described in the last reference of reference 1. Further details regarding YLAM are contained in G. Breit, M. H. Hull, Jr., K. E. Lassila, and K. D. Pyatt, Jr., Phys. Rev. (to be published). A similar designation for n-p fits with YLAN as the first four letters has been partly described in G. Breit, M. H. Hull, Jr., K. Lassila, and K. D. Pyatt, Jr., Phys. Rev. Letters 4, 79 (1960).

[3]Fit YLAN3 is a development on fit YLAN2M with a major difference regarding the sign of ρ_1 through most of the energy range. It includes among other matters the employment of an improved mass treatment for the OPEP as described in the last reference of reference 2 and difference in treatment of OPEP values for $^3F_3$, $^3F_4$, and ρ_4 at different stages of the search.

[4]See last reference of reference 2.

[5]G. Breit and M. H. Hull, Jr., Nuclear Phys. 15, 216 (1960).

DEUTERON PRODUCTION IN HIGH-ENERGY COLLISIONS

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The analysis of the beams of secondary particles resulting from 25-Gev protons hitting several different kinds of targets shows the presence of high-energy deuterons.[1] Deuterons have been observed also in cosmic-ray emulsion work[2] and were interpreted as belonging to the high-energy tail of evaporation.

Though the tail of evaporation and also the nuclear cascade may contribute to the total deuteron yield, it is very probable that the CERN deuterons are mainly due to elementary nucleon-nucleon interactions. If that is so, then deuterons should be produced even in a pure proton-proton collision (and, by the way, not only deuterons but also H^3, He^3, and antideuterons and He^4, if namely one or two antinucleons are created[3]).

It is the aim of the present note to explain this fact and give some numerical results.

We consider a p-p collision and calculate the deuteron production by means of the statistical theory. It can be shown[4] that the statistical theory is derivable from S-matrix theory with the result that (n-particle end state)

$$P_F = S(E, \eta_1 \cdots \eta_n) \times R_F(E, T, T_1 \cdots T_n, S_1 \cdots S_n, m_1 \cdots m_n) \quad (1)$$

is the probability of finding any one of the final states f contained in a set F. Here R_F is the total phase space of all $f \in F$ including isospin, spin, and other statistical factors, whereas $S(E, \eta_1 \cdots \eta_n)$ is the average with respect to F of the matrix element squared. E is the total energy (center-of-mass system) and $\eta_1 \cdots \eta_n$ are those parameters of the n particles which survive the integration over final states (e.g., coupling strength, mass, isospin, structure) and therefore influence still the averaged matrix element squared. Considering the inverse reaction leads to some knowledge of S: Namely, for the inverse reaction the individual matrix elements squared are the same, only the sum over final states of the reaction proper goes over into the average over exactly the same states for the inverse reaction, since initial and final states are now interchanged. The function S is by definition this average, and consequently

$$P_F^{\text{inverse}} = S(E, \eta_1 \cdots \eta_n). \quad (2)$$

On the other hand, the inverse reaction can take place only if the n particles (contained in a normalizing box of volume V) meet in a small volume of order $\Omega = (4\pi/3)(1/\mu)^3$; μ = meson mass ($\hbar = c = 1$); hence $S(E, \eta_1 \cdots \eta_n) \propto (\Omega/V)^{n-1}$. Inserting this into (1) and taking into account the Lorentz contraction leads to the usual formula for the statistical theory, which—with certain refinements—seems to work quite well.

Suppose now that a deuteron appears in the final state. Looking at the reaction proper it seems hard to believe that in Ω a deuteron (which is so much bigger) could be born and escape without being destroyed. But certainly it has its legal place in the possible sets of final states. Now things look better in the inverse reaction: a deuteron comes in and has to join inside Ω the other $n-1$ particles. The condition for this is twofold: It has to be there and, at the same time, it has to be as small as Ω. In a way this "melting in" is a measurement of the size of the deuteron (which of course destroys it—in fact it has to disappear as deuteron) and the probability of finding it as small as Ω (or smaller) is given by the integral over the deuteron wave function:

$$\int_\Omega |\psi_d|^2 dV.$$

Therefore, whereas each pion and nucleon is represented in S by a factor Ω/V, the deuteron gives rise to a factor

$$\frac{\Omega_d}{V} = \frac{\Omega}{V}\int_\Omega |\psi_d|^2 dV. \qquad (3)$$

Taking a Hulthén-type wave function with hard core (the latter has little influence) gives[5] $\int_\Omega |\psi_d|^2 dV \approx \frac{1}{5} - \frac{1}{10}$. With this value for Ω_d one finds the results given in Tables I and II. They show, though they do not apply directly to the experiments (no pp collisions have so far been analyzed with respect to deuterons), that this "elementary production" yields the correct orders of magnitude. The differences may be due to the presence of nuclear matter, to an anisotropy of nucleons in the center-of-mass frame (the transformation c.m. to lab involves the simplifying assumption of isotropy), and to "peripheral" collisions.

It is satisfying that the above picture, which treats the deuteron as a quasi-elementary particle (at least makes no assumptions about how it is formed), can be supported by a kinematical consideration. It can be shown that the above formula can be made plausible also by asking the condition that a neutron and a proton leave the interaction region with a relative momentum, which is "acceptable" as a deuteron internal momentum (it belongs to the Fourier transform of ψ_d). This will be explained elsewhere. Even a detailed final-state interaction treatment[6] leads back to essentially the statistical formula with Ω_d as in (3). The phase-space integrals are computed rigorously (apart from the statistical errors of a Monte Carlo method[7]). All computing work was done on Ferranti-Mercury computers[8] (partly by the author but) mainly by Dr. W. Laskar, University College, London. The author is very grateful to him for not only carrying out the machine runs but also writing up all data tapes.

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Table I. Relative cross sections for deuteron production in pure p-p collisions.

		2.3 Gev	25 Gev
$\sigma_d/\sigma_{\text{inel.}}$	$\Omega_d = \frac{1}{5}$	0.0125	0.024
	$\Omega_d = \frac{1}{10}$	0.006	0.012
$\sigma_{d+\pi}/\sigma_{\text{inel.}}$	$\Omega_d = \frac{1}{5}$	0.0072[a]	10^{-5}
	$\Omega_d = \frac{1}{10}$	0.0036[a]	10^{-5}
$\sigma_{d+\pi}/\sigma_{d+2\pi}$	$\Omega_d = \frac{1}{5}$, $\Omega_d = \frac{1}{10}$	1.73	not calculated

[a]This shows that at this primary energy a strong peak in the deuteron—as well as pion—c.m. spectrum should be observable.

Table II. The (deuteron/proton) ratio in the lab system at 15.9° in pp collisions with 25-Gev primary energy.

Momentum of d and p in Gev/c	$\Omega_d = \frac{1}{5}$	$\Omega_d = \frac{1}{10}$	Experiment p-Al[a]
2	0.002 %	0.001 %	approximately
4	0.6 %	0.3 %	constant, 2 %,[b]
6	2.8 %	1.4 %	between 2.6 and
8	7.8 %	3.9 %	5.5 Gev/c
10	8.2 %	4.1 %	

[a]See reference 1.
[b]Note that the experiment refers to p-Al collisions and the theory to pure p-p collisions.

[1]V. T. Cocconi, T. Fazzini, G. Fidecaro, M. Legros, N. H. Lipman, and A. W. Merrison, Phys. Rev. Letters 5, 19 (1960).
[2]C. F. Powell, P. H. Fowler, and D. H. Perkins, Study of Elementary Particles by the Photographic Method (Pergamon Press, New York, 1959), p. 440.
[3]Calculations including these particles are in progress.
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[5]Y. Yamaguchi (private communication).
[6]L. I. Schiff (private communication), and CERN Report No. 60-32 (unpublished).
[7]F. Cerulus and R. Hagedorn, Suppl. Nuovo cimento 9, 646 and 659 (1958).
[8]R. Hagedorn, CERN Report No. 59-25 (unpublished).