The transition probabilities $W(E2,\gamma)$ are known only for the $2^{+'} \to 0$ transitions; therefore, it is necessary to use the theoretical ratio (1) to determine the relative transition probabilities for use in (2). The relation

$$\mu(2^{+'}; 2^{+'} \to 2^{+}) \equiv \frac{W(E0, e^{-}; 2^{+'} \to 2^{+})}{W(E2, \gamma; 2^{+'} \to 2^{+})} = \alpha - \alpha_2,$$

where α_2 is the theoretical $E2$ conversion coefficient, gives the desired ratio of transition probabilities. The monopole transition probability $W(E0, e^{-}; 2^{+'} \to 2^{+})$ was determined in turn from (1) and the experimental $B(E2; 2^{+'} \to 0)_d$ values. The nuclear monopole strength parameter, ρ, is defined by

$$W(E0, e^{-}) = \Omega \rho^2,$$

where Ω, the electronic factor, is given graphically.[2]

Reiner[3] has treated the problem of monopole enhancement for deformed nuclei and has derived a formula for the ratio of the $E0$ to $E2$ transition probabilities from β-vibrational states. For Th^{232} his formula leads to $\mu(2^{+'}; 2^{+'} \to 2^{+}) = 3.8$, a value which is intermediate between the experimental values obtained by Methods A and B. For U^{238}, however, Reiner's formula gives $\mu(2^{+'}; 2^{+'} \to 2^{+}) = 1.4$ which is a factor of ten larger than the value obtained experimentally. The explanation of the discrepancy is not apparent in terms of the simple vibrational model used by Reiner, since Th^{232} and U^{238} have similar atomic numbers, equilibrium deformations, and vibrational energies which are the only parameters entering into the calculation. The values obtained for the nuclear strength parameter ρ are close to the approximate value of $\frac{1}{5}$ predicted by Church and Weneser for $0^{+} \to 0^{+}$ transitions from vibrational states in spheroidal nuclei.[2] Again, however, it is not clear why the value of ρ should differ by a factor of two between Th^{232} and U^{238}. Perhaps one sees in these discrepancies the influence of the different ground-state configurations.

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NONLOCAL POTENTIAL AND ALPHA-DECAY BARRIER PENETRABILITY

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Since the early works on the alpha disintegration problem, the barrier for the alpha emission has traditionally been taken to be purely Coulombian with an abrupt cutoff at the nuclear boundary defined by some appropriate $A^{1/3}$ law. Alpha decay was first considered[1] by the present author by assuming that the potential barrier is not purely Coulombian and should be taken as $2(Z-2)e^2/r - V$, where V is defined as the short-range interaction between the emitted alpha particle and the residual nucleus. Subsequently, the form of the interaction, V, which is superposed on the Coulomb field, has been taken differently by different authors[2-5] in their calculations of the barrier penetrability.

It is, however, worth while to note that in the above calculations of the penetrability factor, the nuclear part of the barrier has been taken to be purely static, while the careful analyses of the recent experimental findings strongly suggest[6-9] the velocity dependence of both the nucleon-nucleus and the alpha-nucleus interactions. On the other hand, owing to the uncertainties in the knowledge of the effect of the intranuclear dynamics on the half-lives of the alpha-emitting elements, it is important that the calculation of the penetrability factor be made with reasonable certainty by taking all such points into consideration.

The purpose of the present work is therefore to study the effects of the velocity-dependent character of the alpha-nucleus interaction on the magnitude of the alpha-decay barrier penetrability factor. For this purpose we have taken an interaction kernel possessing a nonlocal part represented by a δ function approximated by a Gaussian exponential, so that

$$V(\vec{r},\vec{r}\,') = -V_0 f\left(\left|\frac{\vec{r}+\vec{r}\,'}{2}\right|\right)\delta_b(\vec{r}-\vec{r}\,'), \qquad (1)$$

and

$$\delta_b = \pi^{-3/2} b^{-3} \exp\left[-\left(\frac{r-r'}{b}\right)^2\right], \qquad (1.1)$$

where V_0 and the form function, $f(r)$, of the static part are taken from Igo's potential, viz., $V_0 = 1100$ Mev and $f(r) = \exp[-(r - 1.17A^{1/3})/0.574]$, r being in fermis; and the extent of the nonlocality, b, is given by Frahn[10] for nucleons as $b = 0.902\times 10^{-13}$ cm, which is also assumed by us to be the range of nonlocality of nuclear potential for α particles. Now using the above potential in the integro-differential equation analogous to that taken by Frahn and Lemmer,[11] we find the radial part of the Schrödinger equation for the emitted alpha particle to be

$$d^2u_l/dr^2 + (2\mu/\hbar^2)[\epsilon(r)E - (\hbar^2/2\mu)l(l+1)/r^2$$
$$- 2(Z-2)e^2\epsilon(r)/r + V_0 f(r)\epsilon(r)]u_l$$
$$+ \eta\epsilon(r)\{f''(r)/4 - f'/2r + f'u_l'/u_l\} = 0, \qquad (2)$$

where

$$\epsilon(r) = [1+\eta f(r)]^{-1} \text{ and } \eta = (\mu b^2/2\hbar^2)V_0. \qquad (3)$$

Now in connection with the studies of nuclear energies, Green[12] has shown that the terms corresponding to the part within second braces in (2) contribute negligible shift of the eigenvalues. Therefore, neglecting in Eq. (2) the terms involving derivatives of $f(r)$, and using the WKB method of solution for simplicity, we find that the penetrability factor for the ground state transitions is given by

$$\exp\left\{-2\left(\frac{(2\mu)^{1/2}}{\hbar}\right)\int_{r_i}^{r_0}\left[\frac{2(Z-2)e^2\epsilon(r)}{r} - V_0 f(r)\epsilon(r) - E\epsilon(r)\right]^{1/2} dr\right\}. \qquad (4)$$

The integral in the exponential has been calculated numerically by using Simpson's rule and taking 120 strips. The turning points r_i and r_0 remain unaffected as for a static potential since the modifying factor $\epsilon(r)$ in the integral is common to all the terms.

It is interesting to note that the effect of the nonlocal interaction is quite remarkable. For example, in the case of $_{100}Fm^{254}$, calculation gives a value for the penetrability factor of 5.848×10^{-24}, which is about double the value obtained with the corresponding static potential, viz., 3.524×10^{-24}.

Numerical calculations for the whole series of the alpha-active elements and for their excited-state transitions are in progress and a detailed report will be published in due course.

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