the photon energies 160 Mev and 165 Mev.

The data were averaged over those real photon energies from 150 to 180 Mev which were capable of contributing to the various w^2 bins.

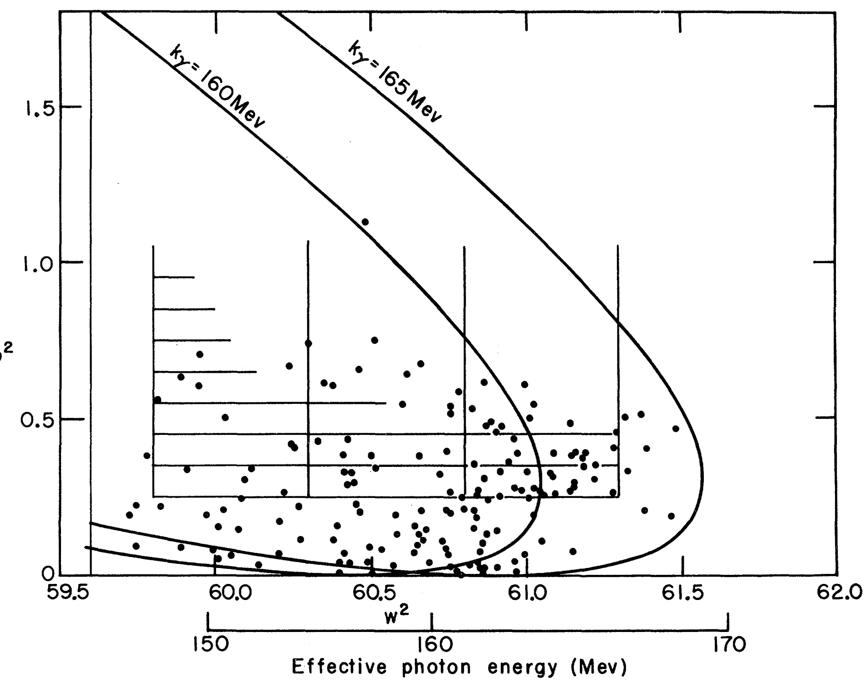

FIG. 1. Polology diagram in p^2 and w^2. Here, p is the momentum of the spectator proton (lower lab kinetic energy), and w is the total internal energy of the remaining ($\pi^- + p$) system ($\hbar = c = \mu = 1$). The curves are kinematical boundaries in these variables within which events from the corresponding real photon energy must fall. The rectangular bins at 61.05 are those used in obtaining the points of Fig. 2. The events plotted here are at photon energy (lab) $k = 160$ to 165 Mev.

The results of extrapolations to the desired cross sections at five effective lab photon energies for the free-nucleon reaction $\gamma + n \rightarrow \pi^- + p$ are presented in Table I. The effective lab photon energy is uniquely specified by a given value of w, the total energy in the π^--p c.m. frame in the final state. A typical extrapolation is shown in Fig. 2. The data lie in the interval $0.3 \leq p^2 \leq 0.9$. Since the form of the extrapolating function is not known, we are somewhat at a disadvantage; however, a simple behavior is expected in this region. Although zero-, first-, and second-order polynomials were tried as extrapolation functions, a straight line was indicated at the three lowest

Table I. Extrapolations for $\sigma(\gamma + n \rightarrow \pi^- + 2p)$.

Extrapolation form	Quantity[a]	Laboratory photon energy (Mev) 153.4 ($w^2 = 60.05$)	158.6 ($w^2 = 60.55$)	163.7 ($w^2 = 61.05$)	168.9 ($w^2 = 61.55$)	174.1 ($w^2 = 62.05$)
One-parameter (weighted average)	N	7	7	7	6	7
	M	6	6	6	5	6
	χ^2/M	0.73	2.46	1.45	0.70	1.19
	S_m	...	...	...	...	...
	$\sigma \times 10^{29}$, cm^2	4.05 ± 0.45[b]	5.53 ± 1.04	8.34 ± 1.16	7.80 ± 1.00	11.83 ± 2.07
Two-parameter (straight line)	N	7	7	7	6	7
	M	5	5	5	4	5
	χ^2/M	0.16	0.86	0.72	0.86	1.26
	S_m	22.64	12.21	7.17	0.04	0.66
	$\sigma \times 10^{29}$, cm^2	7.33 ± 0.72	12.38 ± 2.06	15.23 ± 2.70	8.67 ± 4.45	6.73 ± 6.66
Three-parameter (parabola)	N	7	7	7	6	7
	M	4	4	4	3	4
	χ^2/M	0.19	1.07	0.87	1.13	0.86
	S_m	0.31	0	0.11	0.04	3.33
	$\sigma \times 10^{29}$, cm^2	8.91 ± 2.93	12.18 ± 8.58	11.74 ± 10.85	12.42 ± 18.55	35.84 ± 23.96

[a] N = number of points to be fitted; M = number of degrees of freedom; S_m = parameter for Fisher F test; σ = extrapolated cross section $\sigma(\gamma + n \rightarrow \pi^- + p)$.

[b] See caption of Fig. 3.

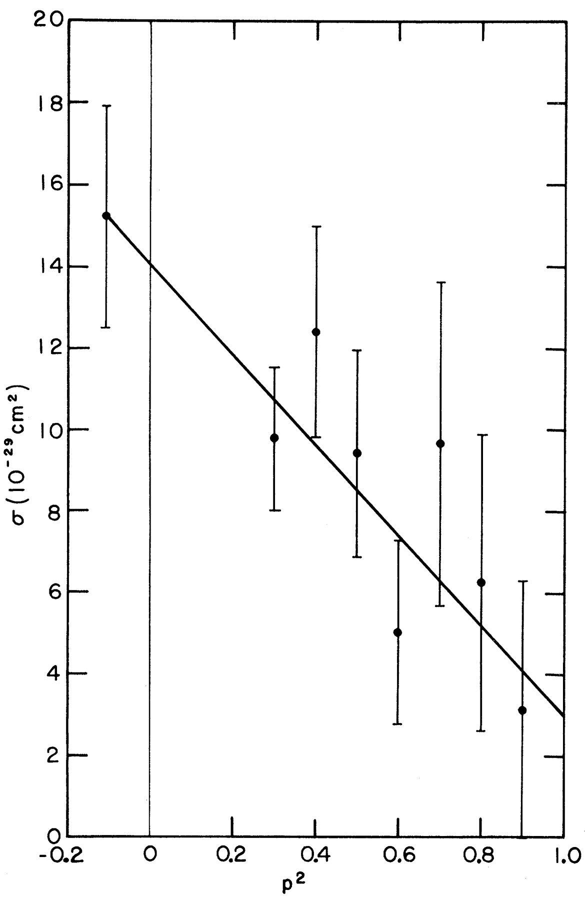

FIG. 2. Polology extrapolation for $w^2 = 61.05$ [k_{eff}(lab) = 163.7 Mev]. The data in the interval $0.3 \leq p^2 \leq 0.9$ are extrapolated by means of a straight line to the nonphysical value $p^2 = -0.107$. At the extrapolated point, the ordinate of the extrapolating function is the cross section $\sigma(\gamma + n \rightarrow \pi^- + p)$.

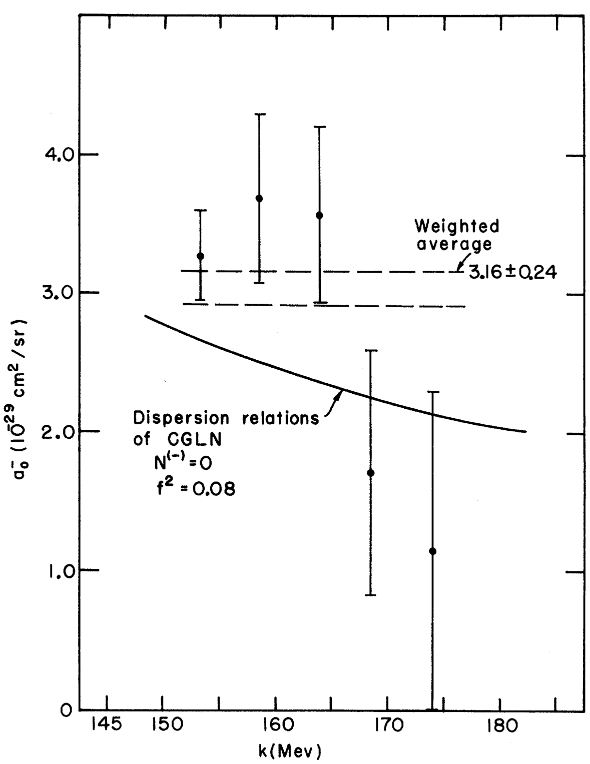

FIG. 3. Values of a_0^- obtained by dividing the extrapolated cross sections of Table I by the phase-space factor $4\pi W$. The curve is based on the dispersion relations of Chew et al. (reference 4), and is taken from the paper of Hamilton and Woolcock (reference 3). The value $\chi^2/M = 0.19$ for the lowest energy point indicates that the purely statistical error may be somewhat too small, and is a reflection of the accidental juxtaposition of the extrapolation points and a straight line.

photon energies by the χ^2 test and the Fisher F test. The two higher-energies gave conflicting results as to the best choice of extrapolation forms. As Table I shows, the errors on the results depend strongly on the polynomial used.

The extrapolated cross sections divided by $4\pi W$, obtained somewhat arbitrarily by using the straight-line extrapolation, are plotted in Fig. 3, where

$$a_0^- = \frac{\sigma(\gamma + n \rightarrow \pi^- + p)}{4\pi W}, \qquad W = \frac{q\omega}{(1 + \nu/M)^2};$$

q is the pion momentum, ω is the pion total energy, and ν is the photon energy, each in the c.m. system, and M is the nucleon mass ($\hbar = c = \mu = 1$). The curve accompanying the points in Fig. 3 is that calculated by Hamilton and Woolcock[3] from the dispersion relation of Chew et al.[4] with $f^2 = 0.08$ and $N^{(-)} = 0$.

When a_0^- is compared with experimental values[5] for a_0^+, an average ratio $R = 1.7 \pm 0.2$ is obtained in the interval from threshold to 174 Mev. These results (not definite) are a first example showing the feasibility (as well as the difficulty) of the Chew-Low extrapolation procedure.

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