Higher Precision Measurement of the hfs Interval of Muonium and of the Muon Magnetic Moment

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New higher precision measurements of the hyperfine Zeeman transitions in the ground state of muonium have been performed with use of the high-stopping-density surface μ^+ beam at the Clinton P. Anderson Meson Physics Facility. The results are $\Delta\nu = 4\,463\text{-}302.88(16)$ kHz (0.036 ppm) and $\mu_\mu/\mu_p = 3.183\,346\,1(11)$ (0.36 ppm). The current theoretical value of $\Delta\nu$ agrees well with experiment within the 0.77-ppm error of $\Delta\nu_{\rm theor}$, which is due principally to inaccuracy in evaluation of the nonrecoil radiative correction term. The most precise current value of m_μ/m_e is obtained from our value of μ_μ/μ_p.

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Muonium (μ^+e^-) is the hydrogenlike atom consisting of a positive muon and an electron. It provides an ideal simple system for determining the properties of the muon and for measuring the muon-electron interaction. With muonium we can sensitively test the theory of quantum electrodynamics for the two-body bound state and can search for effects of weak, strong, or unknown interactions on the electron-muon bound state.[1]

The present paper reports new measurements[2] of the hyperfine structure interval $\Delta\nu$ in the ground state of μ^+e^- and of the magnetic moment of the positive muon, μ_μ, with substantially improved precision. These new experimental values, taken together with recent calculations[3,4] of relativistic recoil and radiative contributions to $\Delta\nu$, provide a considerably more sensitive comparison of theory and experiment for $\Delta\nu$.

The muonium Zeeman transitions $(M_J, M_\mu) = (\frac{1}{2}, \frac{1}{2}) \leftrightarrow (\frac{1}{2}, -\frac{1}{2})$, designated ν_{12}, and $(-\frac{1}{2}, -\frac{1}{2}) \leftrightarrow (-\frac{1}{2}, +\frac{1}{2})$, designated ν_{34}, are observed at strong magnetic field by the microwave magnetic resonance method.[1,5,6] Our increased precision is due principally to the use of the high-intensity, low-momentum "surface" muon beam[7] of the Los Alamos 800-MeV proton linear accelerator [Clinton P. Anderson Meson Physics Facility (LAMPF)]. Measurement of the two transitions ν_{12} and ν_{34} allows the determination of both $\Delta\nu$ and μ_μ, which appear in the relevant Hamiltonian term for μ^+e^-:

$$\mathcal{H} = h\Delta\nu \vec{I}_\mu \cdot \vec{J} - \mu_B{}^\mu g_\mu{}' \vec{I}_\mu \cdot \vec{H} + \mu_B{}^e g_J \vec{J} \cdot \vec{H}, \qquad (1)$$

in which $\vec{I}_\mu$ is the muon spin operator, $\vec{J}$ is the electron total angular momentum operator, $g_\mu{}'$ (g_J) is the muon (electron) gyromagnetic ratio in muonium, $\vec{H}$ is the external static magnetic field, and $\mu_B{}^\mu$ ($\mu_B{}^e$) is the muon (electron) Bohr magneton. The quantities $g_\mu{}'$ and g_J are related[8] to the free muon and electron g values g_μ and g_e by

$$g_\mu{}' = g_\mu\left(1 - \frac{\alpha^2}{3} + \frac{\alpha^2}{2}\frac{m_e}{m_\mu}\right)$$

and

$$g_J = g_e\left(1 - \frac{\alpha^2}{3} + \frac{\alpha^2}{2}\frac{m_e}{m_\mu} + \frac{\alpha^3}{4\pi}\right).$$

A diagram of the experimental apparatus[5] is shown in Fig. 1. The LAMPF stopped-muon channel was tuned to accept and transmit μ^+ originating from the decay of π^+ stopped near the surface

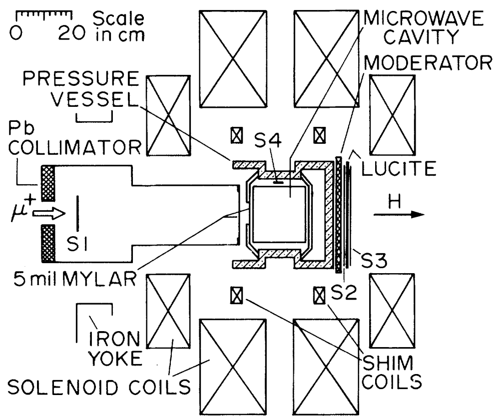

FIG. 1. Schematic diagram of the experimental setup. S1, S2, S3, and S4 are plastic scintillation counters of thicknesses 0.005, 0.25, 0.25, and 0.25 in., respectively. Counter S4 was used to indicate maximum μ^+ stopping rate in center of microwave cavity.

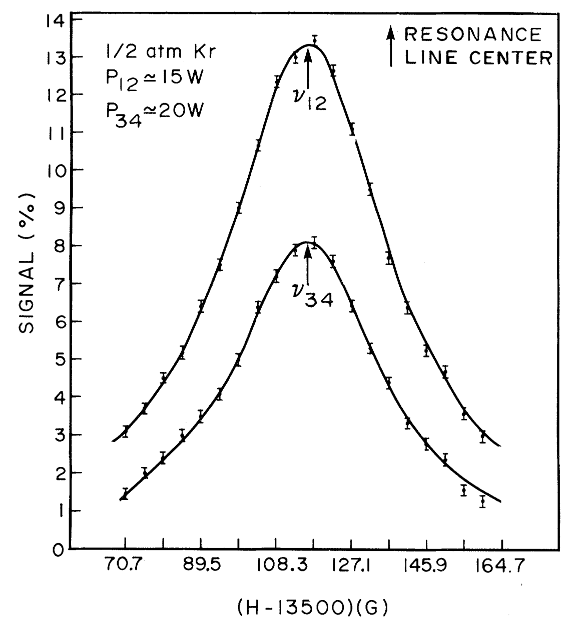

FIG. 2. Typical resonance lines with theoretical line shapes (solid lines) fitted to the data points. Data points were taken alternately on opposite sides of the line center. Data-taking time for each pair of resonance lines was less than 2 h.

of the pion production target in the primary proton beam. With an 800-MeV proton beam of 300 μA average, the μ^+ beam had an instantaneous flux of 3×10^7 s^{-1} (2×10^6 s^{-1} average) after collimation. The muon momenta were between 25 and 28 MeV/c, and the longitudinal polarization was close to 1. The ratio of μ^+ to e^+ in the beam was about 6/1. The μ^+ flux was monitored with a thin (0.12 mm) plastic scintillator S1 by integrating the anode current over the 600-μs proton beam pulse. The target vessel was filled with Kr gas at pressures of $\frac{1}{2}$ or 1 atm, and about half of the incident μ^+ stopped in the gas. Scintillation counters S2 and S3 detected e^+ from μ^+ decays, and the plastic and aluminum moderator downstream from the cavity helped reduce the background from the e^+ contamination in the beam. A central element of the experiment was the high-precision solenoid electromagnet[9] which provided the magnetic field of 13.6 kG, which was homogeneous over the volume of the microwave cavity to about 3 ppm rms and had a long term stability of better than 0.3 ppm. The microwave cavity was resonant in the TM_{110} mode at 1.918 GHz (ν_{12} at 13.6 kG) and in the TM_{210} mode at 2.545 GHz (ν_{34}). It was excited with an input power of ~20 W switched on and off at a repetition period of 160 ms, alternating between modes for successive microwave-on periods.

Typical resonance curves are shown in Fig. 2 and were observed by varying the magnetic field H in small steps with fixed microwave frequency and power. The signal at each value of H is defined by $S=\{(S2\cdot S3/S1)_{\mathrm{rf\,on}}/(S2\cdot S3/S1)_{\mathrm{rf\,off}}\}-1$. The theoretical line shape, which includes as free parameters essentially a resonance line center, a linewidth, and a height, is fitted to the experimental points and determines the resonance magnetic field value. The theoretical line shape incorporates the measured distribution of the magnetic field H, the measured μ^+ stopping distribution, the microwave power distribution over the cavity, and the solid angle for detection of an e^+ from μ^+ decay. A total of 184 resonance lines obtained in about 600 h of data taking (102 at 0.5 atm and 54 at 1 atm in the present experiment, and 18 at 1.7 atm and 10 at 5.2 atm from a previous experiment[5]) were analyzed. The resulting transition frequencies, after adjustment to correspond to a magnetic field H of 13 616.0 G and correction for a small measured quadratic density shift,[10] were extrapolated to zero gas density with use of the linear density dependence $\nu(D)=\nu(0)(1+aD)$. Using[5] the Hamiltonian of Eq. (1), we then obtain

$$\Delta\nu = 4\,463\,302.88(16)\text{ kHz (0.036 ppm)};\qquad \mu_\mu/\mu_p = 3.183\,346\,1(11)\text{ (0.36 ppm)}, \tag{2}$$

in which the one-standard-deviation total error including systematic and random errors is given. Table I lists the sources of error.[11]

The value of $\Delta\nu$ given in Eq. (2) agrees with our earlier measurement[5] of $\Delta\nu$, but the error is less by a factor of 3.3. Our value of μ_μ/μ_p agrees with the most recent muon spin rotation measurement[16] done in liquid bromine which gave $\mu_\mu/\mu_p = 3.183\,344\,1(17)$ (0.53 ppm).

The hfs and g_J fractional density shifts in Kr can be obtained from the density-shift results given above

$$(1/\Delta\nu)(\partial\Delta\nu/\partial D) = -10.57(4)\times10^{-9}\ \mathrm{Torr}^{-1}\ (0\,^\circ\mathrm{C},\mathrm{Kr});$$

$$(1/g_J)(\partial g_J/\partial D) = -1.83(32)\times10^{-9}\ \mathrm{Torr}^{-1}\ (0\,^\circ\mathrm{C},\mathrm{Kr}).$$

Use of our value of μ_μ/μ_p in Eq. (2) together with that of Ref. 16, and of experimental values of g_{μ^+} (Ref. 17) and of $\mu_p/\mu_B{}^e$ (Ref. 18) determines the most precise value for m_μ/m_e, as follows:

$$m_\mu/m_e = (g_\mu/2)(\mu_p/\mu_\mu)(\mu_B{}^e/\mu_p) = 206.768\,259(62)\ (0.30\ \mathrm{ppm}).$$

The current theoretical value for $\Delta\nu$ is given by[1,3,4]

$$\Delta\nu = \tfrac{16}{3}\alpha^2 cR_\infty(\mu_\mu/\mu_p)(\mu_p/\mu_B{}^e)(1+m_e/m_\mu)^{-3}[1+\tfrac{3}{2}\alpha^2+a_e+\epsilon_1+\epsilon_2+\epsilon_3-\delta_\mu'], \tag{3}$$

in which the bracketed term includes the radiative and relativistic corrections to the leading Fermi term, where

$$\epsilon_1 = \alpha^2(\ln 2-\tfrac{5}{2});\quad \epsilon_2 = -(8\alpha^3/3\pi)\ln\alpha[\ln\alpha-\ln 4+\tfrac{281}{480}];\quad \epsilon_3 = (\alpha^3/\pi)(18.4\pm5);$$

$$\delta_\mu' = \frac{3\alpha}{\pi}\frac{m_R}{m_\mu-m_e}\ln m_\mu/m_e + \alpha^2\frac{m_R}{m_\mu+m_e}[2\ln\alpha+8\ln 2-3\tfrac{11}{18}] + (\alpha/\pi)^2 m_e/m_\mu \times[2\ln^2(m_\mu/m_e)-\tfrac{31}{12}\ln(m_\mu/m_e)+(\tfrac{28}{9}+\pi^2/3-1.9)],$$

where $m_R = m_e m_\mu/(m_e+m_\mu)$. The term $\frac{3}{2}\alpha^2$ is a relativistic correction; the terms a_e, ϵ_1, ϵ_2, and ϵ_3 are nonrecoil radiative corrections; the term δ_μ' is relativistic recoil correction, where the first two terms involve recoil only and the third term is a QED radiative recoil contribution including the small hadronic vacuum polarization term. The following values of the fundamental atomic constants are used[18]: $R_\infty = 1.097\,373\,152\,1(11)\times10^5\ \mathrm{cm}^{-1}$ (0.001 ppm)[19]; $c = 2.997\,924\,580(12)\times10^{10}\ \mathrm{cm\ s}^{-1}$ (0.004 ppm); $\alpha^{-1} = 137.035\,963(15)$ (0.11 ppm)[20]; $\mu_p/\mu_B{}^e = 1.521\,032\,209(16)\times10^{-3}$ (0.01 ppm); $\mu_\mu/\mu_p = 3.182\,345\,47(95)$ (0.30 ppm); $m_\mu/m_e = 206.768\,259(62)$ (0.30 ppm); $a_e = (g_e-2)/2 = 1\,159\,652\,200(40)\times10^{-12}$.[21] Hence we ob-

TABLE I. Sources of error in $\Delta\nu$ and μ_μ/μ_p.

Source	$\delta\Delta\nu$ (kHz)	$\delta(\mu_\mu/\mu_p)$ (ppm)
1. Statistical error (e^+ counts)	0.000 ± 0.073	0.000 ± 0.196
2. Random error associated with μ^+ beam monitor	0.000 ± 0.031	0.000 ± 0.084
3. Muon stopping distribution and detector solid angle distribution	0.000 ± 0.008	0.000 ± 0.119
4. Magnetic field measurement (± 0.31 ppm)	0.000 ± 0.000	0.000 ± 0.093
5. Microwave power averaging	+ 0.021 ± 0.035	− 0.102 ± 0.104
6. Gas density	0.000 ± 0.065	0.000 ± 0.001
7. Temperature dependence of a[a]	− 0.073 ± 0.073	− 0.013 ± 0.013
8. Quadratic density shift	0.000 ± 0.041	0.000 ± 0.005
9. Field-dependent line-shape systematics[b]	+ 0.037 ± 0.083	+ 0.356 ± 0.217
10. Bloch-Siegert term and nonresonant states[c]	+ 0.004 ± 0.000	− 0.005 ± 0.000
11. Small approximations in line fitting	0.000 ± 0.000	0.000 ± 0.047
Total correction and one-standard-deviation error	− 0.011 ± 0.160	+ 0.236 ± 0.359

[a] Data of Ref. 5 and of this paper were taken at two temperatures differing by 2.5 °C. Corrections to the data of Ref. 5 were made for the dependence of a on temperature based on experimental (Ref. 12) and theoretical (Ref. 13) information on hydrogen density shifts.

[b] Based on measurements of broadened resonance lines at high microwave power.

[c] Calculation with Refs. 14 and 15.

tain

$$\Delta\nu_{\mathrm{theor}} = 4\,463\,303.7(1.7)(3.0)\ \mathrm{kHz} \quad (0.77\ \mathrm{ppm}), \qquad (4)$$

in which the 1.7-kHz uncertainty comes from combining a 1.3-kHz uncertainty from μ_μ/μ_p with a 1.0-kHz uncertainty from α. The 3.0-kHz theoretical uncertainty is due to ϵ_3.

Comparison of the experimental and theoretical values of $\Delta\nu$ in Eqs. (2) and (4) gives $\Delta\nu_{\mathrm{expt}} - \Delta\nu_{\mathrm{theor}} = -0.8 \pm 3.4$ kHz where the dominant error comes from the theoretical value. The agreement is excellent and provides an important test of the validity of muon electrodynamics.

An alternative approach is to equate $\Delta\nu_{\mathrm{expt}}$ of Eq. (2) to $\Delta\nu_{\mathrm{theor}}$ of Eq. (3) and hence determine α. The result $\alpha^{-1} = 137.035\,974(50)$ (0.37 ppm) is in good agreement with the value of α obtained from the ac Josephson effect.[20]

The standard electroweak theory predicts[22] an axial-vector–axial-vector coupling contribution to $\Delta\nu$ of +0.07 kHz or a fractional contribution of 1.6×10^{-8}. This is about $\frac{1}{2}$ the experimental error in $\Delta\nu$, but about $\frac{1}{50}$ the present theoretical error in $\Delta\nu$. Recent high-energy colliding-beam experiments[23] have measured the charge asymmetry in $e^+e^- \rightarrow \mu^+\mu^-$, which is believed due to this weak neutral-current coupling between leptons. As yet there has been no measurement of a weak-interaction-energy contribution in an atom, and it would be of interest to measure this term in muonium where low momentum transfer is involved.

The contribution of a conjectured Hamiltonian term coupling muonium to antimuonium[24] with a Fermi coupling strength G_{F} would modify the hfs energy levels by about 0.26 kHz. However, because of the small component of antimuonium in the muonium wave function under our experimental conditions with muonium formed in a gas, the effect of such a coupling on our measurements would be negligible.[25]

Improvement in the precision of measurement of $\Delta\nu$ and μ_μ/μ_p by at least a factor of 5 should be possible with the use of a pulsed muon beam which would permit resonance line-narrowing techniques. Improvement in our knowledge of $\Delta\nu_{\mathrm{theor}}$ requires most urgently an improved calculation of the nonrecoil radiative correction term ϵ_3.

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