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[16]M. Demeur, Nuclear Phys. 1, 516 (1956).
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SYMMETRY BETWEEN MUON AND ELECTRON

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(Received July 13, 1960)

As is well known, muons and electrons appear to have identical couplings. Their masses are, however, different. Such a situation seems rather peculiar and has recently received much attention.[1] In this note we shall (1) define a formal operation of muon-electron symmetry; (2) show how the total Lagrangian, excluding weak couplings, can be written in a form exhibiting such a symmetry, if electromagnetic coupling is minimal; (3) show that it is impossible to satisfy such a symmetry when universal weak interactions are included, if only one neutrino exists; (4) show that it is possible to have such a symmetry in a two-neutrino theory; (5) point out the close connection of muon-electron symmetry to a principle forbidding the transformation of muons into electrons.

The present investigation is related to some recent papers[2-4] dealing with the elimination of particular muon-electron couplings. Of the above points, (2) is already contained in reference 3. We shall also make use of the general theorem of reference 4.

We first define a formal operation of muon-electron symmetry. We introduce a two-dimensional e-μ space, which we call L space (lepton space). The e-μ symmetry, or L symmetry, is performed by an unitary operator $\mathcal{S}$, such that

$$\mathcal{S}^{-1}\psi\mathcal{S}=\sigma_1\psi, \qquad (1)$$

where ψ is a vector in L space describing the electron and muon fields and σ_1 is a Pauli matrix in the usual notation. In the representation in which the components of ψ are e and μ the operation (1) just amounts to the substitution $e \rightleftarrows \mu$.

A general renormalizable Lagrangian, excluding weak interactions, can be written as[3,4]

$$\mathcal{L}=-\bar{\psi}[\gamma\cdot\partial(A+\gamma_5 B)+(C+i\gamma_5 D)]\psi+\mathcal{L}_\gamma+\mathcal{L}_S, \qquad (2)$$

where $\mathcal{L}_\gamma$ is the free-photon Lagrangian, $\mathcal{L}_S$ is the strong Lagrangian that we assume does not contain e or μ, and A, B, C, D are Hermitian matrices in L space.[5] The requirement of invariance under L symmetry implies that A, B, C, D all commute with σ_1.

A theorem, whose proof can be found in reference 4, states the existence of a nonsingular matrix T in spin space and L space, such that by transforming according to

$$\psi = T\psi' \tag{3}$$

the Lagrangian takes its usual form in which the electron and muon components of ψ' are not coupled.[6] Thus there exist infinite choices of A, B, C, D that make the Lagrangian (2) manifestly L-symmetric. A particular choice is given in reference 3.

We now add weak interactions to the Lagrangian. We assume that e and μ are coupled identically in the $(1+\gamma_5)$ projection.[7] The matrix T is now restricted from the condition of giving a symmetric description also in terms of ψ'. Writing $T = aR + \bar{a}S$, where $a = \frac{1}{2}(1+\gamma_5)$ and $\bar{a} = \frac{1}{2}(1-\gamma_5)$ and R, S act in L space, such a condition restricts the form of R. R must be of the form

$$R = u + v\sigma_1 + w(\sigma_2 - i\sigma_3), \tag{4}$$

with u, v, w complex numbers.[8] One now sees directly that such a form of R is inconsistent with the assumption that A, B, C, D in Eq. (2) commute with σ_1. From (2) and (3) one sees that R and S must satisfy

$$R^\dagger(A+B)R = 1;\ S^\dagger(A-B)S = 1;\ S^\dagger(C+iD)R = M, \tag{5}$$

where $M = \frac{1}{2}(m_e + m_\mu) + \frac{1}{2}(m_e - m_\mu)\sigma_3$, to obtain, after transformation, the ordinary Lagrangian for muon and electrons. It follows from (5) that R has to satisfy the equations[9]

$$R^\dagger(A+B)R = 1;\ R^\dagger(A-B)^{-1}(C^2+D^2)R = M^2. \tag{6}$$

The first of Eqs. (6) implies $w = 0$ in (4).[10] But then it is impossible to satisfy the second of Eqs. (6) since the left-hand side commutes with σ_1 while M^2 does not.

A different situation occurs if one assumes the existence of two neutrinos, both left-handed: one, ν_e, coupled to the electron, and the other, ν_μ, coupled to the muon.[11] A simple transformation to obtain the desired symmetry consists in introducing new fields e', μ', ν_e', ν_μ', according to

$$e = \frac{1}{\sqrt{2}}(e'+\mu'), \quad \mu = \frac{1}{\sqrt{2}}(e'-\mu'),$$

$$\nu_e = \frac{1}{\sqrt{2}}(\nu_e'+\nu_\mu'), \quad \nu_\mu = \frac{1}{\sqrt{2}}(\nu_e'-\nu_\mu').$$

The total Lagrangian assumes the symmetric form

$$\begin{aligned}\mathcal{L} = &-\bar{e}'(\gamma\cdot\partial+m_+)e' - \bar{\mu}'(\gamma\cdot\partial+m_+)\mu' + m_-[(\bar{e}'\mu')+(\bar{\mu}'e')]\\ &-\bar{\nu}_e'\gamma\cdot\partial\nu_e - \bar{\nu}_\mu'\gamma\cdot\partial\nu_\mu + G[(\bar{e}'\gamma_\lambda a\nu_e')+(\bar{\mu}'\gamma_\lambda a\nu_\mu')\\ &+\ldots][(\bar{\nu}_e'\bar{a}\gamma_\lambda e')+(\bar{\nu}_\mu'\bar{a}\gamma_\lambda\mu')+\ldots]\\ &+\text{(other terms not involving leptons).}\end{aligned} \tag{7}$$

Here $m_\pm = \frac{1}{2}(m_e \pm m_\mu)$, G is the weak-coupling constant, and the contribution to the weak current from baryon and meson terms has not been written down explicitly. The Lagrangian (7) is written for the usual formulation of the A - V theory.[7] Of course, L symmetry here involves also an exchange of ν_e with ν_μ.

Finally we come to the last of the four points mentioned in the introduction. According to general principles we expect that a selection rule be connected to the possibility of L symmetry. One sees that $\mathcal{S}$ in (1) can be taken to satisfy $\mathcal{S}\mathcal{S}^\dagger = 1$, and $\mathcal{S}^2 = 1$, and therefore it is Hermitian, with eigenvalues ± 1. If L symmetry is satisfied, states with eigenvalue $+1$ cannot transform into states with eigenvalue -1. What is the physical meaning of this conservation law? From (1) and (3) one notices that $\mathcal{S}$ can also be represented by a matrix $T^{-1}\sigma_1 T$ acting on ψ'. Such a matrix has the following properties: (a) it is traceless, (b) its square is unity, and (c) it must commute with M, because of the invariance of $\mathcal{L}$. Therefore it can only be $\pm\sigma_3$. It is now evident that the conservation law is one that forbids a muon to transform into an electron and vice versa (unless other particles such as ν_e and ν_μ, bearing quantum number $\mathcal{S}$, are also emitted or absorbed). We may call this law the "law of muonic number conservation". Such a law is not satisfied in the one-neutrino theory and this simple observation may actually be taken as an independent proof of our statement (3) that we derived above by direct algebraic verification. This remark also illustrates the role of minimal electromagnetic coupling in our statement (2), since, by nonminimal coupling, μ-e transitions could well occur.

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[1]See, e.g., J. Schwinger, Ann. Phys. 2, 407 (1957); M. Goldhaber, Phys. Rev. Letters 1, 467 (1958).

[2]N. Cabibbo and R. Gatto, Phys. Rev. 116, 1134 (1959).

[3]G. Feinberg, P. Kabir, and S. Weinberg, Phys. Rev. Letters 3, 527 (1959).

[4]N. Cabibbo, R. Gatto, and C. Zemach, Nuovo cimento 16, 168 (1960).

[5]The μ, e couplings with photons are here obtained,

according to minimal electromagnetic interaction, through the replacement $\partial/\partial x_\mu \rightarrow \partial_\mu = (\partial/\partial x_\mu - ieA_\mu)$. The matrix $A+\gamma_5 B$ is positive definite to assure a positive definite energy.

[6]The physical equivalence implied by (3) is easily seen by considering the mapping of the Hilbert space effected by a nonsingular matrix $\mathcal{T}$ such that $\mathcal{T}^{-1}\psi\mathcal{T}=T\psi$. If P_μ' is the total energy-momentum vector constructed from ψ', the operator $P_\mu' P_\mu'$ has eigenvalues $-m_\mu^2$, which correspond to the electron and the muon. However, $P_\mu P_\mu$, where P_μ is constructed from ψ, must have the same eigenvalues since $P_\mu = \mathcal{T}^{-1} P_\mu' \mathcal{T}$.

[7]R. P. Feynman and M. Gell-Mann, Phys. Rev. 109, 193 (1958); E. C. G. Sudarshan and R. E. Marshak, Phys. Rev. 109, 1860 (1958).

[8]Arbitrary phase changes of the electron and muon fields are unessential to the conclusion.

[9]One notices that $A-B$ is positive definite because $A+\gamma_5 B$ is such, and that A, B, C, D all commute between themselves since they all commute with σ_1.

[10]Write $A+B=p+q\sigma_1$, and take $\chi_\pm=\left[\begin{smallmatrix}1\\ \pm 1\end{smallmatrix}\right]$ as orthogonal base vectors. One finds $R^\dagger(A+B)R\chi_- = (p-q)(u-v)[(u^*-v^*)\chi_- + 2iw^*\chi_+] = \chi_-$, from which $w=0$.

[11]A. Salam, Proceedings of the Seventh Annual Conference on High-Energy Nuclear Physics, 1957 (Interscience Publishers, New York, 1957), p. IX, 44; B. Pontecorvo, J. Exptl. Theoret. Phys. (U.S.S.R.) 37, 1751 (1951).

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Expressions (8) and (9) are too large by a factor 2, as are also Figs. 1 and 2 derived from them. This error was kindly indicated to the authors by Professor L. Michel. The same error appears also in older results on the spin-zero vacuum polarization[1] as has been noted previously.[2]

[1]R. P. Feynman, Phys. Rev. 76, 769 (1949).

[2]A. I. Akhiezer and V. B. Berestetsky, Quantum Electrodynamics (State Technico-Theoretical Literature Press, Moscow, 1959), 2nd ed.