

with the experimental two-body breakup momentum distribution. To investigate this deviation from unity in more detail, a full Faddeev calculation of the coincidence cross section in the kinematics of the present experiment is needed for both the two- and three-body breakup.

New and accurate  ${}^3\text{He}(e, e'p)$  data have been presented here which may serve as a testing ground for  $NN$  potential properties via the calculation of proton momentum distributions in the three-nucleon system. Tables containing the experimental spectral function values in the region  $0 < E_m < 80$  MeV and  $0 < \vec{p} < 310$  MeV/c are available on request.

This work was supported in part by the Dutch Foundation for Fundamental Research on Matter (FOM) and the Swiss National Science Foundation.

<sup>(a)</sup>Present address: Laboratory for Nuclear Sciences, Massachusetts Institute of Technology, Cambridge, Mass. 02139.

<sup>(b)</sup>Present address: DRF/CPN, Centre d'Etudes Nucléaires Grenoble, 85X, 38041 Grenoble Cédex, France.

<sup>1</sup>C. Ciofi degli Atti, E. Pace, and G. Salmè, in Proceedings of the Mini-Conference on the Study of Few-Body Systems with Electromagnetic Probes, Amsterdam, November 1981 (unpublished), and private communication.

<sup>2</sup>R. A. Brandenburg, Y. E. Kim, and A. Tubis, Phys. Rev. C **12**, 1368 (1975).

<sup>3</sup>G. Jacob and Th. A. J. Maris, Rev. Mod. Phys. **38**, 121 (1966).

<sup>4</sup>M. Bernheim *et al.*, Nucl. Phys. **A365**, 349 (1981).

<sup>5</sup>I. S. Shapiro, V. M. Kolybasov, and G. R. Augst, Nucl. Phys. **61**, 353 (1965).

<sup>6</sup>P. Leconte *et al.*, Nucl. Instrum. Methods **169**, 401 (1980).

<sup>7</sup>J. S. McCarthy, I. Sick, and R. R. Whitney, Phys. Rev. C **15**, 1396 (1977).

<sup>8</sup>P. Dunn, to be published, and thesis, Massachusetts Institute of Technology (unpublished).

<sup>9</sup>J. Mougey *et al.*, Nucl. Phys. **A262**, 461 (1976).

<sup>10</sup>L. W. Mo and Y. S. Tsai, Rev. Mod. Phys. **41**, 200 (1969); Y. S. Tsai, Stanford Linear Accelerator Center Report No. 848, 1971 (unpublished).

<sup>11</sup>A. E. L. Dieperink *et al.*, Phys. Lett. **63B**, 261 (1976).

<sup>12</sup>P. Nunberg, D. Prosperi, and E. Pace, Nucl. Phys. **A285**, 58 (1977).

<sup>13</sup>A. Johansson, Phys. Rev. **136**, B1030 (1964); B. F. Gibson and G. B. West, Nucl. Phys. **B1**, 349 (1967).

<sup>14</sup>I. V. Kozlovsky *et al.*, Nucl. Phys. **A368**, 493 (1981).

<sup>15</sup>J. M. Laget, private communication.

## Is the Shell-Model Concept Relevant for the Nuclear Interior?

J. M. Cavedon, B. Frois, D. Goutte, M. Huet, Ph. Leconte, C. N. Papanicolas,<sup>(a)</sup>  
X.-H. Phan, S. K. Platchkov, and S. Williamson  
*Département de Physique Nucléaire à Haute Energie, Centre d'Etudes Nucléaires de Saclay,  
F-91191 Gif-sur-Yvette Cédex, France*

and

W. Boeglin and I. Sick

*Department of Physics, University of Basel, Basel, Switzerland*

(Received 19 July 1982)

The 3s radial wave function  $R(r)$  has been determined by electron scattering from  ${}^{206}\text{Pb}$  and  ${}^{205}\text{Tl}$ . The shape of  $R(r)$  in the central region of the nucleus is used to test the validity of the independent-particle shell model at large nuclear density.

PACS numbers: 21.65.+f, 21.60.Cs, 25.30.Cg

The shell model was introduced quite late<sup>1</sup> in the historical development of nuclear theory. The strong repulsive core of the nucleon-nucleon interaction, the resulting nucleon-nucleon correlations, and the related saturation of nuclear densities had discouraged the idea of treating nucleons as independent particles moving in an

average potential. The success of the shell model came as quite a surprise. The applicability of the shell model is still one of the striking features of finite nuclei.

The foundation of the shell model has been extensively studied since. Nuclear matter theory<sup>2</sup> has enabled us to understand the short healing

distance ( $\sim 1$  fm) of correlated nucleon wave functions and the role of the Pauli principle, which prevents the short-range nucleon-nucleon interaction from scattering nucleons into higher, already occupied states. The concept of independent (quasi-)particles has been then quite well established for states near the Fermi surface. Nevertheless, the popularity of the shell model rests mainly on its success in explaining experimental observables.

Experimental verification in the high-density region of nuclei is, however, severely limited. Most observables are determined by using strongly interacting probes that are readily absorbed by nuclei. Experimental observations therefore reflect mostly surface properties. This is obvious for all nuclear reactions involving composite particles ( $d$ ,  $^3\text{He}$ ) or pions, and is true even for high-energy nucleons that in principle give better access to the nuclear interior. The large body of available data has little relevance to a test of the validity of the independent-particle model in the high-density region of nuclei. While the absence of absorption makes electrons the best tool to study the nuclear interior, electron scattering has provided little information specific to the present context. The charge and transition densities mainly measure collective properties, and the information provided up to now on independent-particle wave functions in the nuclear interior is quite indirect.

In this paper, we study the isotonic charge-density difference  $\Delta\rho(r)$  between  $^{205}\text{Tl}$  and  $^{206}\text{Pb}$  as a test of shell-model wave functions in the high-density central region of a heavy nucleus. According to the shell model, Pb and Tl differ by a  $3s$  proton. The  $3s$  radial wave function has a very special character: a pronounced maximum at the center of the nucleus, with two nodes at  $r=2.5$  and  $5$  fm. This unique radial structure can be unmistakably distinguished from other more complicated contributions to  $\Delta\rho(r)$ , and can be used as a crucial test of the shell-model wave functions at high nuclear density.

We have measured elastic electron scattering cross sections for  $^{205}\text{Tl}$  and  $^{206}\text{Pb}$  at an incident energy of  $502$  MeV. The data taken between  $30^\circ$  and  $70^\circ$  scattering angle cover a momentum-transfer range from  $1.4$  to  $3.2$   $\text{fm}^{-1}$ . The experiment was performed in the HE1 end station<sup>3</sup> at the Saclay linear accelerator with a beam intensity up to  $20$   $\mu\text{A}$ . The targets were  $100$   $\text{mg}/\text{cm}^2$  thick and isotopically enriched to  $>95\%$ ; their thickness and homogeneity were determined by

x-ray absorption. To avoid melting, the targets were wobbled following a Lissajous pattern and cooled by a jet of hydrogen gas of  $1$  Torr pressure. Two ferrite toroid monitors and a Faraday cup were used to integrate the beam with an accuracy of  $0.2\%$ .

The scattered electrons were analyzed by the  $900\text{-MeV}/c$  magnetic spectrometer. This experiment is the first one to use the new dispersion matching system "Stradivarius."<sup>4</sup> It consists of a quadrupole dispersing the beam after the momentum analysis, followed by a five-quadrupole rotator to match the dispersion plane of the magnetic spectrometer. The system has a momentum acceptance  $\Delta E/E = 2 \times 10^{-3}$  for a resolution  $\Delta E/E = 10^{-4}$  in energy loss. For this experiment, the energy resolution used,  $2 \times 10^{-4}$ , was quite sufficient to separate the elastic peak from the first excited state ( $\frac{3}{2}^+$ ) of  $^{205}\text{Tl}$  at an excitation energy of  $203.7$  keV. The scattered electrons were detected with a new vertical drift chamber<sup>5</sup> with  $0.15\text{-mm}$  spatial resolution. The trigger was given by two rows of plastic scintillators and a Lucite Cherenkov counter. Absolute cross sections were obtained by normalizing with the efficiency of the detector system. This efficiency was determined to be  $0.96 \pm 0.03$  by the measurement of  $^{12}\text{C}$  cross sections in the region of the second diffraction maximum ( $q \cong 2.1$   $\text{fm}^{-1}$ ). The targets, mounted on a rotating wheel, were measured in rapid succession without change of experimental conditions. With this procedure, systematic errors largely cancel, yielding only an overall  $1\%$  uncertainty in the measurement of one isotope relative to another.

Our results for the cross-section ratio of  $^{205}\text{Tl}$  to  $^{206}\text{Pb}$  are shown in Fig. 1 together with the results at  $119$  and  $199.5$  MeV measured by Eutenauer, Friedrich, and Voegler.<sup>6</sup> (We have omitted the  $289\text{-MeV}$  data for which we find the same inconsistencies observed earlier.<sup>7</sup>) The ratio of cross sections shows a striking variation in the region of momentum transfer  $q = 2$   $\text{fm}^{-1}$ , which is the effect of the  $3s_{1/2}$  proton hole in  $^{205}\text{Tl}$ . The  $3s_{1/2}$  wave function has a very pronounced maximum at  $r=0$  with a shape similar to a Bessel function  $j_0(q'r)$ , where  $q' \cong 2$   $\text{fm}^{-1}$ . Consequently, the ratio of the cross sections of  $^{205}\text{Tl}$  to  $^{206}\text{Pb}$  exhibits a strong peak close to a  $\delta(q' - q)$  function. Thus, the peak region in the ratio of cross sections directly yields information on the shape of the  $3s_{1/2}$  wave function. The core polarization, which corresponds mainly to a size increase of the nucleus and has a much smoother shape, in-

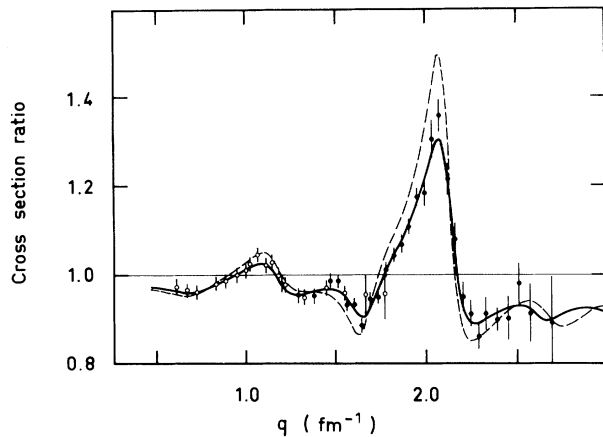


FIG. 1. The experimental  $^{205}\text{Tl}$ -to- $^{206}\text{Pb}$  cross-section ratios from the present experiment (closed circles) and Ref. 6 (open circles) as a function of momentum transfer. The best fit is given by a solid line. The mean-field calculations of Dechargé and Gogny (Ref. 11) and Campi and Sprung (Ref. 10) (dashed line) are virtually indistinguishable.

duces variations at smaller momentum transfer.

The charge density has been determined from a fit to the electron scattering cross sections combined with five moments deduced from the measurement of muonic x-ray transitions.<sup>8</sup> A model-independent analysis was made with an expansion of the charge density in a sum of Gaussians.<sup>9</sup> The width parameter of this expansion has been determined by a fit to Hartree-Fock calculations ( $\gamma = 1.388$ ).

The resulting charge-density difference is shown in Fig. 2 together with the experimental uncertainty which includes the contributions of statistical, systematic, and model errors. The shape determined by the experiment is strikingly close to the one expected for a  $3s_{1/2}$  wave function.

We have compared our experimental results to the predictions of two density-dependent-Hartree-Fock (DDHF) calculations.<sup>10,11</sup> These calculations assume that the nucleons move independently in an average potential determined by the interaction with the other nucleons. This effective interaction has a density dependence which reflects the short-range correlations between nucleons. In both calculations, the ground state of  $^{205}\text{Tl}$  is described as a pure  $(3s_{1/2})_{\pi}^{-1}(3p_{1/2})_{\nu}^{-2}$  hole state in  $^{208}\text{Pb}$ . Figure 1 shows that these calculations explain qualitatively the structure of the data. However, the amplitude of the observed peak at  $2 \text{ fm}^{-1}$  is significantly smaller than the

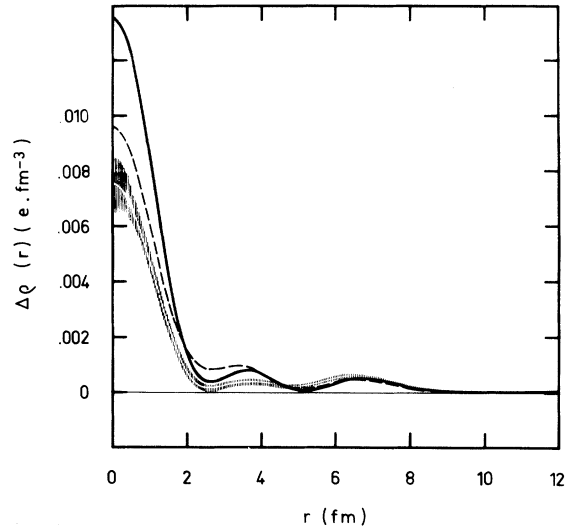


FIG. 2. Experimental charge-density difference between  $^{206}\text{Pb}$  and  $^{205}\text{Tl}$  together with prediction of mean-field theory (Ref. 10) (solid line) and adjusted calculation (Ref. 15) (dashed line).

theoretical one.

Such disagreement is not surprising, since DDHF calculations are not sufficient for odd-even nuclei. Calculations for  $^{205}\text{Tl}$  show that the ground-state configuration is more complicated. Both studies<sup>12</sup> of coupling of the  $3s$  proton to collective excitations of  $^{206}\text{Pb}$  as well as shell-model calculations<sup>13</sup> for the  $(3s_{1/2})^{-1}(2d_{3/2})^{-2}$  state show that the  $3s$ -hole strength in  $^{205}\text{Tl}$  amounts to  $s = 0.7-0.9$ . The remainder of the strength is mainly found in the  $2d_{3/2}$  configuration. These predictions are confirmed by the results of transfer

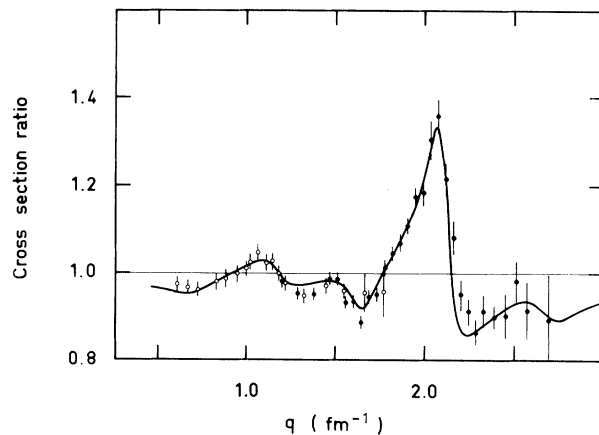


FIG. 3. Same as Fig. 1. The solid curve represents the adjusted calculation (Ref. 15) performed using a  $3s$  hole strength of 0.7.

reactions.<sup>14</sup> Proton pickup from  $^{206}\text{Pb}$  measures only 70% of the  $l=0$  strength found in  $^{208}\text{Pb}$ . Accordingly, a new DDHF calculation<sup>15</sup> has been performed with the hole strength constrained to  $s_{3s}=0.7$  and  $s_{2d}=0.3$ . The resulting prediction for the cross-section ratio is shown in Fig. 3. The experimental data are amazingly well reproduced.

In configuration space there is a small deviation between data and dashed curve smoothly decreasing in the region from 0 to 7 fm. This can probably be attributed to a too simple treatment of core polarization effects, i.e., the neglect of many small wave-function admixtures beyond  $2d_{3/2}$ . The shape of the oscillatory behavior of  $\Delta\rho(r)$ , however, is very well accounted for. This shows that the 3s radial wave function is very similar in shape to the one predicted by mean-field calculations.

The present work provides clear evidence that the concept of a shell-model wave function in the center of a heavy nucleus is well founded. Even in this high-density region, the independent-particle picture retains an impressive degree of validity.

---

<sup>(a)</sup>Present address: Department of Physics, University of Illinois at Urbana-Champaign, Urbana, Ill. 61820.

<sup>1</sup>O. Haxel, J. H. D. Jensen, and H. E. Suess, *Phys. Rev.* **75**, 1766 (1949); M. G. Mayer, *Phys. Rev.* **75**, 1969 (1949).

<sup>2</sup>H. A. Bethe, *Ann. Rev. Nucl. Sci.* **21**, 93 (1971).

<sup>3</sup>P. Leconte *et al.*, *Nucl. Instrum. Methods* **169**, 401

(1980).

<sup>4</sup>C. Grunberg and P. Leconte, to be published.

<sup>5</sup>W. Bertozzi, M. V. Hynes, C. P. Sargent, C. Creswell, P. C. Dunn, A. Hirsch, M. Leitch, B. Norum, F. N. Rad, and T. Sasanuma, *Nucl. Instrum. Methods* **141**, 457 (1977).

<sup>6</sup>H. Euteneuer, J. Friedrich, and N. Voegler, *Nucl. Phys.* **A298**, 452 (1978).

<sup>7</sup>B. Frois, J. B. Bellicard, J. M. Cavedon, M. Huet, P. Leconte, P. Ludeau, A. Nakada, X.-H. Phan, and I. Sick, *Phys. Rev. Lett.* **38**, 152 (1977).

<sup>8</sup>H. L. Anderson, C. K. Hargrove, F. P. Hincks, J. D. McAndrew, R. J. McKee, R. D. Barton, and D. Kessler, *Phys. Rev.* **187**, 1565 (1969); D. Kessler, H. Mes, A. C. Thomson, H. L. Anderson, M. S. Dixit, C. K. Hargrove, and R. S. McKee, *Phys. Rev. C* **11**, 1719 (1975); H. Backe, R. Engfer, V. Jahnke, E. Kankeleit, R. M. Pearce, C. Petitjean, R. Schellenberg, R. Schneuwly, W. U. Schröder, H. K. Walter, and A. Zehnder, *Nucl. Phys.* **A189**, 472 (1972); R. Engfer, H. Schneuwly, J. L. Vuilleumier, H. K. Walter, and A. Zehnder, *At. Data Nucl. Data Tables* **14**, 509 (1974).

<sup>9</sup>I. Sick, *Nucl. Phys.* **A218**, 509 (1974).

<sup>10</sup>X. Campi and D. W. L. Sprung, *Nucl. Phys.* **A194**, 401 (1972).

<sup>11</sup>J. Dechargé and D. Gogny, *Phys. Rev. C* **21**, 1568 (1980).

<sup>12</sup>L. Zamick, V. Klemt, and J. Speth, *Nucl. Phys.* **A245**, 365 (1975); A. Covello and G. Sartoris, *Nucl. Phys.* **A93**, 481 (1967); N. Azziz and A. Covello, *Nucl. Phys.* **A123**, 681 (1969).

<sup>13</sup>B. Silvestre-Brac and J. P. Boisson, *Phys. Rev. C* **24**, 717 (1981); M. Tomaselli, D. Herold, and L. Grünbaum, *Nuovo Cimento* **A46**, 468 (1978).

<sup>14</sup>E. R. Flynn, R. A. Hardekopf, J. D. Sherman, J. W. Sunier, and J. P. Coffin, *Nucl. Phys.* **A279**, 394 (1977); S. Hinds, R. Middleton, J. H. Bjerregaard, O. Hansen, and O. Nathan, *Nucl. Phys.* **A83**, 17 (1966).

<sup>15</sup>X. Campi, private communication.