ISOTOPIC SPIN DEPENDENCE OF THE PION-NUCLEON HIGHER RESONANCE

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In a recent letter[1] Carruthers has suggested that the "spectrum" of higher resonances in pion-nucleon interactions is different for the $T=1/2$ and $T=3/2$ states; in particular suggesting the existence of a D_{33} resonance in the neighborhood of the third π^-p maximum. This seems difficult to reconcile with the experimental results for the branching ratios of the various possible charge states produced by interaction of π^- with protons. The results[2] at 960 Mev are shown in Table I. Their most significant feature is the small total charge exchange scattering. As has been pointed out elsewhere,[3] the ratio of direct elastic to charge-exchange elastic scattering cannot exceed 2, when the elastic $T=3/2$ (π^+p) scattering is not negligible, unless there is appreciable interference between the $T=1/2$ and $T=3/2$ amplitudes for π^-p scattering, and hence unless the same angular momentum and parity states contribute to both, with approximately the same phase. Since presumably some of the charge-exchange scattering is inelastic, the inequality is quite strongly violated, while if none is inelastic then this is evidence of similar very strong interference terms in the inelastic amplitude. The energy dependence of the π^+p total cross section[4] has a point of inflection at the third π^-p maximum (900 Mev), which can be reproduced by adding a small fraction of third π^-p peak to a cross section rising steadily towards the fourth maximum at 1.4 Bev. In Carruthers' picture this small extra peak in the π^+p cross section should be due to a D_{33} resonance, and he uses interference between the low-energy portion of this and the D_{13} resonance peaked at 600 Mev to explain the suppression of charge-exchange scattering in the 500-800 Mev region. However, the fact that the charge-exchange scattering is still strongly suppressed, and hence that there is still strong interference, up to 1 Bev is hard to understand on this picture. It seems more reasonable that the interference at this energy should be with the third (possibly $F_{1/2\,5/2}$) resonance, and that the resonant spectrum is the same for both total isotopic spin states.

Table I. Experimental branching ratios in π^-p interactions at 960 Mev.

Channel	Cross section (mb)
π^-+p	19.0
π^0+n, $\pi^0+\pi^0+n$	7.6
$\pi^0+\pi^-+p$	6.8
$\pi^++\pi^-+n$	9.5
3π + nucleon	2.7
strange particles	1.3
	(Errors are ~10 %)

One possible way of understanding the simultaneous occurrence of the resonances in both isotopic spin states is by attributing their existence[3] to strong interactions in pairs of the particles in final states with two pions. The second and third maxima (600 and 900 Mev) are associated with the interaction of a pion and nucleon through the 33 resonance [Fig. 1(a)]; the fourth maximum (1.4 Bev) is associated with a pion-pion interaction in a state with $t_{\pi\pi}=1$, $j_{\pi\pi}=1$ [Fig. 1(b)]. In each case it is assumed that the charge of the third particle is relatively unimportant and hence that both total isotopic spin states can exhibit the resonance, while the phase of the transition amplitude should not depend on T. At 960 Mev both (a) and (b) should occur and we may expand the state:

$$|\pi\pi N\rangle = a_3\,|T=\tfrac{3}{2},\ t_{\pi N}=\tfrac{3}{2}\rangle + a_1\,|T=\tfrac{1}{2},\ t_{\pi N}=\tfrac{3}{2}\rangle + b_3\,|T=\tfrac{3}{2},\ t_{\pi\pi}=1\rangle + b_1\,|T=\tfrac{1}{2},\ t_{\pi\pi}=1\rangle.$$

These four charge states must each be multiplied by an appropriate angular momentum state: one for a_1 and a_3 and a different one for b_1 and

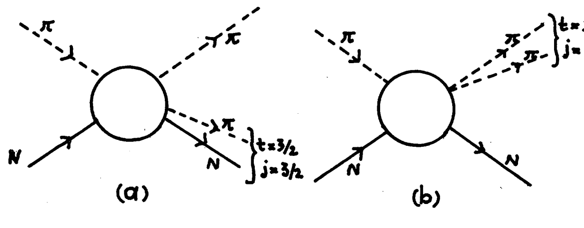

FIG. 1. Mechanisms for the second and third resonances (a) and the fourth resonance (b).

Table II. Theoretical cross sections.

Channel	Relative cross section
$\pi^+ + \pi^- + n$	$\frac{1}{135}[26\|a_3\|^2+50\|a_1\|^2+14(10)^{1/2}\mathrm{Re}(a_3^*a_1)]+\frac{1}{9}[2\|b_3\|^2+2\|b_1\|^2-4\mathrm{Re}(b_3^*b_1)]$
$\pi^0 + \pi^- + p$	$\frac{1}{135}[17\|a_3\|^2+20\|a_1\|^2-10(10)^{1/2}\mathrm{Re}(a_3^*a_1)]+\frac{1}{9}[\|b_3\|^2+4\|b_1\|^2+4\mathrm{Re}(b_3^*b_1)]$
$\pi^0 + \pi^0 + n$	$\frac{1}{135}[2\|a_3\|^2+20\|a_1\|^2-4(10)^{1/2}\mathrm{Re}(a_3^*a_1)]$

b_3. The two states [(a) and (b)] are not strictly orthogonal, but the two configurations are so different that their overlap should be small and we neglect their interference in the total cross sections, which are given in Table II.

Experimentally, the distributions for the $n\pi^+\pi^-$ events are well described by the conventional isobar model,[5] which corresponds to $b_1 = b_3 = 0$. From the fact that the π^+ seems to be the "recoil" pion (with higher momentum) about 60% of the time, the assumption made above that the phases of a_1 and a_3 are the same leads to the result that $a_3 \cong \frac{1}{4}a$. But this would predict a branching ratio of 5:1:1.5 for $(n\pi^+\pi^-):(p\pi^-\pi^0):(n\pi^0\pi^0)$ which is quite unlike the observed ratio (Table I). In addition, the distributions for the $\pi^0\pi^-p$ events seem to show evidence of a pion-pion correlation.[6] These facts can all be reconciled if we assume $b_1 \cong b_3 \cong 0.7a_1$, contributing only to $\pi^0\pi^-p$.

There have been several other discussions of possible effects of pion-pion interactions in these processes.[6-8] However, they have mainly been concerned with the absorption mechanism via interaction with the virtual pions emitted by the nucleon. If it is assumed that the interaction pions are emitted directly[6] (Fig. 2), then $b_1 = -\sqrt{2}b_3$, which would predict pion-pion correlations primarily for $n\pi^+\pi^-$ events, contradictory to experiment. The other discussions[7,8] allowing pion-nucleon interactions to take place after the absorption via a pion-pion interaction are particular cases of isobar models of the type (a), and are relevant to the explanation of the relative magnitudes of a_1 and a_3 (or analogous amplitudes in the various models considered) but not to the explanation of final-state pion-pion correlations.

The data at present seem consistent with a picture in which the final states are dominated by the two types of final-state interaction (a) and (b), and in which the phase of the amplitude is independent of T, but the way in which the final states are generated is not understood.

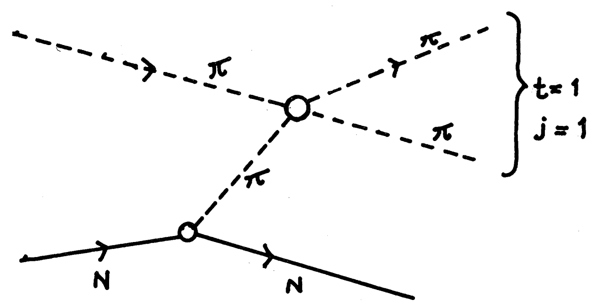

FIG. 2. Direct pion-pion interaction.

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