EXTENSION OF THE ISOBARIC NUCLEON MODEL FOR PION PRODUCTION IN PION-NUCLEON AND NUCLEON-NUCLEON COLLISIONS*

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We have previously proposed an isobaric nucleon model of pion production in pion-nucleon and nucleon-nucleon collisions.[1,2] In this model, it is assumed that the nucleon is excited to an isobaric level which subsequently decays into a nucleon and one or several pions. The isobaric state which has been considered in our previous calculations is the $T=J=3/2$ state, to be denoted by N_1^*, which corresponds to the resonance of the π^+-p system at 180-Mev pion energy. For π-N interactions, we have considered the following reactions:

$$\pi+N\to\mathfrak{N}_0^*\to N_1^*+\pi, \qquad (1)$$

$$N_1^*\to N+\pi, \qquad (2)$$

where $\mathfrak{N}_0^*$ is the initial compound state of the π-N system, which subsequently decays into $N_1^*+\pi$. The probability $P_{3/2}\,dm_I$ for exciting the isobar N_1^* in the mass range between m_I and m_I+dm_I was obtained from

$$P_{3/2}(m_I)dm_I=C\sigma_{3/2}(m_I)F\,dm_I, \qquad (3)$$

where $\sigma_{3/2}(m_I)$ is the total π^+-p cross section at the appropriate π^+ energy, F is the two-body phase space factor, and C is a constant. This model has been found to be in reasonable agreement with most of the dominant features of a number of experiments on inelastic π-N and N-N interactions.[3-8]

It has been recently shown[9,10] that the $T=1/2$ cross section for the π-N system, $\sigma_{1/2}$, has two pronounced maxima in the region of 500-1000 Mev pion kinetic energy T_π, one at $T_\pi\cong600$ Mev, the other at $T_\pi\cong880$ Mev. If we assume that these maxima are due to higher isobaric states of the nucleon, to be denoted by N_{2a}^* and N_{2b}^*, then we can expect that pion production will proceed via these states, in addition to the N_1^* processes (1) and (2). We will consider together the two states N_{2a}^* and N_{2b}^*, and will therefore refer to the group of the $T=1/2$ states as N_2^*. Thus we have the following possible reactions:

$$\pi+N\to\mathfrak{N}_0^*\to N_2^*+\pi, \qquad (4)$$

$$N_2^*\to N+\pi, \qquad (5)$$

$$N_2^*\to N_1^*+\pi;\;\; N_1^*\to N+\pi. \qquad (6)$$

Reactions (5) and (6) correspond to alternative modes of decay of N_2^*. In these reactions, it is assumed that any transition between isobaric states proceeds by emission of a single pion.

In analogy with our previous work,[2] we obtain the probability $P_{1/2}(m_I)$ for exciting the isobars N_2^* from an expression similar to Eq. (3), in which $\sigma_{3/2}(m_I)$ is replaced by $\sigma_{1/2}(m_I)$. The use of the experimental cross section $\sigma_{1/2}(m_I)$ for all possible mass values m_I implies that we include the so-called background between the resonances as in a sense representing possible unresolved levels in the vicinity, or the tails of levels centered at other energies.

In the following, we consider only the reactions (4) and (5) leading to single pion production via N_2^*. The momentum spectra for the recoil pion from (4) and for the decay pion from (5) will be called $J_{\pi,3}$ and $J_{\pi,4}$, respectively. These spectra are determined in the same manner as the spectra $J_{\pi,1}$ [decay pion from (2)] and $J_{\pi,2}$ [recoil pion from (1)] obtained in II [see Eqs. (4)-(8)].

Figure 1 shows the spectra $J_{\pi,1}$-$J_{\pi,4}$ for an incident pion energy $T_{\pi,\text{inc}}=1.0$ Bev. The fact that the decay pions from $N_2^*(J_{\pi,4})$ have a large average momentum is a direct consequence of the large mass difference between N_{2a}^* and the nucleon.[11] The mass m_I which corresponds to

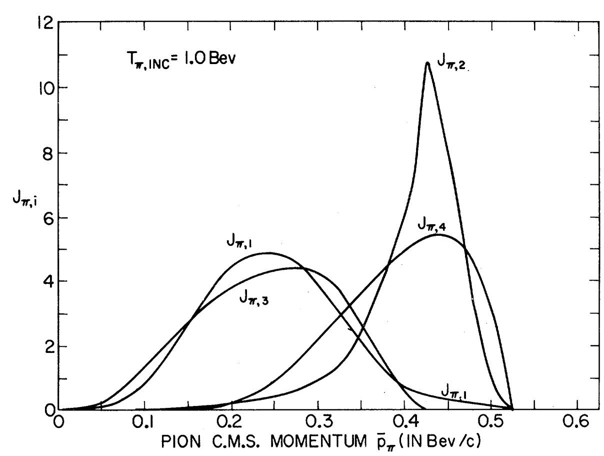

FIG. 1. Center-of-mass momentum spectra $J_{\pi,1}$-$J_{\pi,4}$ for an incident pion energy $T_{\pi,\text{inc}}=1.0$ Bev. $J_{\pi,1}$ and $J_{\pi,2}$ pertain to the N_1^* reactions (1) and (2); $J_{\pi,3}$ and $J_{\pi,4}$ pertain to the N_2^* reactions (4) and (5).

$T_\pi = 600$ Mev is $m_I = 1.51$ Bev, so that $m_I - m_N = 0.57$ Bev. The fact that the recoil pions from reaction (4) ($J_{\pi,3}$) have a relatively low momentum is also a consequence of the large mass of N_{2a}^*, which leaves relatively little kinetic energy for the recoil pion when N_{2a}^* is excited with a mass $m_I \sim 1.5$ Bev.

We have obtained the momentum spectra of the π^+, π^0, and π^- mesons from the reactions $\pi^- + p \to \pi^- + \pi^+ + n$, and $\pi^- + p \to \pi^- + \pi^0 + p$. These reactions will be denoted by (I) and (II), respectively. The cross section for producing $N_\alpha^* + \pi$ ($\alpha = 1$ or 2) [Eqs. (1) and (4)] in the state of total isotopic spin T will be denoted by $\sigma_{2T,\alpha}$. There are four cross sections of this kind: σ_{11}, σ_{12}, σ_{31}, and σ_{32}. We define ρ_1 and ρ_2 by $\rho_1 \equiv \sigma_{31}/2\sigma_{11}$; $\rho_2 \equiv \sigma_{32}/2\sigma_{12}$. The phase angles between the $T = 3/2$ and $T = 1/2$ matrix elements will be denoted by φ_1 and φ_2 for reactions (1) and (4), respectively.[12]

The total cross sections for reactions (I) and (II) are given by

$$\sigma^{(\mathrm{I})} = \tfrac{2}{3}\sigma_{11}(\tfrac{5}{9} + \tfrac{26}{45}\rho_1 + \tfrac{7}{9}a) + \tfrac{4}{9}A\sigma_{12,s}, \quad (7)$$

$$\sigma^{(\mathrm{II})} = \tfrac{2}{3}\sigma_{11}(\tfrac{2}{9} + \tfrac{17}{45}\rho_1 - \tfrac{5}{9}a) + \tfrac{2}{9}\sigma_{12,s}(A + 2B), \quad (8)$$

where

$$A \equiv \tfrac{2}{3} + \tfrac{1}{3}\rho_2 + \tfrac{2}{3}\sqrt{2}\,\rho_2^{1/2}\cos\varphi_2, \quad (9)$$

$$B \equiv 1 + \rho_2 - A, \quad (10)$$

$a \equiv (2/5^{1/2})\rho_1^{1/2}\cos\varphi_1$, and $\sigma_{12,s}$ is the part of σ_{12} which goes into single pion production.

The differential cross section as a function of center-of-mass momentum $\bar{p}_\pi$ for the π^- from reaction (II) is given by

$$(d\sigma/d\bar{p}_\pi)^{(\mathrm{II})}(\pi^-) = \tfrac{2}{3}\sigma_{11}[(\tfrac{1}{9} + \tfrac{1}{45}\rho_1 - \tfrac{1}{9}a)J_{\pi,1} + (\tfrac{1}{9} + \tfrac{16}{45}\rho_1 - \tfrac{4}{9}a)J_{\pi,2}] + \tfrac{2}{9}\sigma_{12,s}(AJ_{\pi,3} + 2BJ_{\pi,4}). \quad (11)$$

The π^0 differential production cross section is obtained from (11) by interchanging $J_{\pi,1}$ with $J_{\pi,2}$, and $J_{\pi,3}$ with $J_{\pi,4}$.

For reaction (I), the π^+ mesons made via N_2^* are predominantly fast (term in $d\sigma/d\bar{p}_\pi \propto J_{\pi,4}$), whereas the π^- mesons are on the average of lower energy (spectrum $\propto J_{\pi,3}$). Hence the introduction of an admixture of N_2^* reactions will not affect the agreement with experiment previously obtained[2] for the π^- and π^+ from reaction (I).

There are three experiments on inelastic $\pi^- - p$ interactions in the region of $T_{\pi,\mathrm{inc}} \cong 1.0$ Bev.[6-8] Concerning the reaction $\pi^- + p \to \pi^- + \pi^0 + p$, for the Bologna (Alles-Borelli et al.[7]) and Brookhaven (Ayer, Pickup, and Salant[8]) experiments at $T_{\pi,\mathrm{inc}} = 0.96$ Bev, the average momentum of the π^- is appreciably higher than that of the π^0 mesons, whereas for the Göttingen (Derado and Schmitz[6]) experiment at 1.0 Bev, there is a preponderance of slow π^- and fast π^0 mesons. Our previous isobar model work involving N_1^* only gives good agreement with the Bologna-Brookhaven spectrum of π^- and π^0. If the difference between the Göttingen and Bologna-Brookhaven experiments should indeed be real, we can approximately fit the Göttingen experiment by means of the present extended isobar model by allowing a relatively large admixture of N_2^* processes and taking $\varphi_2 = 0°$. However, it is perhaps reasonable to add together the pion momentum distributions from the three experiments, and we have obtained an approximate fit to the combined distributions, using the present isobar model, with $\sigma_{12}/(\sigma_{11} + \sigma_{12}) = 0.2$.

From the total inelastic cross sections for $\pi^- - p$ (~22 mb),[6] and for $\pi^+ - p$ (~10 mb),[13] from the cross section for double pion production (~3 mb),[6,7] and from the requirement that the ratio $R \equiv \sigma^{(\mathrm{II})}/\sigma^{(\mathrm{I})}$ should equal the experimental value of 0.58 for the combined experiments at 1.0 Bev, we obtain the following values for the parameters: $\sigma_{11} = 22$ mb, $\sigma_{12} = 5.6$ mb, $\sigma_{12,s} = 3.1$ mb, $\rho_1 = \rho_2 = 0.18$, $\varphi_1 = 105°$, $\varphi_2 = 0°$, which gives: $A = 1.13$, $B = 0.05$; $\sigma^{(\mathrm{I})} = 10.3$ mb, and $\sigma^{(\mathrm{II})} = 6.0$ mb.

The resulting theoretical spectra for the π^- and π^0 from reaction (II) are shown in Figs. 2(a) and 2(b), together with the combined histograms from the 3 experiments at 1.0 Bev. For the π^- spectrum, the theoretical curve lies somewhat below the histogram at low momenta ($\bar{p}_\pi = 0.15$-0.3 Bev/c), and correspondingly, the present isobar model predicts too many high-energy π^- ($\bar{p}_\pi = 0.4$-0.5 Bev/c). For the π^0 mesons, the theoretical curve from the isobar model is in good agreement with the experimental histogram.[14]

We have also obtained the branching ratios for the various pion-producing reactions for p-p and n-p collisions. The following reactions were considered: $p + N \to N + N_1^*$; $p + N \to 2N_1^*$; $p + N \to N + N_2^*$; $p + N \to N_1^* + N_2^*$; $p + N \to 2N_2^*$. The first two reactions have been previously investigated in I. The present extension of the model allows up to four-pion final states in π-N and up to eight-pion final states in N-N interactions.

For pion production in nucleon-antinucleon in-

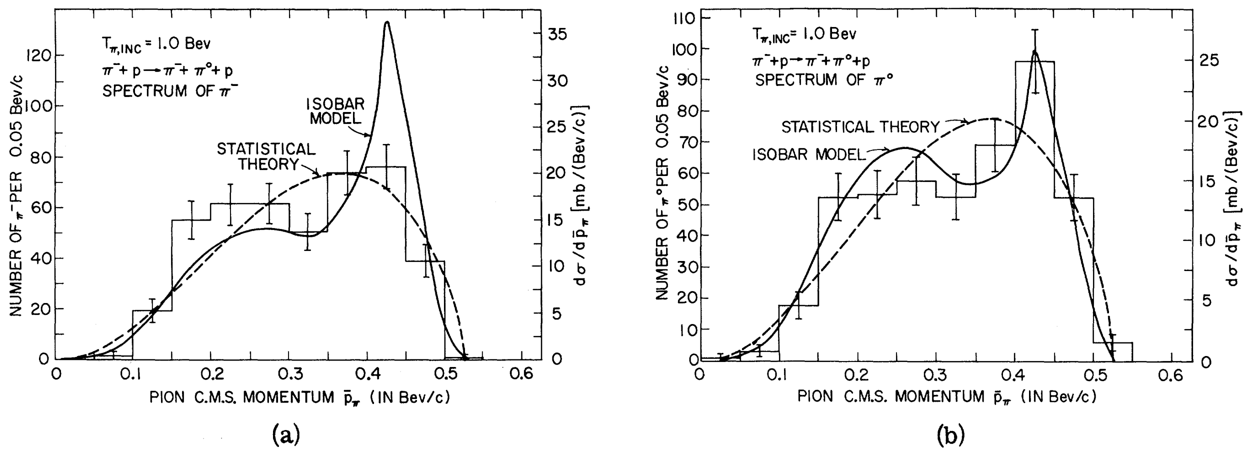

FIG. 2. Center-of-mass momentum spectra of the π^- and π^0 mesons from the reaction $\pi^- + p \to \pi^- + \pi^0 + p$ at $T_{\pi,\,\mathrm{inc}} = 1.0$ Bev. (a) π^- mesons; (b) π^0 mesons. In each figure, the histogram represents the combined data of the experiments of Derado and Schmitz (reference 6), Alles-Borelli et al. (reference 7), and Ayer, Pickup, and Salant (reference 8). The solid curve was obtained from the present isobar model. The dashed curve gives the result of the Fermi statistical theory. The theoretical curves and the histogram are normalized to the same area.

teractions without annihilation, we can assume that the reaction may proceed via the formation of an anti-isobar $\bar{N}_\alpha{}^*$ which is the antiparticle of the isobar $N_\alpha{}^*$. Specifically, for single pion production, we have the following two reactions, which have equal cross sections:

$$\bar{N} + N \to \bar{N}_1{}^* + N \to \bar{N} + N + \pi, \qquad (12)$$

$$\bar{N} + N \to N_1{}^* + \bar{N} \to \bar{N} + N + \pi, \qquad (13)$$

where $\bar{N}$ = antinucleon. For $\bar{p}$-p collisions, the probabilities P for the four possible final states are: $P(\bar{p}n\pi^+) = P(\bar{n}p\pi^-) = 1/6$; $P(\bar{p}p\pi^0) = P(\bar{n}n\pi^0) = 1/3$. The pion spectrum is given by $I_{\pi,s}$ as defined in I [Eq. (21) and Fig. 3]. The spectrum for either the nucleon or the antinucleon from each of the four reactions is $\frac{1}{2}(I_{N,1} + I_{N,2})$ in the notation of I [Eq. (47) and Figs. 7, 8].

A more complete account of the present work will be given in a future publication.

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[12]We note that ρ_1 and φ_1 are the same as the quantities ρ and φ, respectively, defined in II [Eqs. (20) and (21)]. The interference between the $T = 1/2$ and $T = 3/2$ channels is exactly taken into account. The approximation is made (as previously) that the isobar lifetime is long enough so that interference effects due to its decay are small. See also S. Bergia, F. Bonsignori, and A. Stanghellini, Nuovo cimento (to be published).

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