PION CLOUD EFFECTS AND THE TWO-CENTER MODEL OF COSMIC-RAY JETS

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(Received July 12, 1960)

Recent investigations of high-energy nuclear interactions[1-6] have shown that at energies above 1 Gev an important part is played by peripheral collisions involving only one or two pions in the meson cloud of the colliding nucleons. In the present note, an attempt is made to interpret the two-cone structure of cosmic-ray jets along these lines.

Indeed, it has been shown[7,8] that most jets can be interpreted as two bunches of particles emerging from two centers moving in opposite directions each with a Lorentz factor $\bar{\gamma}$ in the center-of-mass system of the colliding nucleons. Until now, $\bar{\gamma}$ has been treated as a free parameter, supposed to be related in some way or other to the impact parameter and providing essentially an estimate of the inelasticity of the collision. Perhaps the most remarkable fact of the results obtained until now by means of the two-center model is the cluster of values of $\bar{\gamma}$ about a mean value ~1.5 - 1.6.

On the other hand, the pion-cloud effects detected at lower energy ought to appear even more pregnantly at the high energies of the cosmic-ray jets, owing to the very short deBroglie wavelengths of the colliding particles. This means that at reasonable impact parameters, each of the two nucleons has a large chance to collide with one single pion in the meson cloud of its collision partner. The process is then essentially a pair of head-on pion-nucleon collisions.[9,10] Owing to the fact that the incoming nucleon and its pion cloud move with the same velocity in the lab system (Lorentz factor γ_0), the kinematics of the two quasi-independent pion-nucleon collisions automatically lead to a fixed value of $\bar{\gamma}$ at high γ_0 values (say, above 10^{11} ev).

Let γ_1 be the Lorentz factor for the collision of the incoming nucleon with the pion at rest in the lab system, and γ_2 the corresponding quantity for the collision of the incoming pion with the target nucleon. We have then (see reference 10)

$$\gamma_1 = (\gamma_0 + \mu)(1 + \mu^2 + 2\mu\gamma_0)^{-1/2}, \tag{1}$$

$$\gamma_2 = (\mu\gamma_0 + 1)(1 + \mu^2 + 2\mu\gamma_0)^{-1/2}, \tag{2}$$

where μ is the pion rest mass ($M = c = 1$). For $\gamma_0 \gg 1$ these expressions reduce to

$$\gamma_1 \approx (\gamma_0/2\mu)^{1/2}, \tag{3}$$

$$\gamma_2 \approx (\mu\gamma_0/2)^{1/2}. \tag{4}$$

Therefore, obviously,

$$\gamma_1\gamma_2 \approx \gamma_0/2 \approx \gamma_c^{\,2}, \tag{5}$$

where γ_c is the Lorentz factor of the c.m. system of the colliding nucleons with respect to the laboratory frame,

$$\gamma_1/\gamma_2 = (\gamma_0 + \mu)/(\mu\gamma_0 + 1) \approx \mu^{-1}, \text{ (for large } \gamma_0) \tag{6}$$

Table I. Comparison of computed and observed quantities for the two-center model. All experimental γ values are Castagnoli estimates[a]; for $\gamma_c \geq 7$, they have to be corrected by a factor[b, c] $f \sim 0.7$. At lower γ_c values this correction factor approaches unity.[b] Since $\bar{\gamma}$ varies as $\gamma_c^{1/2}$, the corresponding correction factor has been taken as $\sim f^{1/2}$. In the range $5 < \gamma_c \leq 10$ the interpolation formula deduced in reference 1 has been used. The expected values are weighted averages.

Energy range	$\langle\gamma_c\rangle$	$\bar{\gamma}$ Computed	$\bar{\gamma}$ Observed	$\bar{\gamma}$ Corrected	γ_1/γ_2 Computed	γ_1/γ_2 Observed	γ_1/γ_2 Corrected
$1 < \gamma_c \leq 5$	3.7	1.37	1.35 ± 0.05	...	5.20	5.15 ± 0.58	...
$5 < \gamma_c \leq 10$	7.0	1.43	1.56 ± 0.06	1.39 ± 0.05	6.45	8.00 ± 1.18	6.40 ± 0.95
$\gamma_c > 10$	44.3	1.49	1.89 ± 0.13	1.59 ± 0.11	6.67	13.70 ± 2.42	9.56 ± 1.69
All jets	19.3	1.49	1.63 ± 0.06	1.47 ± 0.05	6.67	9.45 ± 0.06	6.61 ± 0.75

[a]C. Castagnoli, G. Cortini, C. Franzinetti, A. Manfredini, and D. Moreno, Nuovo cimento 10, 1539 (1953).
[b]E. M. Friedländer, Nuovo cimento 12, 483 (1959).
[c]P. Jain, E. Lohrmann, and M. Teucher, Phys. Rev. 115, 643 (1959).

and

$$\bar{\gamma} = (\mu+1)\gamma_c(1+\mu^2+2\mu\gamma_0)^{-1/2} \approx \tfrac{1}{2}(\mu^{1/2}+\mu^{-1/2}), \quad (7)$$

in the same approximation as above.

Hence, at large energies we would expect

$$\gamma_1/\gamma_2 \approx 6.67, \quad (8)$$

and

$$\bar{\gamma} \approx 1.49. \quad (9)$$

At low energies (say, $\gamma_c < 7$ or $\gamma_0 < 100$), both γ_1/γ_2 and $\bar{\gamma}$ rapidly decrease and approach unity.

According to the above ideas, an analysis has been made of 78 jets (p, n, and α events)[11] measured in the Prague Laboratory. The results are shown in Table I. The agreement between the expected and the observed values and their trends is obvious.

Further applications of this method may be thought of for the case of nucleon-nucleus and nucleus-nucleus collisions, where peripheral interactions are expected to be sometimes superimposed on head-on nucleon-nucleon collisions.

The author is very indebted to Dr. J. Pernegr, who kindly put at his disposal detailed data of the jets recorded and measured in the Prague part of Po-Stack No.1.

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PROOF OF THE MANDELSTAM REPRESENTATION IN PERTURBATION THEORY*

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(Received August 9, 1960)

I wish to report a proof that the Mandelstam representation[1] for a scattering amplitude is valid for every term in the perturbation series for the amplitude. The proof applies to any system of interacting particles that does not have anomalous thresholds. The latter requirement can be expressed by the mass conditions obtained in fourth order. In this Letter I will outline the main steps of the proof, using the equal-mass case as an illustration. The details are contained in two papers[2,3] in the course of publication and a third[4] shortly to be submitted.

The invariant energies squared will be denoted s, t, and u; a term in the expansion of the amplitude $A(s,t)$ will be written $F(s,t)$. The main steps include the following:

(1) The physical branch of F in a physical scattering region has the representation

$$F(s,t) = \lim_{\epsilon\to 0} c\int_0^1 d\alpha_1\cdots d\alpha_n \frac{\delta(1-\sum\alpha_i)[C(\alpha)]^{n-2l-1}}{[D_\epsilon(\alpha,s,t)]^{n-2l}},$$

where we have (I, Sec. 6),[2]

$$D_\epsilon(\alpha,s,t) = sf(\alpha)+tg(\alpha)-m^2K(\alpha)+i\epsilon C(\alpha)\sum\alpha_i$$
$$= D(\alpha,s,t)+i\epsilon C(\alpha)\sum\alpha_i.$$

Singularities (i.e., branch points) of F will occur when D_ϵ has end-point zeros or coincident zeros (pinching the integration contour) in each α variable as ϵ approaches 0. End-point zeros can be reinterpreted by using reduced diagrams.

(2) $D(\alpha,s,t)$ is negative for real positive α in the region $s>0$, $t>0$, $u>0$. Hence F is real in this region [I, Secs. 4 and 8 (F)].

(3) The only straight lines of singularities of the physical branch of F are normal thresholds, since they must intersect a physical scattering region (I, Sec. 5).

(4) Curves of singularities $\Gamma(s,t)$ of F in the real s,t plane have slope (I, Sec. 7)

$$dt/ds = -f(\alpha)/g(\alpha),$$

and they have normal thresholds as asymptotes.

(5) From (2), (3), and (4) it can be shown (III)[4]