

Various mechanisms involving carriers of high k_H have been considered but appear unlikely because they would tend to produce opposite shifts in the [100] and [111] orientations.⁴ It seems most likely that the heavy-hole shifts are related to the "quantum effects"⁵ in this resonance, and this conjecture is presently under investigation.

As a final remark, it should be mentioned that no shifts were observed in the electron resonances.

Thanks are due to E. M. Conwell for the suggestion of the problem and for helpful discussions, to J. Kugler for able assistance in the experimentation, and to K. Arnold and Miss M. Tilton for

the germanium crystal. G. Weibel, R. Harrison, H. Malamud, and C. Fallier gave much assistance in the design and construction of the apparatus.

¹R. N. Dexter, H. J. Zeiger, and B. Lax, Phys. Rev. 104, 637 (1956).

²E. O. Kane, J. Phys. Chem. Solids 1, 83 (1956).

³E. M. Conwell, J. Phys. Chem. Solids (to be published).

⁴G. Dresselhaus, A. F. Kip, and C. Kittel, Phys. Rev. 98, 368 (1955).

⁵R. C. Fletcher, W. A. Yager, and F. R. Merritt, Phys. Rev. 100, 747 (1955).

THEORY OF MAGNETIC RESONANCE IN $\alpha\text{Fe}_2\text{O}_3$ †

P. Pincus

Department of Physics, University of California, Berkeley, California

(Received May 26, 1960)

For many years, the problem of the magnetic properties of hematite ($\alpha\text{Fe}_2\text{O}_3$) have gone unexplained. Recently, however, Dzyaloshinsky¹ has shown that the weak ferromagnetism exhibited by $\alpha\text{Fe}_2\text{O}_3$ above the transition discovered by Morin² at -14°C could be explained by an anisotropic exchange interaction of the form $\vec{D}' \cdot \vec{S}_1 \times \vec{S}_2$. In a two-sublattice model of antiferromagnetism, such an interlattice interaction will cant the sublattices toward one another and hence create a weak magnetic moment of the order of $(D'/J)M$, where $J > 0$ is the antiferromagnetic exchange integral, and M is the static sublattice magnetization. Subsequently Moriya³ has derived just such an exchange term from first principles and has shown the constant vector \vec{D}' to be of first order in spin-orbit coupling. However, the magnetic resonance experiments performed by Anderson et al.⁴ and Kumagai et al.⁵ have remained unexplained. Here, we should like to show how the Dzyaloshinsky-Moriya interaction can explain these microwave results.

$\alpha\text{Fe}_2\text{O}_3$ is a rhombohedral crystal with the [111] direction coinciding with the ternary axis. The (111) planes form alternating layers of O^{2-} and Fe^{3+} ions with alternating Fe^{3+} planes magnetized oppositely. The susceptibility and torque measurements of Smith,⁶ Néel and Pauthenet,⁷ and Lin⁸ indicate that above the Morin transition

the [111] axis is a hard direction of magnetization and that there is very little anisotropy in the basal plane. These results have been substantiated by the neutron diffraction work of Shull et al.⁹

We shall now derive the magnetic resonance frequencies in the molecular field approximation for a two-sublattice antiferromagnet with a Dzyaloshinsky-Moriya interaction, where \vec{D} is directed along the ternary axis. We shall also assume a fairly strong uniaxial anisotropy tending to make the [111] axis a hard direction. Both Turov and Borovik-Romanov¹⁰ have derived the spin-wave dispersion law, including $k = 0$, for such an antiferromagnet, but they have neglected the small but extremely important anisotropy in the (111) plane. As we shall see, it will be necessary to include such an anisotropy term in order to explain the resonance experiments. For the sake of simplicity, we shall ignore the fact that there exist really three equivalent two-fold axes in the (111) plane. Also, to make the equations easier to write down, we shall assume that the external field \vec{H}_0 lies in the basal plane at right angles to the easy direction of magnetization. In Fig. 1 we indicate our coordinate system and the equilibrium positions of the sublattice magnetizations.

Under the assumption of these interactions,

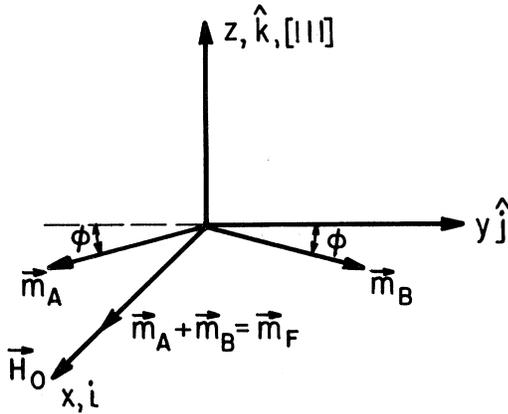


FIG. 1. Equilibrium positions of A and B sublattice magnetizations, making an angle φ with easy direction in (111) plane. The net moment \vec{M}_F is directed along the x axis.

the equations of motion for the A and B sublattices may be written in the following form:

$$\begin{aligned} d\vec{M}_A/dt &= -\lambda\vec{M}_A \times \vec{M}_B + \gamma\vec{M}_A \times \vec{H}_0 - \vec{M}_A \times (\vec{D} \times \vec{M}_B) \\ &\quad + KM_A^z \hat{k} \times \vec{M}_A + K'M_A^y \hat{j} \times \vec{M}_A, \\ d\vec{M}_B/dt &= -\lambda\vec{M}_B \times \vec{M}_A + \gamma\vec{M}_B \times \vec{H}_0 + \vec{M}_B \times (\vec{D} \times \vec{M}_A) \\ &\quad + KM_B^z \hat{k} \times \vec{M}_B - K'M_B^y \hat{j} \times \vec{M}_B, \end{aligned} \quad (1)$$

where $\lambda > 0$ is the Weiss molecular field constant for the antiferromagnetic interaction, $K > 0$ is the anisotropy constant giving rise to the hard direction, and K' gives the small in-plane anisotropy. Upon setting the torques equal to zero, we can determine the equilibrium positions of the sublattice magnetizations which are found to be given by

$$\varphi \cong (H_0 + H_{DM}) / (2H_e + H_{A'}), \quad (2)$$

where we have taken $H_e = \lambda M \gg H_0 + H_{DM} = H_0 + DM$, and $H_{A'} = K'M$. In the absence of any external field, the net magnetic moment M_F is then given by

$$M_F \cong 2M\varphi \cong M(D/\lambda), \quad (3)$$

as previously stated. From Pauthenet's and Lin's experiments, we can estimate $D/\lambda \cong M_F/M \approx 3 \times 10^{-3}$. Thus, for an exchange field $H_e \approx 5 \times 10^6$ oe, $H_{DM} \approx 1.5 \times 10^4$ oe.

Now, we work in two new coordinate systems

defined by the equilibrium positions of the A and B sublattice magnetizations. We linearize (1), assuming small deviations from the equilibrium positions, and determine the eigenfrequencies for the normal modes.

$$(\omega_1/\gamma)^2 \cong H_0(H_0 + H_{DM}) + 2H_e H_{A'}, \quad (4a)$$

$$(\omega_2/\gamma)^2 \cong H_{DM}(H_0 + H_{DM}) + 2H_e H_{A'}. \quad (4b)$$

We must recall when considering these equations that H_A may be quite large ($\approx 10^4$ oe) while $H_{A'}$ is rather small (≈ 1 oe). The mode (4a) roughly corresponds to an oscillation of the weak moment M_F about its equilibrium position in the basal plane, while mode (4b) is just the usual anti-ferromagnetic resonance. We believe that the magnetic resonance experiments of Anderson and Kumagai have observed the low-frequency modes (4a).

The resonant frequency for the high-frequency mode can be determined only if we have an estimate of H_A . Tachiki and Nagamiya¹¹ have calculated the dipole contribution to the energy and

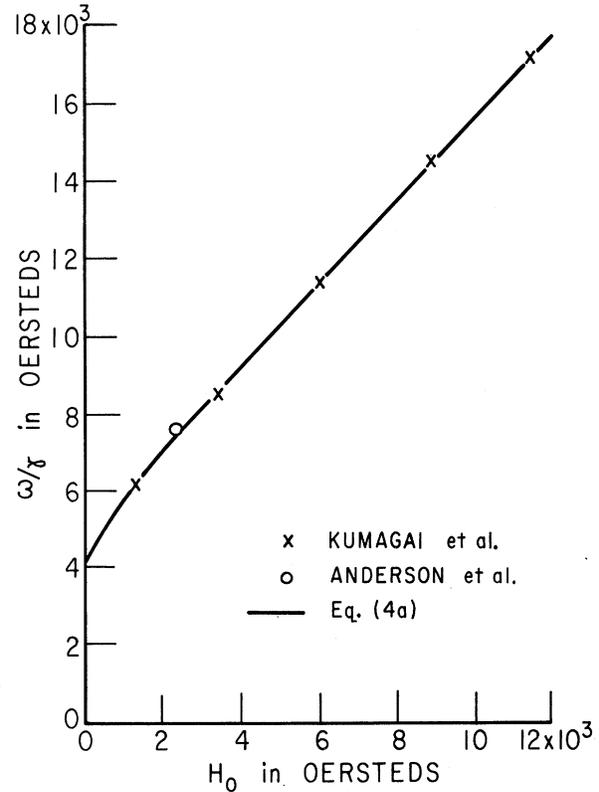


FIG. 2. Comparison of microwave resonance results for ω/γ vs H_0 with calculation using Eq. (4a) with $H_{DM} = 1.3 \times 10^4$ oe, $2H_e H_{A'} = 1.7 \times 10^7$ (oe)², and $g = 2$.

found it to be $1.15 \cos^2 \theta \text{ cm}^{-1}$, which would give an anisotropy field of about 9×10^3 oersteds. This would correspond to a resonant frequency in the 10^3 -kMc/sec range for the mode (4b). If we allow \vec{H}_0 to vary with polar angle θ from the [111] axis, (4a) becomes

$$(\omega_1/\gamma)^2 \cong H_0 \sin \theta (H_0 \sin \theta + H_{DM}) + 2H_e H_A', \quad (5)$$

which is the angular dependence observed by Anderson and Kumagai. Kumagai has found the resonance field to vary linearly with frequency. In Fig. 2 we plot ω vs H_0 for $g \approx 2$, $H_{DM} \approx 1.3 \times 10^4$ oe, and $2H_e H_A' \approx 1.7 \times 10^7$ (oe)². We see that the resonance experiments in $\alpha\text{Fe}_2\text{O}_3$ can be rather simply explained in terms of the Dzyaloshinsky-Moriya interaction.

The author would like to express his appreciation to Professor C. Kittel for suggesting this problem and for several valuable discussions concerning its solution. Thanks are also due Professor A. Portis, Professor F. Keffer, and

Professor T. Nagamiya for their helpful discussions.

[†]Supported in part by the National Science Foundation.

¹I. Dzyaloshinsky, *J. Phys. Chem. Solids* **4**, 241 (1958).

²F. J. Morin, *Phys. Rev.* **78**, 819 (1950).

³T. Moriya, *Phys. Rev. Letters* **4**, 228 (1960).

⁴P. W. Anderson, F. R. Merritt, J. P. Remeika, and W. A. Yager, *Phys. Rev.* **93**, 717 (1954).

⁵H. Kumagai, H. Abe, K. Ôno, I. Hayashi, J. Shimada, and K. Iwanaga, *Phys. Rev.* **99**, 1116 (1955).

⁶T. Smith, *Phys. Rev.* **8**, 721 (1916).

⁷L. Néel and R. Pauthenet, *Compt. rend.* **234**, 2172 (1952).

⁸S. T. Lin, *Phys. Rev.* **116**, 1447 (1959).

⁹C. G. Shull, W. Strauser, and E. O. Wollan, *Phys. Rev.* **83**, 333 (1951).

¹⁰E. A. Turov, *J. Exptl. Theoret. Phys. U.S.S.R.* **36**, 1254 (1959) [translation: *Soviet Phys.-JETP* **36**(9), 890 (1959)]. A. S. Borovik-Romanov, *J. Exptl. Theoret. Phys. (U.S.S.R.)* **36**, 766 (1959) [translation: *Soviet Phys.-JETP* **36**(9), 539 (1959)].

¹¹M. Tachiki and T. Nagamiya, *J. Phys. Soc. (Japan)* **13**, 452 (1958).

OBSERVATION OF A SHORT-LIVED COSMIC-RAY SOLAR FLARE INCREASE WITH A HIGH-COUNTING-RATE MESON DETECTOR*

R. A. R. Palmeira[†] and K. G. McCracken[‡]

Physics Department and Laboratory for Nuclear Science, Massachusetts Institute of Technology, Cambridge, Massachusetts

(Received June 6, 1960)

We report here observations of a very abrupt and short-lived cosmic-ray increase which was observed at M.I.T. (geographic coordinates $41^\circ 23' \text{N}$, $71^\circ 08' \text{W}$, sea level) on May 4, 1960. These data were obtained from three large meson telescopes, of total sensitive area 10 m^2 , and total counting rate $\approx 900 \text{ counts sec}^{-1}$. The telescopes have been described elsewhere,¹ and it suffices to state that there are now 7.5 cm of lead between the scintillators. The minimum μ -meson momentum accepted by the telescope is now approximately $202 \text{ Mev}/c$.

The data from the telescopes are recorded as follows. (1) The counting rate from each telescope is accumulated over intervals of six minutes, and punched on tape in the form of binary numbers. (2) A printing register records the hourly totals from each telescope as decimal numbers. (3) The outputs from the three telescopes are combined, accumulated over intervals of 30 seconds, and the accumulated count registered on a chart recorder.

Figure 1 displays the 30-second counting rate data obtained between 1010 and 1110 U.T. on May 4, 1960. The total number of counts recorded in each 30-second interval is plotted as a percentage relative to the mean counting rate

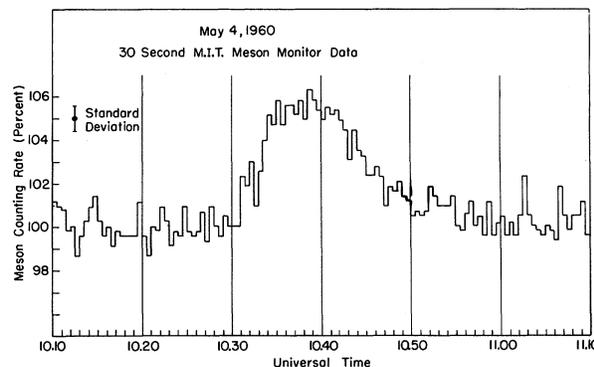


FIG. 1. The 30-second meson data observed at M.I.T. subsequent to the production of cosmic rays by the sun on May 4, 1960.