PHYSICAL REVIEW LETTERS

VOLUME 5 | JULY 1, 1960 | NUMBER 1

SCATTERING OF THERMAL ENERGY IONS IN SUPERFLUID LIQUID He BY PHONONS AND He^3 ATOMS*

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(Received June 8, 1960)

We have recently reported work[1,2] designed to use ions, produced in liquid He by ionization with α particles, as microscopic probes to study the properties of the superfluid. A time-of-flight method permitted direct measurements of the drift velocity u of the ions in the presence of an applied electric field $\mathcal{E}$. When $\mathcal{E}$ is kept sufficiently small, the ion energy never exceeds thermal energy appreciably; the ion mobility $\mu \equiv u/\mathcal{E}$ is then independent of the field $\mathcal{E}$. The temperature dependence of μ is shown in Fig. 1. Below 2°K it is accurately given by an equation of the form

$$\mu = \alpha \exp(\Delta'/kT), \tag{1}$$

Δ'/k being 8.8°K for positive and 8.1°K for negative ions. This result was interpreted in terms of the scattering of ions by rotons. The number density of thermally excited rotons $n_\gamma \propto \exp(-\Delta/kT)$ decreases rapidly as the temperature is reduced, and $\mu \propto n_\gamma^{-1}$. The energy Δ necessary to create a roton is deduced from neutron scattering experiments[3] as being $\Delta/k = 8.65$°K, in good agreement with the values used in (1).

Figure 1 shows, however, that, below about 0.65°K for positive and 0.8°K for negative ions, a plot of $\ln\mu$ vs T^{-1} begins to deviate below the straight line of Eq. (1). This suggests that at these low temperatures n_γ has become so small ($n_\gamma \approx 2\times10^{16}$ cm^{-3} at 0.6°K) that some other scattering processes become predominant. The two possibilities which suggest themselves are (a) scattering of ions by the phonon excitations of the fluid and (b) scattering by the few He^3

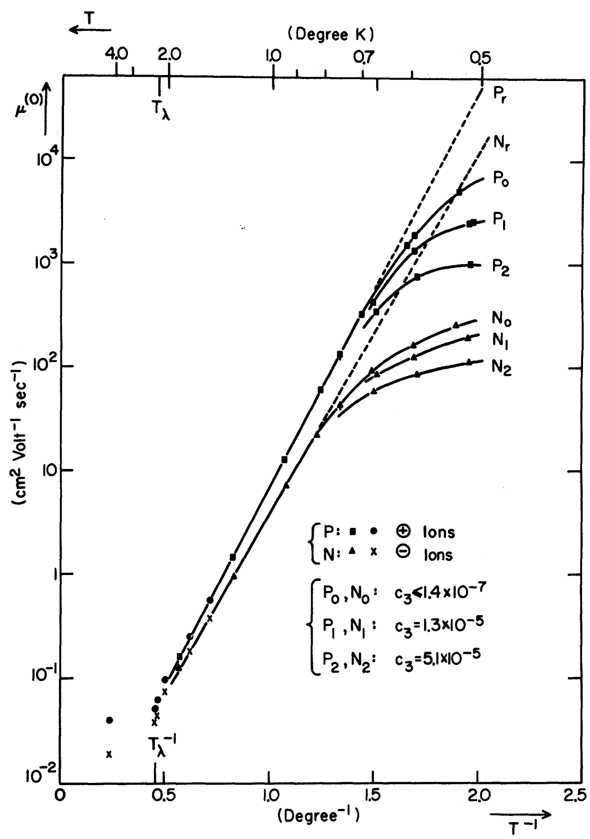

FIG. 1. Experimental values of the ion mobility μ as a function of absolute temperature T in liquid helium containing various atomic concentrations c_3 of He^3.

atoms present in the liquid helium. (In well helium the isotopic abundance of He^3 is 1.4×10^{-7}; this results in $n_3 = 3\times10^{15}$ He^3 atoms being present per cm^3 of liquid helium.) To distinguish between these two possibilities we first tried to purify some ordinary helium from He^3 by superflow through porous Vycor glass.[4] The degree of purification thus achieved could unfortunately not be ascertained by mass spectrometers available to us because of the very low He^3 concentrations involved. Experiments on ion mobilities in the helium thus purified led to results substantially the same as those in ordinary helium. We therefore decided to carry out experiments in helium to which known small amounts of pure He^3 had been added. The atomic concentration c_3 of He^3 in the helium could be computed from the proportions of the gases mixed and was subsequently checked approximately by mass spectroscopic analysis. Results of two sets of ion mobility measurements in liquid He containing He^3 concentrations $c_3' = 1.3\times10^{-5}$ and $c_3'' = 5.1\times10^{-5}$, respectively, are shown in Fig. 1. The experiments show clearly that He^3 atoms in superfluid liquid He^4 do indeed act like impurities which constitute additional scattering centers for the ions. The magnitude of the observed effects also shows that scattering by the isotopic abundance of He^3 in ordinary He is too small to account for any appreciable deviations of the ion mobilities from the straight lines of Eq. (1). These deviations are then presumably due to scattering of the ions by phonons.

If the scattering centers are assumed to act independently of each other, the total probability τ^{-1}, per unit time, of an ion being scattered is the sum of the probabilities that it is scattered by rotons, phonons, and He^3 atoms. Since $\mu = (e/M)\tau$, M being the effective mass of the ion, this implies that $\mu^{-1} = \mu_r^{-1} + \mu_p^{-1} + \mu_3^{-1}$. Here μ_s denotes the ion mobility in the presence of scattering centers of type s only, and $s = r, p$, and 3 refers to rotons, phonons, and He^3 atoms, respectively. Hence one can use a subtraction procedure to obtain from the data estimates (of reduced accuracy) for μ_p and μ_3. Specifically, if μ_0 denotes the measured mobility in ordinary or purified helium and μ_r the mobility extrapolated from the straight lines of Eq. (1), then μ_p can be estimated from $\mu_p^{-1} = \mu_0^{-1} - \mu_r^{-1}$. Also, if μ_x denotes the measured mobility in helium enriched with He^3, then μ_3 for this particular concentration of He^3 can be deduced from $\mu_3^{-1} = \mu_x^{-1} - \mu_0^{-1}$. Results of this analysis are summarized in Table I. Estimated values of the mobilities at 0.55°K are shown in the column labeled μ_s. The ratio of the mobilities μ_3 calculated for the two enriched samples is in satisfactory agreement with the ratio 3.9 of the respective He^3 concentrations. From μ_s one can estimate the ion-scatterer collision cross section σ_{is} for an assumed effective mass M of the ion since the number density of scatterers is known. [For $c_3 = 5.1\times10^{-5}$, $n_3 = 1.1\times10^{18}$ He^3 atoms/cm^3; from the velocity of sound $C = 237$ m/sec, one computes $n_p = 2.4(4\pi)(kT/hC)^3$ $= 3.4\times10^{18}$ phonons/cm^3 for $T = 0.55$°K.] Values of $D_{is} = (\sigma_{is}/\pi)^{1/2}$ calculated for $M = M_{He}$ are listed in Table I. (M_{He} = mass of He atom.) It is likely that[2,5] $M \gg M_{He}$; then one has approximately[6] $D_{is} \propto M^{-1/2}$. For comparison, the data at about 0.9°K, where roton scattering is still predominant, yield estimates (for $M = M_{He}$) of $D_{ir} = 30$ A for positive and $D_{ir} = 42$ A for negative ions. These results suggest that a roton is a more effective scattering center for an ion than a He^3 atom, and that the negative ion is likely to be a larger and more massive entity than the positive one. Finally, the temperature dependence of the mobilities is of the form[7] $\mu_s \propto T^{-k}$ with the values of k listed in the table. Thus μ_3 is approximately temperature independent as one would expect from a hard-sphere interaction between ion and He^3 atom. The approximate dependence $\mu_p \propto T^{-3}$ suggests, since $n_p \propto T^3$, that σ_{ip} is roughly temperature independent. On the basis of an ion-phonon interaction basically equivalent to the scattering of a rigid sphere by a long-wavelength sound wave, one would predict[8,2] a very different result, $\mu_p \propto T^{-9}$.

Table I. Ion mobilities μ_s, at 0.55°K, due to scattering of ions (i) by phonons ($s = p$) and by He^3 atoms ($s = 3$) present in the two concentrations $c_3' = 1.3\times10^{-5}$ and $c_3'' = 5.1\times10^{-5}$. $D_{is} = (\sigma_{is}/\pi)^{1/2}$ denotes the ion-scatterer collision diameter estimated for $M = M_{He}$. The temperature dependence is of the form $\mu_s \propto T^{-k}$, with the values of k listed in the last column.

Ion (i)	Scatterer (s)	μ_s (cm^2 v^{-1} sec^{-1})	D_{is} (A)	k
+	phonon	5900	1.3	3.3 ± 0.3
+	He^3: c_3'	4350	...	≈0
+	He^3: c_3''	1200	8.8	≈0
-	phonon	240	6.2	2.4 ± 0.4
-	He^3: c_3'	560	...	≲1
-	He^3: c_3''	180	22.9	≲1

We wish to thank Dr. B. Steiner for performing the mass spectrographic analyses and Dr. M. E. Norberg of the Corning Glass Company for providing us with porous Vycor glass.

*This work was supported by the Atomic Energy Commission and, in the case of one of the authors (F. R.), also by the Alfred P. Sloan Foundation.

[1]L. Meyer and F. Reif, Phys. Rev. 110, 279 (1958).
[2]F. Reif and L. Meyer, Phys. Rev. (to be published).
[3]J. L. Yarnell, G. P. Arnold, P. J. Bendt, and E. C. Kerr, Phys. Rev. 113, 1379 (1959).
[4]K. R. Atkins, H. Seki, and E. U. Condon, Phys. Rev. 102, 582 (1956). The onset temperature for superfluidity was about 1.35°K in the Vycor used.
[5]K. R. Atkins, Phys. Rev. 116, 1339 (1959).
[6]More exactly, $D_{i3} \propto (M+M^2/M_3)^{-1/4}$, where M_3 is the effective mass of a He^3 atom. A reasonable estimate is $M_3 = 2M_{He}$.
[7]This is deduced from the temperature range from about 0.6 to 0.5°K where the mobility differences are sufficiently large for the subtraction analysis to be feasible.
[8]I. M. Khalatnikov and V. N. Zharkov, J. Exptl. Theoret. Phys. U.S.S.R. 32, 1108 (1957) [translation: Soviet Phys.-JETP 5, 905 (1957)].

SHIFT OF AN ELECTRON INTERFERENCE PATTERN BY ENCLOSED MAGNETIC FLUX

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(Received May 27, 1960)

Aharonov and Bohm[1] have recently drawn attention to a remarkable prediction from quantum theory. According to this, the fringe pattern in an electron interference experiment should be shifted by altering the amount of magnetic flux passing between the two beams (e.g., in region *a* of Fig. 1), even though the beams themselves pass only through field-free regions. Theory predicts a shift of *n* fringes for an enclosed flux Φ of nhc/e; it is convenient to refer to a natural "flux unit," $hc/e = 4.135\times10^{-7}$ gauss cm^2. It has since been pointed out[2] that the same conclusion had previously been reached by Ehrenberg and Siday,[3] using semiclassical arguments, but these authors perhaps did not sufficiently stress the remarkable nature of the result, and their work appears to have attracted little attention.

Clearly the first problem to consider, experimentally, is the effect on the fringe system of stray fields not localized to region *a* but extending, e.g., over region *a′* in Fig. 1. In addition to the "quantum" fringe shift due to the enclosed flux, there will then be a shift due simply to curvature of the electron trajectories by the field. A straightforward calculation shows that in a "biprism" experiment,[4] such a field should produce a fringe displacement which exactly keeps pace with the deflection of the beams by the field, so that the fringe system appears to remain undisplaced relative to the envelope of the pattern. A field of type *a*, on the other hand, should leave the envelope undisplaced, and produce a fringe shift within it. In the Marton[5] interferometer, conditions are different, and a field of type *a′* should leave the fringes undisplaced in space. This explains how Marton et al.[5] were able to observe fringes in the presence of stray 60-cps fields probably large enough to have destroyed them otherwise; this experiment thus constitutes an inadvertent check of the existence of the "quantum" shift.[2]

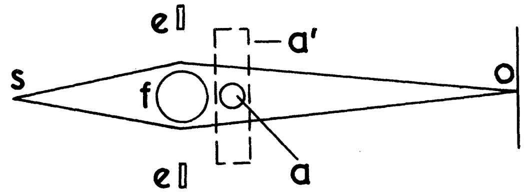

FIG. 1. Schematic diagram of interferometer, with source *s*, observing plane *o*, biprism *e*, *f*, and confined and extended field regions *a* and *a′*.

To obtain a more direct check, a Philips EM100 electron microscope[6] has been modified so that it can be switched at will from normal operation to operation as an interferometer. Fringes are produced by an electrostatic "biprism" consisting of an aluminized quartz fiber *f* (Fig. 1) flanked by two earthed metal plates *e*; altering the positive potential applied to *f* alters the effective angle of the biprism.[4] The distances *s*-*f* and *f*-*o* (Fig. 1) are about 6.7 cm and 13.4 cm, respectively. With this microscope it was not possible to reduce the virtual source diameter below about 0.2 μ, so that it was necessary to use a fiber *f* only about 1.5 μ in diameter and a