PRODUCTION OF PARTICLE BEAMS AT VERY HIGH ENERGIES*

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We consider the production of beams of very-high-energy strongly interacting nuclear particles. By studying the transition amplitudes near their poles corresponding to "real" one-particle intermediate states, we show that photons are very effective in initiating collimated beams of high-energy charged pions and K mesons, (anti-) nucleons, etc. A photon of energy k incident on a nuclear target produces a high-energy charged pion, say, of energy $\omega_q > \frac{1}{2}k$ ($\hbar = c = 1$) in a forward cone of opening angle $\theta_{1/2} \sim \mu/\omega_q$ with a cross section that is reduced by roughly the fine structure constant, $\alpha = 1/137$, from geometric, $\sim 1/\mu^2 = 20$ mb. For nucleon-initiated processes, on the other hand, although one avoids the fine structure constant, the statistical model predicts that very high energy secondaries emerge in only a very small fraction of the collisions.

This result is of significance for predicting and comparing yields from very high energy electron and proton accelerators. Experiments can be performed on existing machines to check the validity of this "pole analysis" which, as applied here to general inelastic processes, is an extension of the work of Chew, Goebel, and particularly of Chew and Low.[1]

Figure 1 illustrates the process being considered, with X the nuclear target and (n) anything else produced in addition to the high-energy pion. Our basic approximation now, in calculating the cross section when the pion is produced with energy $\omega_q \sim k$ and within the forward angular cone at an angle $\theta_q \leq \theta_{1/2} \sim \mu/\omega_q$, is to assume that the amplitude corresponding to Fig. 2 makes the major contribution to this process. In Fig. 2 the photon produces a pair of charged pions, one of which emerges directly with ω_q, θ_q while the other ploughs into X initiating reactions to various final states (n). The cross section of interest is the sum of squares of the amplitudes for all possible states (n). The above approximation is made on the following grounds:

(i) We are near the pole in the production amplitude for Fig. 2 corresponding to a real pion in transit between (a) and (b). The propagator for this pion is

$$-[(k-q)^2 - \mu^2]^{-1} = (2q\cdot k)^{-1} = [2\omega_q k(1-\beta_q \cos\theta_q)]^{-1}, \quad (1)$$

where $k_\mu = (k, \vec{k})$ and $q_\mu = (\omega_q, \vec{q})$ are the photon and pion four-momenta, respectively. It is large for high energy $\omega_q \sim k \gg \mu$ and small angle $\theta_q \lesssim \mu/\omega_q$, equalling approximately

$$\{\mu^2(k/\omega_q)[1+(\theta_q\omega_q/\mu)^2]\}^{-1} \approx 1/\mu^2.$$

This emphasizes the importance of this diagram relative to others for two or more pions or heavier particles propagating from (a) to (b). The next lightest state connecting (a) and (b) contains two pions and the continuous sequence of poles (the branch cut) for this state starts at a mass $(2\mu)^2$, so that the propagator for the two-pion state then is less than $1/4\mu^2$. Another way of saying this is that the emitted high-energy pion is distributed over a cone of broader angle $\theta' > 2\mu/\omega_q$ for heavier intermediate states and so the flux per solid angle at $\theta_q \sim \mu/\omega_q$ is reduced by a factor $< 1/4$.

(ii) We are led to a similar conclusion in considering processes in which more than one pion emerges directly from the electromagnetic

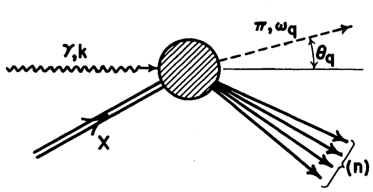

FIG. 1. Diagram for production amplitude $\gamma + X \to \pi + (n)$, where (n) denotes all particles produced in the final state, other than the observed π, from the target X.

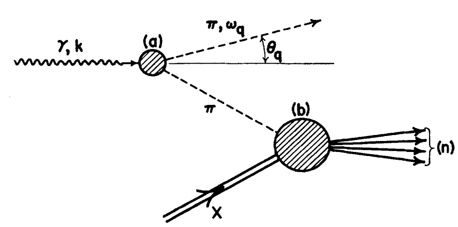

FIG. 2. Diagram for one-pion contribution to the production amplitude $\gamma + X \to \pi + (n)$. This dominates near the pole $(k-q)^2 \approx \mu^2$.

vertex (a). Integration over the undetected final-particle group (p) spreads the cone into which the observed pion emerges to a broad angle of width $\approx$ [energy in particle group (p)]$/\omega_q \gg \mu/\omega_q$.

(iii) Lastly we may consider the amplitude corresponding to one pion being exchanged between (a) and (b) but with the observed high-energy pion emerging from the nuclear part of the interaction (b). We come to the same conclusion as above, namely that the cone into which the observed pion emerges is spread over a larger angle than μ/ω_q.

For these reasons we consider Fig. 2 a first approximation to Fig. 1 for $\omega_q \sim k \gg \mu$ and $\theta_q \lesssim \mu/\omega_q$. We can then compute the production cross section in terms of the pion electromagnetic vertex for a real photon multiplied by the total cross section for a pion of energy $k-\omega_q$ incident on target X leading to all final states (n). This gives directly,[2] with $\beta_q \equiv |\vec{q}|/\omega_q \to 1$,

$$d^2\sigma_{\gamma,\pi^\pm}(k,\omega_q,\theta_q) = \frac{\alpha}{2\pi}\frac{\sin^2\theta_q}{(1-\beta_q\cos\theta_q)^2}\frac{d\Omega_q}{4\pi}\frac{\omega_q(k-\omega_q)d\omega_q}{k^3} \times\sigma_{\pi^\mp+X,\,\mathrm{total}}(k-\omega_q), \qquad (2)$$

for the cross section that a $\pi^\pm$ in energy interval $d\omega_q$ about ω_q and angle interval $d\Omega_q$ about θ_q is produced, along with anything else, by an incident unpolarized quantum of energy k on a target X.

Equation (2) leads to a sizable flux of pions of energy $\omega_q \sim k$ within a cone of angle $\theta_{1/2} = \mu/\omega_q$. Let us compare it with other electromagnetic and nonelectromagnetic mechanisms for producing a π beam. First consider the electromagnetic production of $\pi^\pm$ pairs in the Coulomb field of a charge Ze (Fig. 3). The cross section expressed in terms of the angle and energy distribution of one of the pions is[3]

$$d^2\sigma_{\gamma,\pi^+,\pi^-}(k,\omega_q,\theta_q) = \frac{\alpha}{2\pi}\frac{[(\mu/\omega_q)^4+\sin^4\theta_q]}{(1-\beta_q\cos\theta_q)^4}\frac{d\Omega_q}{4\pi}\frac{\omega_q(k-\omega_q)d\omega_q}{k^3} \times(Z\alpha)^2\frac{2\pi}{\omega_q^{\,2}}\ln(\omega_q/\mu). \qquad (3)$$

In (3) we have neglected $1 < \ln(\omega_q/\mu)$ and taken the relativistic limit for the pions. Equations (2) and (3) are comparable in magnitude if in (2) we

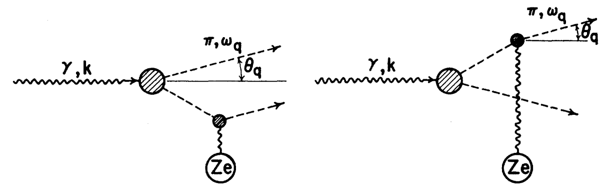

FIG. 3. Diagrams for electromagnetic production of $\pi^\pm$ pairs.

use $\sigma_{\pi+X,\,\mathrm{total}} \approx 30$ mb, as is appropriate for total pion-nucleon cross sections in the Bev range, while in (3) we insert $Z=82$. However, for any given target, process (2) will dominate for $\theta_q \sim \mu/\omega_q$ and $\omega_q \sim k$. Before leaving (3) we call attention to a difference in its derivation from that for (2). Both diagrams in Fig. 3 are necessary in order to arrive at the Pauli-Weisskopf cross section, whereas rescattering of the high-energy pion is not included in Fig. 2 and in the derivation of (2), on the basis of our earlier arguments. This is because the Coulomb scattering of the virtual pion in Fig. 3 concentrates both final pions in the forward cone of $\theta_{1/2} \sim \mu/\omega$ and the two diagrams there are of comparable magnitude and opposite sign.

Next consider production of pions by an incident proton beam on a hydrogen target. Hagedorn and von Behr[4] have calculated pion yields from such collisions on the basis of the statistical model; they predict the fraction of interactions leading to pions produced in given angular and momentum intervals. Their results are presented in graphical form in the CERN tables.[4] Since we are primarily concerned here with the production of high-energy collimated pion beams, we want to compare (2) and the statistical model for $\theta_q \sim \mu/\omega_q$ and $k \sim \omega_q$. Choosing $k = 25$ Bev, $\omega_q = 20$ Bev, and $\theta_q = 1°$ to get an idea of the numbers, we find from (2)

$$d^2\sigma_{\gamma,\pi}/d\Omega_q d\omega_q \cong 10^{-2}\sigma_{\pi+X,\,\mathrm{total}}(5\text{ Bev}) \cong 0.3\text{ mb/sr-Bev},$$

for X a target proton. This is a factor of ten larger than the prediction of the statistical theory,[5]

$$\cong 7\times10^{-4}\sigma_{p+p,\,\mathrm{total}}(25\text{ Bev}) \cong 0.03\text{ mb/sr-Bev}.$$

To be sure the predictions of the statistical model are least reliable at the high-energy tip of the spectrum of produced particles. It is also true that quantitative validity of (2) can be established by experiment alone. Equation (2) may be precise at the pole (1) in the nonphysical region $\cos\theta_q = (1/\beta_q) > 1$, but we cannot theoretically anticipate whether or not there are appreciable corrections in the physical region at $\theta_q \sim \mu/\omega_q$. At present the recent Cornell measurements[6] provide qualitative support for (2) in the energy-angle region of $k \sim 700$-1000 Mev, $\omega_q \sim 500$ Mev, and $\theta_q \sim 15°$. The peaking of their cross sections for mesons at forward angles is naturally explained by the retardation denominator in (2). Also, the total yields as well as the ratio of the π^- to the π^+ yield can be approximately accounted for by known $\pi^\pm$-nucleon cross sections occurring in (2). More detailed studies at larger ω_q and smaller θ_q are desirable.[7]

If the predictions of the statistical model for very high energy pion production in nucleon-nucleon collisions are verified, we conclude from (2) that a photon is more effective than a proton in initiating a high-energy beam of pions.[8]

Finally we turn to K-meson and baryon production. Equation (2) may be transcribed directly to the production of K^+ and K^- beams by changing the pion mass μ to μ_K for the K mesons, and the pion cross sections to $\sigma_{K^\pm + X}(k - \omega_q)$. Since K mesons are heavier than pions, several pions can accompany the K meson from (a) to (b) in Fig. 2 without appreciably broadening the cone of emission of the high-energy $K^\pm$. In this application to K mesons then, Eq. (2) is to be given only qualitative significance in predicting the K beam intensity. It can be checked experimentally in terms of known total $K^\pm$ cross sections at energies $k - \omega_q$. The ratio of intensities of high-energy K^+ to K^- mesons is just the ratio of K^- to K^+ interaction cross sections.

Going further and more recklessly afar from the pole, we can calculate also the production of nucleon and hyperon (anti-) particle beams by the mechanism of Fig. 2, obtaining in the high-energy limit

$$d^2\sigma_{\gamma,\eta}(k,\omega_q,\theta_q) = \frac{\alpha}{2\pi}\,\frac{\omega_q(k-\omega_q)d\omega_q}{k^3}\,\frac{d\Omega_q}{4\pi}\,\sigma_{\eta+X,\,\mathrm{total}}(k-\omega_q)$$
$$\times\left\{\frac{(2-k/\omega_q)(a+\lambda)^2\sin^2\theta_q + \lambda^2\sin^2\theta_q[2-(k\omega_q/2M^2)\sin^2\theta_q]}{(1-\beta_q\cos\theta_q)^2}\right\}, \tag{4}$$

where M is the mass of the particle η, a is its charge in units of e, and λ its Pauli anomalous magnetic moment in units of the Bohr magneton ($a = +1$ for the proton, $\lambda = -1.91$ for the neutron, etc.). We wish to point out in connection with (4) only that it predicts a considerable yield. For example, the cross section to produce antineutrons at $\theta_q = M/\omega_q = 3°$, with $k = 25$ Bev and $\omega_q = 20$ Bev, is

$$5\times10^{-3}\,\sigma_{N+X,\,\mathrm{total}}(5\ \mathrm{Bev})\,\mathrm{sr}^{-1}\,\mathrm{Bev}^{-1},$$

and exceeds the prediction of the statistical theory calculation[4] for a proton-proton collision by a factor of ≈ 50. This is, however, a dangerous and perhaps misleading application.

Very valuable discussions, especially with D. Ritson and J. Ballam, have contributed to these remarks.

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[1]G. F. Chew, Phys. Rev. 112, 1380 (1958); C. J. Goebel, Phys. Rev. Letters 1, 337 (1958); G. F. Chew and F. E. Low, Phys. Rev. 113, 1640 (1959).

[2]The factors are arrived at as follows: $\frac{1}{2}\sum_{s=1}^{2}(2e\vec{\epsilon}_s\cdot\vec{q})^2 = 8\pi\alpha\beta_q^2\omega_q^2\sin^2\theta_q$ for the polarization average of the pion electromagnetic vertex, times $[(q-k)^2-\mu^2]^{-2} = [2\omega_q k(1-\beta_q\cos\theta_q)]^{-2}$ for the square of the propagator, times $d^3q/(2\pi)^3 2\omega_q = \beta_q\omega_q d\omega_q d\Omega_q/16\pi^3$ for the phase space of the observed pion, times $k^{-1}(k-\omega_q)\sigma_{\pi+X,\,\mathrm{total}}(k-\omega_q)$ for the absorption to all states (n) of the intermediate pion, where the factor $(k-\omega_q)/k$ takes into account that the normalization factor for the incident photon is $1/2k$, and $1/2(k-\omega_q)$ for the pion is missing in extracting the cross section $\sigma_{\pi+X,\,\mathrm{total}}(k-\omega_q)$.

[3]W. Pauli and V. Weisskopf, Helv. Phys. Acta 7, 709 (1934); the formula in this reference is a factor of 4 larger than Eq (3) above. W. Pauli, Revs. Modern Phys. 13, 203 (1941); the result in this reference is a factor of 2 larger than Eq. (3) above. Equation (3) is derived for a point Coulomb field and is valid only for small angles $\theta_q < \mu/\omega_q$ where the momentum transfers are $\approx\mu^2/k$. The Bethe-Heitler formula for the electromagnetic pair production of spin 1/2 Dirac par-

ticles of the same mass μ is larger by a factor ~ 6 at $\theta_q \sim \mu/\omega_q$ due to the spin degree of freedom.

[4]R. Hagedorn, Nuovo cimento 15, 434 (1960); J. von Behr and R. Hagedorn, CERN Report 60-20, 1960 (unpublished).

[5]Pion and nucleon total cross sections are comparable above a few Bev and both are $\sim 30-40$ mb for target protons. [See report of V. I. Veksler to Ninth Annual High-Energy Physics Conference, Kiev, U.S.S.R., 1959 (unpublished); also private communication from J. Adams to B. Cork.]

[6]B. M. Chasan, G. Cocconi, V. T. Cocconi, R. M. Schechtman, and D. H. White, Phys. Rev. 119, 811 (1960).

[7]An experiment is now in progress at Stanford University, by L. Hand and W. K. H. Panofsky.

[8]Using the equivalent photon spectrum for a high-energy electron, one can directly compare the yields of pions from incident electron and proton beams. These calculations have been made by J. Ballam, Stanford University High-Energy Physics Laboratory Report M200-8 (unpublished).