one hand, that $y[=\frac{1}{2}\ln(X_2/X_1)]$ and $Z^2(=X_1X_2)$ are also orthogonal coordinates, the latter being defined again on a logarithmic scale. Furthermore, since one has $1-u^2=x_t^2/Z^2$, definite values of u and, as well, of a given area in the (u,y) plane are obtained, for different values of x_t, by simply performing a translation along the Z^2 axis (first diagonal). On the other hand, since $d\ln(X)=dX/X$, a given area on this plot gives directly the integral of $(dX_1/X_1)(dX_2/X_2)$ over the corresponding X_1, X_2 ranges. Thus, as long as the constant area, defined in the (u,y) plane by the angular acceptance, remains within the limits given by the phase space, $\iint[(1-u^4)/u](dX_1/X_1)(dX_2/X_2)$ stays constant and $G(x_t^2)$ will vary only slightly as a result of the smooth variations of the factor $N(X_1)N(X_2)$ in the translations; when the phase-space limits become effective [i.e., for $x_t>\tan(\theta_0/2)$], $G(x_t^2)$ will decrease more sharply.

In conclusion, we notice that the simple and transparent DEPA can be successfully used (see Fig. 3), at least for nontagging measurements. In particular, it allows one to easily understand and compute the p_t dependence through a scaling function of $x_t=2p_t/\sqrt{s}$ and of the angular cutoff in the acceptance of the central detector.

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Zeros in Amplitudes: Gauge Theory and Radiation Interference

Stanley J. Brodsky [a]
Institute for Advanced Study, Princeton, New Jersey 08540

and

Robert W. Brown [b]
Fermi National Accelerator Laboratory, Batavia, Illinois 60510

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It is found that any complete single-photon tree amplitude has a general canonical form which vanishes in certain kinematical zones provided that any derivative couplings are as prescribed by gauge theory. The location of these zones depends only on the external charges and momenta. Their occurrence is based on classical radiation interference that is a generalization of the well-known absence of dipole radiation by colliding particles with the same charge-to-mass ratio. Weak-boson amplitude zeros are explained.

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We have found amplitude zeros in a very general class of single-photon tree graphs. The physical basis for these zeros is the complete destructive interference of the classical radiation of the incoming and outgoing charged lines and is the generalization of the well-known nonrelativistic result where electric dipole radiation vanishes in collisions of particles with the same charge-to-mass ratio and where magnetic dipole radiation vanishes for such collisions if in addition their g factors are identical. What is remarkable is that the same cancellation can go through, at arbitrary photon momenta, for quantum tree amplitudes which include spin and contact (seagull) currents and internal line radiation.

This result can be stated as a theorem. Let T_G denote a tree graph with n external lines labeled by particle four-momenta p_i, charges Q_i, and masses m_i. The external and internal lines can be scalar, Dirac, or vector particles (spin ≤ 1). The vertices of T_G are taken to correspond to local interactions involving any number of fields with constant or single derivative couplings, and the derivative couplings must be of gauge-theory form.[1] In particular, the photon-particle couplings, which are central to the theorem, must correspond to the same gyromagnetic ratio, $g=2$, for all spinning particles.

Theorem: If M_γ is the single-photon emission amplitude which is the sum generated by making photon attachments (four-momentum q) in all possible ways[2] onto T_G, then

$$M_\gamma = 0 \tag{1}$$

if the ratios $Q_i/p_i \cdot q$ are all equal.

Comment.—The conditions for (1) to be valid are independent of the orientation of any spin and can be written as the $n-2$ equalities

$$\frac{Q_i}{p_i \cdot q} = \frac{Q_1}{p_1 \cdot q}, \quad i = 2, \ldots, n-1, \tag{2}$$

where we have chosen $i=1$ as a standard and $i=n$ as that ratio determined in terms of the rest by charge and momentum conservation. Equations (2) and momentum conservation define a kinematic region of amplitude zeros which we call the *null radiation zone* of M_γ.

Proof: The theorem is proven first in the special case where T_G is an arbitrary vertex, V_G, and then generalized to include internal lines. The external-line current factors $Q_j \cdot \epsilon / p \cdot q$ which are identified with photon emission with charge Q flowing along momentum p in a Feynman graph are[3] outgoing,

$$\frac{Q}{p \cdot q} \tilde{\chi} [p \cdot \epsilon + \text{spin current} + \text{contact current}] \ldots, \tag{3a}$$

and incoming,

$$\ldots [-p \cdot \epsilon - \text{spin current} - \text{contact current}] \chi \frac{Q}{p \cdot q}, \tag{3b}$$

for particle wave function χ and photon polarization ϵ. The "contact" current arises from both the momentum change (due to photon emission) in and the gauging (seagull) of any derivative coupling for the external leg. For the cases of interest, $\chi = \{1;\, u(p);\, \eta_\alpha(p) = g_{\alpha\beta}\eta^\beta(p)\}$, $\tilde{\chi} = \{1;\, \bar{u}(p);\, \eta_\alpha{}^\dagger(p) = g_{\alpha\beta}\eta^\beta(p)^\dagger\}$, and the spin current is $\{0;\, \frac{1}{4} i \sigma^{\beta\alpha}\omega_{\beta\alpha};\, \omega_{\alpha\beta}\}$ for a {scalar; Dirac; vector} particle, respectively, where

$$\omega_{\mu\nu} = q_\mu \epsilon_\nu - \epsilon_\mu q_\nu \tag{4}$$

and η_α is the vector polarization. If there is a derivative coupling $\partial^\alpha = g^{\alpha\beta}\partial_\beta$, the contact current is $\omega^{\alpha\beta}$.

The one-photon amplitude $M_\gamma(V_G)$ generated from V_G is calculated using (3) and the relevant vertex coupling. Clearly the convection currents cancel separately when Eqs. (2) are satisfied by momentum conservation and transversality ($q \cdot \epsilon = 0$). Except for the Yang-Mills vertex, the cancellation of the spin and contact terms is a consequence of an interesting relationship between Lorentz transformations and the photon-particle couplings. Namely, we find that the spin currents are proportional to the relevant first-order wave-function corrections corresponding to the Lorentz transformation $\Lambda_{\mu\nu} = g_{\mu\nu} + \lambda\omega_{\mu\nu}$, where λ is infinitesimal, while the contact terms are proportional to the first-order change in a derivative coupling (in momentum space) also due to the same $\Lambda_{\mu\nu}$. Thus when Eqs. (2) are satisfied $M_\gamma(V_G)$ is proportional to $\delta M(V_G)$, the first-order change in the vertex amplitude $M(V_G)$ corresponding to V_G. Since $M(V_G)$ is Lorentz invariant $\delta M(V_G) = 0$ and the theorem follows. The theorem also follows for radiation by a vector particle coming from a Yang-Mills vertex because of a crucial cancellation in the terms quadratic in q which appear in this instance.[4]

The generalization of the proof to include internal lines follows from the fact that if Eqs. (2) are satisfied then $Q_I/p_I \cdot q = Q_1/p_1 \cdot q$, where Q_I and p_I refer to any internal line of T_G that is not in a closed loop, and from a novel decomposition of the internal-line emission into two quasi-external-line forms. That is, any internal-line emission factor involving propagators D can be placed in the general form

$$\frac{Q}{p\cdot q}\left[D(p-q)(p\cdot\epsilon+\text{spin}+\text{contact})-(p\cdot\epsilon+\text{spin}+\text{contact})D(p)\right], \tag{5}$$

where as with (3) all indices have been suppressed. A simple (scalar) example of (5) is

$$\frac{1}{(p-q)^2-m^2}\,Q(2p-q)\cdot\epsilon\,\frac{1}{p^2-m^2}=\frac{Q}{p\cdot q}\left[\frac{1}{(p-q)^2-m^2}\,p\cdot\epsilon-p\cdot\epsilon\,\frac{1}{p^2-m^2}\right]. \tag{6}$$

The identity (5) is crucial since it implies, under the conditions of the theorem and on the basis of the same invariance arguments used for V_G, that the currents which are represented by the square brackets of (3) and (6) cancel as we sum the contributions of the photon attachments throughout T_G. Namely, as a consequence of (5), $M_\gamma(T_G)$ is basically a sum of gauge-invariant quasivertex amplitudes for which the theorem holds individually. This completes the proof of the theorem.

Radiation representation.—The theorem and the linearity of $M_\gamma(T_G)$ in the charges imply that there is an $(n-2)$-dimensional first-order zero in the space of the variables $Q_i/p_i\cdot q$. Irrespective of whether this null radiation zone lies in the physical region, the theorem implies that $M_\gamma(T_G)$ has the representation

$$M_\gamma(T_G)=\sum_{i=2}^{n-1}\left(\frac{Q_i}{p_i\cdot q}-\frac{Q_1}{p_1\cdot q}\right)\boldsymbol{F}_i. \tag{7}$$

Furthermore, it is straightforward to show that $M_\gamma(T_G)$ can be expressed in a new canonical "double-difference" form. For the special case $M_\gamma(V_G)$ it is

$$M_\gamma(V_G)=\sum_{i=2}^{n-1}p_i\cdot q\left(\frac{Q_1}{p_1\cdot q}-\frac{Q_i}{p_i\cdot q}\right)\left(\frac{j_n}{p_n\cdot q}-\frac{j_i}{p_i\cdot q}\right), \tag{8}$$

where j_i is the product of the current for photon emission by the ith leg and the remainder of the amplitude.[5] The general form for $M_\gamma(T_G)$ is obtained by use of the quasivertex expansion discussed in the previous paragraph.

Example.—Let us illustrate the theorem by a simple $n=4$ scalar particle example where T_G is a t-channel exchange graph with constant vertices. By the identity (6), the five graphs of M_γ can be rearranged to read

$$M_\gamma=\frac{1}{(p_3-p_2)^2-m_5^2}\left[\frac{Q_4}{p_4\cdot q}p_4\cdot\epsilon-\frac{Q_1}{p_1\cdot q}p_1\cdot\epsilon+\frac{Q_1-Q_4}{(p_1-p_4)\cdot q}(p_1-p_4)\cdot\epsilon\right]$$
$$+\frac{1}{(p_1-p_4)^2-m_5^2}\left[\frac{Q_3}{p_3\cdot q}p_3\cdot\epsilon-\frac{Q_2}{p_2\cdot q}p_2\cdot\epsilon+\frac{Q_2-Q_3}{(p_2-p_3)\cdot q}(p_2-p_3)\cdot\epsilon\right], \tag{9}$$

within an overall constant factor and with $p_1+p_2=p_3+p_4+q$. The internal particle has mass m_5 and charge $Q_5=Q_1-Q_4=Q_3-Q_2$.

It is seen that (9) vanishes under the conditions (2). The two terms in (9) correspond to the two quasivertices and vanish separately as advertised. For $Q_5=0$, the two terms combine to cancel.

Classical correspondence.—The relativistic amplitude for radiation during collisions is found from the classical current

$$\vec{\mathrm{j}}(\vec{\mathrm{x}},t)=\left[\theta(-t)\sum_{i=1}^{k}+\theta(t)\sum_{i=k+1}^{n}\right]Q_i\vec{\mathrm{v}}_i\,\delta(\vec{\mathrm{x}}-\vec{\mathrm{v}}_it-\vec{\mathrm{r}}_i(0))+[\text{small-distance, small-time corrections}], \tag{10}$$

where k initial particles scatter into $n-k$ final particles with uniform velocities $\vec{\mathrm{v}}_i=\dot{\vec{\mathrm{r}}}_i$ up to or after a time τ of collision, say, $-\tau/2\leq t\leq\tau/2$. Spin currents are ignored. Then the classical amplitude for radiation in the direction $\hat{n}$ by this current for low frequency, $\omega\tau\ll 1$, is[6]

$$A=\frac{1}{\omega}\left[\sum_{1}^{k}-\sum_{k+1}^{n}\right]\frac{Q_i}{1-\hat{n}\cdot\vec{\mathrm{v}}_i}\vec{\mathrm{v}}_i\cdot\vec{\epsilon}\exp[-i\omega\hat{n}\cdot\vec{\mathrm{r}}_i(0)]. \tag{11}$$

It is seen from (11) that the sudden disappearance/appearance of charges provides the correct infrared limit $A \to A_{\rm IR}$ as $\omega \to 0$, where

$$A_{\rm IR} = \Big[\sum_{1}^{k} - \sum_{k+1}^{n}\Big] \frac{Q_i}{\omega(1-\hat{n}\cdot\vec{v}_i)} \vec{v}_i \cdot \vec{\epsilon}. \qquad (12)$$

Nonrelativistically, $A_{\rm IR}$ reduces to the electric dipole amplitude and indeed for common charge-mass ratios,

$$A_{\rm IR} \xrightarrow[v_i \to 0]{} \frac{Q_1}{\omega m_1} \vec{\epsilon} \cdot \Big[\sum_{1}^{k} - \sum_{k+1}^{n}\Big] m_i \vec{v}_i = 0. \qquad (13)$$

We discover a relativistic generalization of (13) by rewriting (12) in four-vector notation,

$$A_{\rm IR} = \Big[\sum_{k+1}^{n} - \sum_{1}^{k}\Big] \frac{Q_i}{p_i \cdot q} p_i \cdot \epsilon. \qquad (14)$$

With this expression, it is obvious that $A_{\rm IR} = 0$ for common $Q_i/p_i \cdot q$, precisely the conditions of the theorem.

Finally, the theorem leads to a number of results and remarks[3]:

(1) The null radiation zone always lies in the physical domain for a given process if all the incoming and outgoing charges are of the same sign and if $n-1$ masses are neglected. If we then consider increasing these masses to some arbitrary values, the zone may move out of the physical region depending on the ratios Q_i/m_i.

(2) The $n=3$ case is precisely the zero discussed previously for weak-boson reactions,[7,8] and we have thereby identified the physical origin of such "gauge zeros." For the same case, the factorization of Ref. 9 is reproduced in (8).

(3) Through the generality of the theorem we discover previously unnoticed zeros in ancient radiative processes such as

$$e^- + e^- \to e^- + e^- + \gamma, \qquad (15)$$

where the zero occurs for the photon at right angles to the beams in the c.m. system and for the final electrons at equal energies. It is a two-dimensional zone consisting of the common electron energy and a final electron azimuth relative to the photon axis. Other reactions such as hard quark scattering, $q+q \to q+q+\gamma$, or τ radiative decays serve as examples in which the existence of radiative zeros depends crucially on the particle magnetic moments, as in the W reactions.[7,8]

(4) A physical null radiation zone can exist for a subset of neutral external particles provided that they are massless, they can propagate along the photon's direction, and their spin terms vanish in that configuration. Vector particle couplings must involve conserved currents and avoid elastic forward scattering; thus $\bar{\nu}_e + e^- \to W^- + \gamma$ has a gauge zero (Ref. 7) while Compton scattering, $\gamma + e^- \to \gamma + e^-$, does not.

(5) Other massless gauge bosons are known[9,10] to give rise to $n=3$ amplitude zeros. The present theorem can be generalized to the determination of the canonical form and possible null radiation zones for arbitrary gauge groups. The "charges" now involve the representations of the particles and the amplitude must be invariant under the transformations of the corresponding internal symmetry group. Unfortunately, along with color the amplitude zeros are neutralized by the necessary averaging/summing that goes on when the quark and gluon reactions are hadronized.

(6) At the heart of the theorem is the close, elegant connection between the electromagnetic gauge couplings and a Lorentz transformation of the particles' spin. The same connection is responsible for $g=2$, i.e., the identity of the orbital and spin precession frequencies of a charged particle in a uniform field if its couplings are given by gauge-theory tree graphs.[11] In each case, the null radiation zones and the value $g=2$ are destroyed by quantum corrections from loop graphs. We also note that $g \neq 2$ destroys the spin-current cancellation by adding terms that are no longer a universal Lorentz transformation of the fields.

(7) The theorem implies a low-energy theorem for *any* complete photon amplitude, including closed loops, since the leading terms[4] in q vanish under the conditions (2).

(8) It is well known that gauge theory couplings can be derived[12] by assuming a unitarity constraint on the high-energy behavior of tree graph amplitudes. By turning our argument on its head, electromagnetic gauge theory couplings can be derived by the constraint that the canonical form (8) is maintained in tree graph approximation.

(9) Although we have not investigated systems with spin > 1 it is possible to build a spin-J system out of a composite of $2J$ spin-$\frac{1}{2}$ collinear fermions. It is interesting to note that in the tree graph approximation gauge couplings and identical $Q_i/p_i \cdot q$ for collinear constituents translate into an effective gauge coupling for the spin-J composite which preserves the null zone.

(10) In view of the intimate relationship between internal symmetry and space-time which is crucial to our results, an interesting open question is the possibility of the extension of the canonical form, (8), to graviton radiation and even to super-

symmetry.

(11) Local gauge theory exposes an intimate relationship between internal symmetry and space-time. Null zones, including the ancient dipole result, show another face of this relationship, leading to more equations involving the internal variables (e.g., charge) and space-time (e.g., masses and angles).

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[(a)]On leave from Stanford Linear Accelerator Center, Stanford University, Stanford, Cal. 94305.

[(b)]On leave from Case Western Reserve University, Cleveland, Ohio 44106.

[1]We mean by this that the couplings involve no derivatives of Dirac fields and at most single derivatives of scalar and vector fields. Products of single derivatives of distinct scalar fields are allowed. All vector derivative couplings must be of the Yang-Mills trilinear variety or products thereof. Such couplings include all renormalizable theories of current physical interest as well as an infinite class of nonrenormalizable theories corresponding to unrestricted numbers of fields.

[2]Attachments are made onto all charged lines and onto vertices with derivative couplings (seagulls).

[3]A much more detailed discussion will be presented elsewhere: R. W. Brown, K. L. Kowalski, and S. J. Brodsky, to be published.

[4]Apart from the $(p\cdot q)^{-1}$ factors the convection terms are of order q^0 while in the usual cases the spin and contact terms are linear in q. When Dirac or vector particles encounter a derivative coupling involving their own field, quadratic terms appear and generally these violate the theorem. An important exception occurs for the Yang-Mills vertex, where a cancellation occurs due to the cyclic nature of the gauge coupling.

[5]The appearance of the current differences is easy to understand by a complementary version of the theorem. Namely, suppose that all of the $j_i/p_i\cdot q$ factors were equal, then $M_\gamma(V_G)$ vanishes by charge conservation, if we define all particles as outgoing.

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Locally Supersymmetric Grand Unification

A. H. Chamseddine, R. Arnowitt, and Pran Nath
Department of Physics, Northeastern University, Boston, Massachusetts 02115

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A locally supersymmetric grand unification program is proposed which couples the $N=1$ supergravity multiplet to an arbitrary grand unified gauge group with any number of left-handed chiral multiplets and a gauge vector multiplet. A specific model is discussed where it is shown that not only do the gravitational interactions eliminate the degeneracy of the vacuum state encountered in global supersymmetry, but simultaneously they can break both supersymmetry and SU(2)⊗U(1) down to a residual $SU(3)^c$⊗U(1) symmetry at ~300 GeV.

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Recently much interest has been devoted to supersymmetric grand unified theories.[1-5] All existing supersymmetric grand unified models are based on *global* supersymmetry. In such theories it is generally easy to break spontaneously the internal, e.g., SU(5), symmetry, but more difficult to break supersymmetry itself. In this paper we propose a new type of supersymmetric grand unified model based on *local* supersymmetry. We consider here $N=1$ supergravity[6] coupled to left-handed chiral scalar[7] and gauge multiplets.[8] We will see that the supergravity couplings automatically produce a spontaneous breaking which removes the degeneracy of the

