COLLECTIVE OSCILLATIONS AND GIANT DENSITY FLUCTUATIONS*

R. Brout
Los Alamos Scientific Laboratory, University of California, Los Alamos, New Mexico,
and Department of Physics and Laboratory of Atomic and Solid-State Physics,
Cornell University, Ithaca, New York
(Received July 29, 1960)

It is the purpose of this note to interpret an instability in compressional modes found by Glassgold, Heckrotte, and Watson (GHW).[1] We show that their instability is related to the Hartree-Fock (HF) treatment of many-body systems by Overhauser,[2] who claims that HF theory always predicts a ground state in a Fermi gas with a stationary density fluctuation of a given wavelength present. Kohn and Nettel (KN)[3] give a demonstration that for three dimensions this cannot in general be the case for weak interactions. The present work is in support of KN. However, in addition to their negative result, we find that in a linearized theory, a sufficiently strong attractive interaction gives rise to stable HF solutions with a built-in density ripple, as well as a GHW blow-up. For either repulsive or weak attractive interactions there is neither instability in GHW nor a self-consistent density ripple. This is in complete analogy to the elegant analysis of Wolff[4] on the spin density case. In neither of these cases is one involved in a BCS type transition which takes place for infinitesimal attractive interactions.[5] Further the present method seems inapplicable to study the well-known lattice type transition of the low-density electron gas. A possible application, discussed at the end of this note, is the theory of freezing of He^3.

Our starting point is Overhauser's SCF analysis.[6] Since in this work we discuss density fluctuations we shall, as do KN, confine ourselves to the Hartree field alone. This is permissible if spin is unimportant and brings out the qualitative aspects of the situation. The Schrödinger equation for a state $\vec{k}$ with energy $E_{\vec{k}}$ and wave function $\phi_{\vec{k}}$ (normalized in a box Ω) in Hartree theory is

$$[-(\hbar^2/2m)\nabla^2 + V(\vec{r})]\phi_{\vec{k}} = E_{\vec{k}}\phi_{\vec{k}}, \tag{1}$$

$$V(\vec{r}) = \sum_{\vec{k}} \int \phi_{\vec{k}}^*(\vec{r}')v(\vec{r}-\vec{r}')\phi_{\vec{k}}(\vec{r}')d^3r'. \tag{2}$$

Here $v(\vec{r})$ is a two-body potential assumed to have a range. With Overhauser we search for density fluctuations of wave number $\vec{q}$; i.e., $V(\vec{r})$ has only a single Fourier component. In perturbation theory, we have

$$\phi_{\vec{k}} = \Omega^{-1/2}\Big\{e^{i\vec{k}\cdot\vec{r}} + [g^+/\omega_{\vec{q}}(\vec{k})]e^{i(\vec{k}+\vec{q})\cdot\vec{r}} + [g^-/\omega_{-\vec{q}}(\vec{k})]e^{i(\vec{k}-\vec{q})\cdot\vec{r}}\Big\}, \tag{3}$$

$$\omega_{\vec{q}}(\vec{k}) = [(\vec{k}+\vec{q})^2 - \vec{k}^2]/2m, \tag{4}$$

$$g^{\pm} = \int e^{-i(\vec{k}\pm\vec{q})\cdot\vec{r}} V(\vec{r}) e^{i\vec{k}\cdot\vec{r}} d^3r = \int e^{\pm i\vec{q}\cdot\vec{r}} V(\vec{r}) d^3r. \tag{5}$$

Substituting the solution (3) with the definitions (4) and (5) into Eq. (2) and carrying linear terms in $g^+(\vec{k})$ only (which is consistent with our general linear development), we have

$$g^+ = [v(\vec{q})/\Omega]\sum_{\vec{k}'}[g^-/\omega_{-\vec{q}}(\vec{k}') + g^+/\omega_{\vec{q}}(\vec{k}')]. \tag{6}$$

Multiplying Eq. (6) by $1/\omega_{\vec{q}}(\vec{k})$, combining with the equation for $g^-(\vec{k})$, and summing on $\vec{k}$ gives the necessary condition that a nontrivial solution exists:

$$1 = [v(\vec{q})/\Omega]\sum_{\vec{k}}{}'[1/\omega_{\vec{q}}(\vec{k}) + 1/\omega_{-\vec{q}}(\vec{k})]. \tag{7}$$

The sum on $\vec{k}$ is for $|\vec{k}|<k_F$; $|\vec{k}+\vec{q}|>k_F$; k_F = Fermi momentum; $v(\vec{q}) = \int v(\vec{r})e^{i\vec{q}\cdot\vec{r}}d^3r$. With this remark Eq. (7) is transformed to

$$1 = [v(\vec{q})/\Omega]\sum_{\vec{k}}\{[n(\vec{k}+\vec{q}) - n(\vec{k})]/\omega_{\vec{q}}(\vec{k})\}, \tag{8}$$

which is the same as KN Eq. (12).

Before we discuss Eq. (8), we first summarize the theory of collective oscillations in random phase approximation (RPA). A convenient approach is the dielectric constant method.[7] In response to a driving force of time dependence $e^{i\omega t}e^{-\epsilon|t|}$, the "dielectric constant," defined as the ratio of induced density fluctuation to driving force, in RPA is

$$\epsilon_{\vec{q}}(\omega) = 1 + 4\pi\alpha_{\vec{q}}(\omega) = 1 - [v(\vec{q})/\Omega]\sum_{\vec{k}}\frac{n(\vec{k}+\vec{q}) - n(\vec{k})}{\omega_{\vec{q}}(\vec{k}) - \omega + i\epsilon}. \tag{9}$$

The root of $\epsilon_{\vec{q}}(\omega)$ gives the frequency $\omega(\vec{q})$ for

which waves propagate freely in the system. In RPA, these, generally, will not be damped.

For $q \ll k_F$, the function $4\pi\alpha_{\vec{q}}(\omega)$ takes on the simple well-known form

$$4\pi\alpha_{\vec{q}}(\omega) = -v(q)g(\epsilon_F)\left[1+(\xi/2)\ln\left|\frac{1-\xi}{1+\xi}\right| - \frac{\pi i}{2}\xi[1-\theta(\xi)]\right], \tag{10}$$

where $v_F = k_F/m$. The term $[(\pi i/2)\xi]$ arises for $\xi < 1$ in the limit $\epsilon \to 0$, hence the step function $[1-\theta(\xi)]$; $g(\epsilon_F)$ = [density of states at the Fermi surface]/Ω; $\xi = (\omega/v_F q)$. In Fig. 1, the function $-\mathrm{Re}[4\pi\alpha_{\vec{q}}(\xi)/v(q)g(\epsilon_F)]$ is plotted for real ξ. In the figure three situations are indicated. For repulsive forces, there is always a solution with $\mathrm{Re}\xi > 1$; $\mathrm{Im}\xi = 0$. These are plasmons for the case of Coulomb forces and "zero sound" for the case of forces with range. For attractive interactions with $g(\epsilon_F)|v(q)| < 1$, no root exists and no free oscillation occurs. For attractive forces with $g(\epsilon_F)|v(q)| > 1$, if a solution exists it would have $\mathrm{Re}\xi < 1$ in which case we have to include the imaginary term in Eq. (10). In this case the solution in both the above examples is easily seen to be $\mathrm{Re}\xi = 0$; $\mathrm{Im}\xi = [1 + 1/gv]$. In particular the sign of $\mathrm{Im}\xi$ changes as gv passes through -1 with the stable side being for weak interactions ($g|v| < 1$). It is thus found that a critical attractive force exists ($|v(q)| = [g(\epsilon_F)]^{-1}$) in order for instability to occur. This is identical to the case of exchange interactions where a critical ferromagnetic exchange force was necessary for a ferromagnetic instability.[4]

Comparison with Eq. (8) then shows that it cannot be a consistent equation unless $g(\epsilon_F)v(q) = -1$. For $v(q)$ repulsive or $g(\epsilon_F)|v(q)| < 1$, the only solution of Eq. (6) is $g^{\pm}(k) = 0$. For $g(\epsilon_F)v(q) < -1$, a Hartree solution with a density ripple occurs.

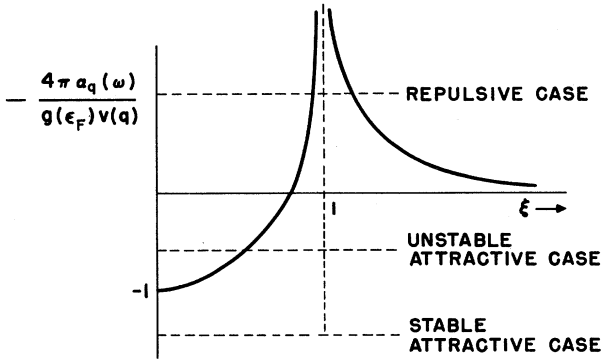

FIG. 1. Plot of polarizability in dimensionless units vs frequency in dimensionless units.

It is manifested by a blow-up in zero sound about the spherical Fermi sea and according to Overhauser[6] has a lower energy than the latter.

A possible application of the above is to note that when He^3 is put under pressure, there is a certain critical pressure (~30 atm) for which it freezes. Now He^3 is rather an open liquid with [interparticle distance] >> [position in minimum of $v(r)$]. A careful calculation (using "t" matrices for v matrices) should then show how $v(q)$ grows with pressure. Presumably this will occur fastest for $q \approx$ [interparticle distance]$^{-1}$. At the point of instability, the gas-like quasi-particle picture will break down and freezing will set in.

One last comment on the mathematics of the instability is in order. There are cases when collective modes have complex frequencies with imaginary parts which damp the oscillation even in RPA. A case in point is phonons in metals.[8] Here for small q the phonon is given by the roots of

$$\epsilon_{\vec{q}}(\omega) = 1 - \omega_M^2/\omega^2 - 4\pi\alpha_{\vec{q}}(\omega), \tag{11}$$

where $\omega_M^2 = 4\pi ne^2/M$; M = ion mass; n = ion density; $\alpha_{\vec{q}}(\omega)$ = electron polarizability. One finds a root at $\mathrm{Re}\omega = cq$, where $c = (m/3M)^{1/2}v_F \ll v_F$. Hence this root has $\xi \ll 1$. Nevertheless it is damped rather than divergent. This is because the sign of the imaginary part is determined by the derivative of $\epsilon_{\vec{q}}(\omega)$ at the root[9] which in the present case is determined by the ω_M^2/ω^2 term in (11). In the previous case it was determined by $\alpha_{\vec{q}}(\omega)$.

Finally, it should be pointed out that Thouless[10] and Pomeranchuk[11] have given general criteria for the stability of HF solutions which are no doubt related to the topics discussed here.

---

*Work performed under the auspices of the U. S. Atomic Energy Commission.

[1]A. E. Glassgold, W. Heckrotte, and K. M. Watson, Ann. Phys. 6, 1 (1959).

[2]A. W. Overhauser, Phys. Rev. Letters 4, 415 (1960).

[3]W. Kohn and S. J. Nettel, Phys. Rev. Letters 5, 8 (1960).

[4]P. A. Wolff (to be published).

[5]L. Van Hove, Physica 25, 849 (1959).

[6]A. W. Overhauser, Phys. Rev. Letters 3, 414 (1959).

[7]P. Nozieres and D. Pines, Nuovo cimento 9, 470 (1958).

[8]T. Staver, Princeton thesis, 1952 (unpublished).

[9]V. Glaser and G. Källén, Nuclear Phys. 2, 706 (1956/57).
[10]D. Thouless (to be published).
[11]I. Ia. Pomeranchuk, J. Exptl. Theoret. Phys. (U.S.S.R.) 35, 524 (1958) [translation: Soviet Phys.-JETP 35(8), 361 (1959)].

ABSORPTION SPECTRA AND ZEEMAN EFFECT OF COPPER AND ZINC IMPURITIES IN GERMANIUM*

P. Fisher and H. Y. Fan
Department of Physics, Purdue University, Lafayette, Indiana
(Received July 29, 1960)

The absorption spectra of group V and group III impurities in germanium have been reported previously.[1,2] For these impurities of small ionization energy, theoretical treatment based on the effective-mass formalism should be a good approximation, and the experimental results, especially in the case of donors, bear this out. Experimental investigation has been now extended to two acceptor impurities, Cu and Zn, of higher ionization energies. The ionization absorption of Cu and Zn in germanium has been seen before[3,4] but no observation of the excitation lines has been reported. Each of these impurities can bind more than one hole. Studies have been made on the neutral and the singly charged states. It may be expected that the effective-mass formalism would not apply for the ground state but may still be valid for the excited states. The absorption spectra provide the means to check this assumption and to reveal effects due to differences in the ground states.

Measurements were made at liquid helium temperature. For the observation of the Cu^- and Zn^- absorptions, samples suitably compensated by Sb impurity were used. The solid curve in Fig. 1 shows the absorption spectrum of Cu. The observed lines are labeled according to the designation used for the spectra of group III impurities.[2] As seen in Table I, the energy spacings between the *D* line and the various other lines are very close to the corresponding spacings in the group III spectra. This applies also to the spectrum observed for Zn, the data for which are included in Table I. In the case of Zn, addi-

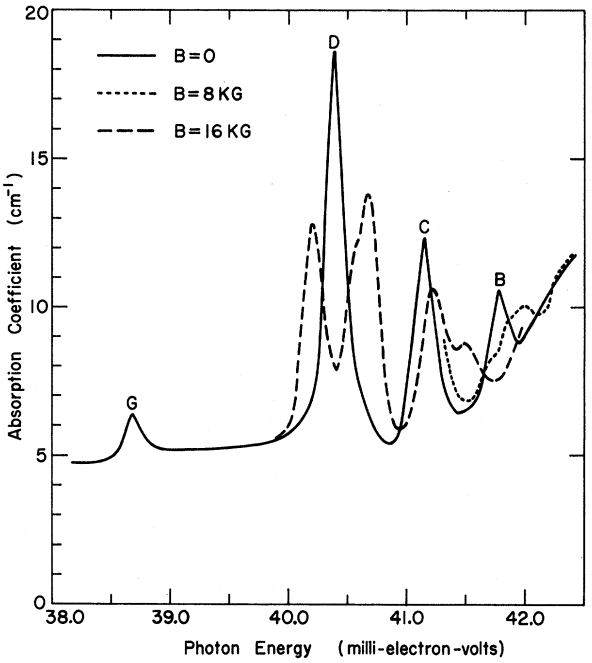

FIG. 1. Absorption spectrum and transverse Zeeman effect of Cu impurity in germanium. Copper concentration is 1.2×10^{16} cm^{-3}. Magnetic field is parallel to [111] in the (110) plane.

Table I. Ionization values and energy spacings between the excitation lines for various impurities in germanium, in units of 10^{-3} ev.

	Group III (average values)	Cu	Zn	Zn^-
B-D	1.388	1.398 ± 0.020	1.382 ± 0.051	•••
C-D	0.722	0.745 ± 0.016	0.729 ± 0.033	3.33 ± 0.06
D-G	1.73	1.72 ± 0.03	1.84 ± 0.03	•••
Ionization energies	Optical	42.8	32.6	85.8
	Thermal	40	30	90