Upper Critical Field of Regular Superconductive Networks

Jorge Simonin, Daniel Rodrigues, and Arturo López
Centro Atómico Bariloche, Comisión Nacional de Energía Atómica–Instituto Balseiro, Comisión Nacional de Energía Atomica and Universidad Nacional de Cuyo, 8400-Bariloche, Argentina

(Received 25 June 1982)

The de Gennes–Alexander theory of superconductive networks is used to study the upper critical fields of two-dimensional square lattices built from N equally spaced infinite wires joined by transverse strands. Phase diagrams and current-flow patterns for representative cases are shown. A critical value is found of the magnetic flux per square below which the current flow resembles the Meissner state, and above which an ordered array of vortices appears, in general incommensurate with the underlying lattice. The critical flux decreases for increasing N.

PACS numbers: 74.20.De, 74.50.+r.

Important aspects of disorder in inhomogeneous superconductors have been considered from a percolative viewpoint by Alexander, using the linearized Ginzburg-Landau (GL) equations for nets of thin wires,[1,2] which behave as weak links[3] joining the nodes of the network. The experimental realization of ordered arrays of this kind may be near, as recent experiments on two-dimensional arrays of Josephson junctions show.[4] In this paper we study the magnetic phase boundary and the mechanism of nucleation in such periodic structures of flux-locking elements.

We consider "ladder" structures, consisting of N infinitely long wires, equally spaced and joined by transverse strands. For finite N we find a critical value φ_1 of φ (flux per square) below which the order parameter at the nodes is uniform, with a current distribution resembling the shielding currents of a slab in a parallel field. When $\varphi > \varphi_1$, a finite wave vector q_m is dominant and the current distribution has a vortex structure,[5] modulated with a wavelength $2\pi/q_m$ which may be incommensurate with the underlying lattice. As for Abrikosov's solution for type-II superconductors, nonlinear terms in the GL equations, including vortex-vortex interactions, should stabilize a given structure, possibly commensurate. The behavior of φ_1 for $N = 20$, 30, and 40 indicates that $\varphi_1 \to 0$ as N increases.

The de Gennes–Alexander theory of superconductivity[1-3] on networks is based on the solutions of the GL equations on one-dimensional branches ab, bc, $cd, \ldots$, which join at the nodes a, b, $c, \ldots$. The order parameter along branch ab of length $l = L/\xi$ is given by

$$\psi(s) = \frac{\exp(i\gamma_{as})}{\sin l} \times [\psi_a \sin(l-s) + \psi_b \exp(-i\gamma_{ab}) \sin s], \quad (1)$$

where s is the curvilinear coordinate along the branch from a, $\gamma_{as} = (2\pi/\varphi_0)\int_a^s \vec{A}(l')\cdot d\vec{l}'$, A is the vector potential of the applied field, φ_0 is the flux quantum, and $\psi_a = |\psi_a| e^{i\alpha}$ and $\psi_b = |\psi_b| e^{i\beta}$ are the values of ψ at a and b. When the quantum mechanical current associated with the order parameter (1) is supplemented with a generalized Kirchhoff current law,[1-3] one obtains linear equations for the order parameters at the nodes, whose compatibility condition leads to the phase diagram.

For a simple ladder, the nodal equations read

$$\begin{aligned} &3\cos l\, \psi_n^{\uparrow} - e^{i\gamma}\psi_{n-1}^{\uparrow} - e^{-i\gamma}\psi_{n+1}^{\uparrow} - \psi_n^{\downarrow} = 0, \\ &3\cos l\, \psi_n^{\downarrow} - e^{-i\gamma}\psi_{n-1}^{\downarrow} - e^{i\gamma}\psi_{n-1}^{\downarrow} - \psi_n^{\uparrow} = 0, \end{aligned} \quad (2)$$

where n indicates the node, $(\uparrow, \downarrow)$ refers to the upper or lower branch, and $2\gamma = 2\pi\varphi/\varphi_0$. We look for a solution in the form $\psi_n^{\alpha} = f_q^{\alpha} e^{iqn}$ $(\alpha = \uparrow, \downarrow)$. Then Eqs. (2) reduce to a 2×2 system which has a solution if $\beta_-\beta_+ = 1$, where $\beta_{\pm} = 3\cos l - 2\cos(\gamma \pm q)$, implying

$$\cos l = \tfrac{1}{3}\{2\cos\gamma \cos q \mp (4\sin^2\gamma \sin^2 q + 1)^{1/2}\}. \quad (3)$$

Equation (3) gives l (temperature) as function of γ (applied field) for each q. The phase diagram is determined by asking that l should be minimum (highest T) for a given field. Equation (3) has solutions corresponding to metastable states, not considered here. The condition of minimum l implies (a) $\sin q = 0$, (b) $\cos q = \pm \cos\gamma\, (1 + \frac{1}{4}\sin^2\gamma)^{1/2}$. For $\varphi < \varphi_1 = 0.215\varphi_0$ the solution is given by (a) and for $\varphi > \varphi_1$ by (b). Superconductivity condenses in a uniform mode (at the nodes) for $\varphi < \varphi_1$, whereas a modulated structure is energetically more favorable for $\varphi > \varphi_1$. Figure 1 gives the phase diagram. The phase boundary is the envelope of the curves corresponding to different q's. Only a few q values are shown, but since for

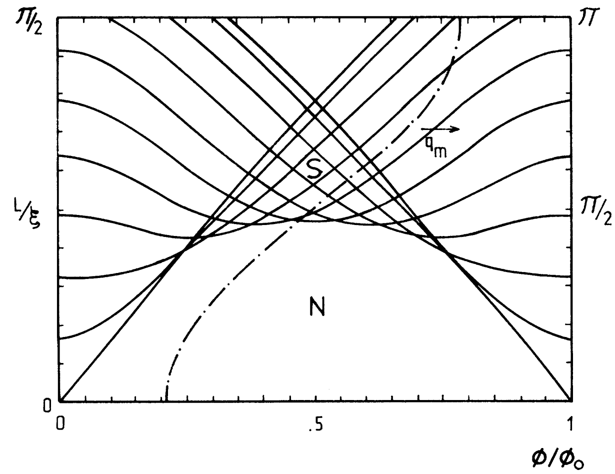

FIG. 1. Phase diagram of infinite ladder for q from 0 to π, in steps of $\pi/10$. The vertical axis is $l = L/\xi$ $[\xi = \xi_0/(1-t^2)^{1/2}]$. The physical phase boundary is the envelope of the curves shown and is periodic in φ/φ_0. Also shown is q_m, the wave vector of the condensed state at the phase boundary.

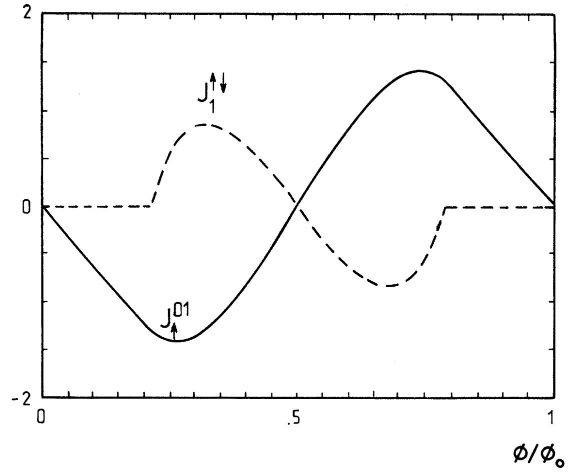

FIG. 2. Currents in the first longitudinal branch $J_\uparrow^{01}$ and in the first transverse branch $J_1^{\uparrow\downarrow}$. For normalization see text.

an infinite ladder q is continuous between $-\pi$ and π, the phase boundary is the envelope of a continuous set of curves. Also shown is the value of q_m, which dominates the superconducting structure at the transition. The whole physical response of the system is periodic in φ/φ_0.

The order parameter and the vortex structure can be similarly obtained. With use of normalized eigenvectors the general form of ψ is

$$\psi_n = \begin{pmatrix} \psi_n{}^{\uparrow} \\ \psi_n{}^{\downarrow} \end{pmatrix} = \frac{1}{(1+\beta^2)^{1/2}} \left| \begin{pmatrix} 1 \\ \beta \end{pmatrix} e^{iqn} + \begin{pmatrix} \beta \\ 1 \end{pmatrix} e^{-iqn} \right|. \quad (4)$$

This solution gives rise (in general) to a net transport of supercurrent along the ladder, except for l on the phase boundary. The currents on each branch can be evaluated from Eq. (4). Figure 2 gives the current in the upper branch from node 0 to 1 ($J_\uparrow^{01}$) and the current along the first transverse branch ($J^{\uparrow\downarrow}$). In the linearized

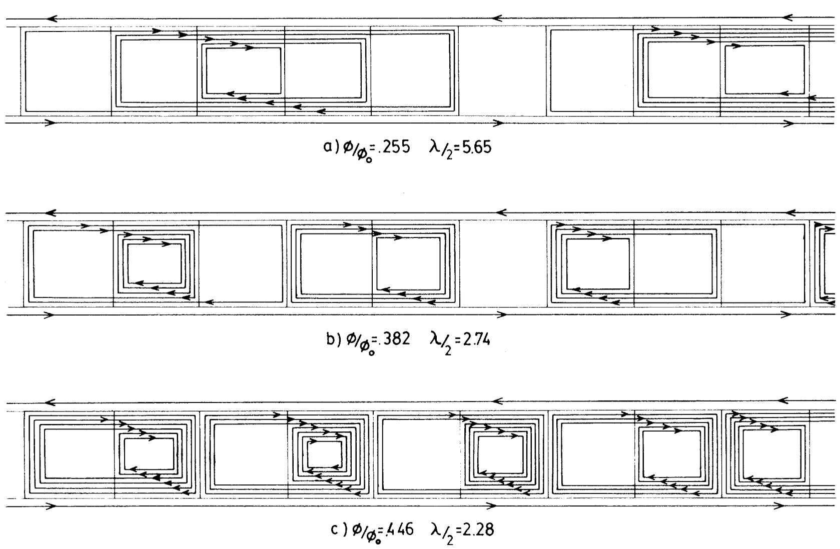

FIG. 3. Vortex structure in the ladder showing the evolution of current distribution as field is increased. The net current in each branch is the sum of all current lines shown. The external flow lines represent the shielding currents. Normalization is arbitrarily defined for each graph and a suitable discretization was applied.

approximation the amplitude of the order parameter is undetermined. Figure 2 was drawn by using an arbitrary normalization; the general features of these curves will be preserved even if the nonlinearities are included.

Figure 3 is a plot of the current distribution for different field values. For $\varphi < \varphi_1$ there are currents only along the longitudinal branches acting as shielding currents. The critical field is the field of "first flux penetration" at which the transverse currents appear. The current structure gives a vortex pattern of wavelength $\lambda/2 = (\pi/q)L$, which for an infinite ladder can be incommensurate with the network. For higher fields, the wavelength decreases and the number of vortices per unit length increases.

Figure 3(c) shows the case where $\varphi/\varphi_0 \lesssim \frac{1}{2}$. The corresponding q, from Fig. 2, is close to $\pi/2$. As φ/φ_0 goes through $\frac{1}{2}$, the order parameter vanishes at the center of every second transverse branch, and the currents go through zero. Fluxoid quantization is preserved because the smallest circuit must include two squares. Beyond $\varphi/\varphi_0 = \frac{1}{2}$ the currents increase again, in reversed sense and this situation prevails until $\varphi/\varphi_0 = 1$.

The case of N parallel wires can be similarly dealt with. Translational invariance leads to a $N \times N$ tridiagonal determinant. Figure 4 gives the phase diagram for $N=4$. It is seen that φ_1 decreases to $0.065\varphi_0$. The solutions for different q's lie between the lines l_m and l_M. For a given field there is a single nucleation mode of fixed q which is shown; for φ/φ_0 between 0.33 and 0.35, q_m stays constant and equal to π. From 0.35 to 0.5, q_m drops to $\pi/2$.

The current flow is given in Fig. 5 for $\varphi/\varphi_0 = 0.1$ and 0.35. For $\varphi > \varphi_1$ currents in the transverse branches appear whose flow lines can be interpreted in terms of vortices penetrating the network. As φ goes from φ_1 to $0.33\varphi_0$, and q_m varies from 0 to π, the vortex size shrinks to one square. Associated with the plateau in q is a shallow minimum in l_m, indicating a relative stability of the vortex structure of Fig. 5(b). For $\varphi/\varphi_0 \geqslant 0.35$, as q drops to $\pi/2$, a more complex vortex structure appears. For $\varphi/\varphi_0 = 0.5$ all currents go through zero and reverse sign.

The phase diagrams for $N=20$, 30, and 40 are given in Fig. 6. It is seen that l_m remains unchanged whereas l_M tends to a well-defined limit. The numerical results show that φ_1 tends toward zero. For $N > 2$, l_m goes through a smooth maximum at $\varphi/\varphi_0 = 0.41$, and through a minimum at $\varphi/\varphi_0 = 0.5$, related to the vanishing of the currents at this point. The eigenvectors show that the order parameter is maximum at the surface and vanishes in a distance $\sim 2\xi$. Thus for the wide strands we are always dealing with surface effects, associated with the highest eigenvalue. The vortex structure arises from the combination of two degenerate solutions for q and $-q$, as in Ref. 5. These results are not related to those for the infinite square lattice.[2] To find a relation

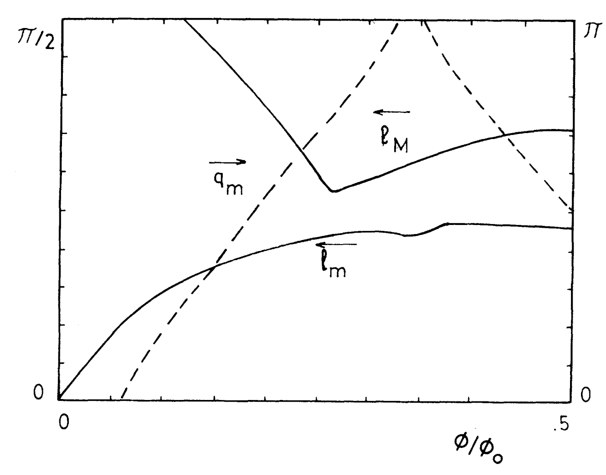

FIG. 4. Phase diagram l_m for $N=4$. Also shown are l_M and q_m. Note the shallow minimum in l_m in coincidence with the plateau in q_m. Only half a period in φ/φ_0 is shown.

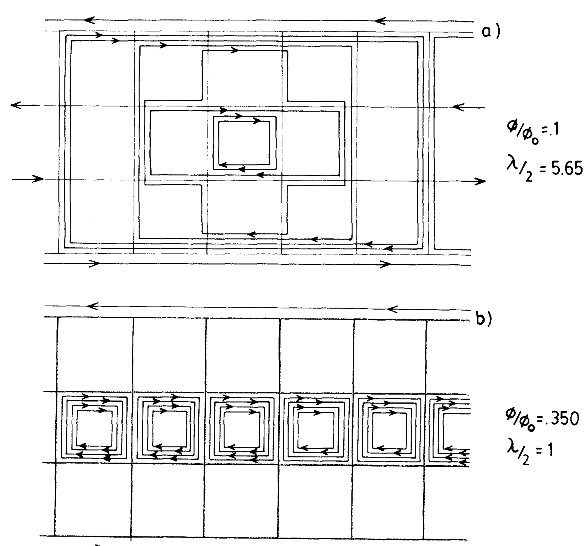

FIG. 5. Vortex structure in the $N=4$ network for two field values.

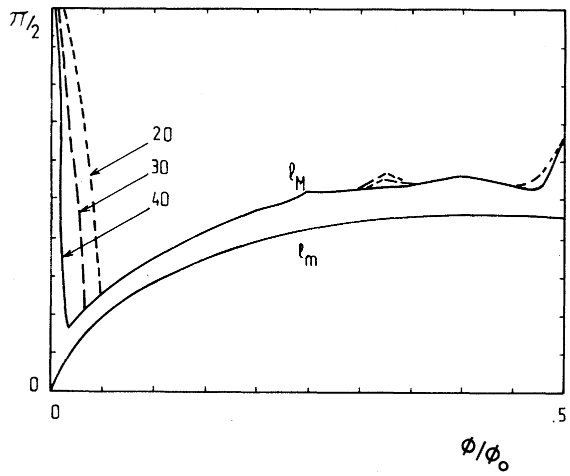

FIG. 6. Phase diagram for $N=20$, short dashed line; $N=30$, long dashed line; $N=40$, continuous line. The lower line is l_m which remains almost unchanged. The upper line, l_M, shows how the limit $N \rightarrow \infty$ is approached.

one should look for lower eigenvalues which are insensitive to boundary conditions, as for H_{c_2}. This might prove difficult since localization in this system is given by a power law.[6] The square lattice will not show uniform condensation for other reasons: Since the field must penetrate the structure, vortices will appear for arbitrarily small fields, as for an infinite slab in a perpendicular field. Experimental work on these systems would help further to understand their properties.

We acknowledge fruitful conversations with F. de la Cruz, R. Maynard, R. Rammal, J. Riess, and B. Giovannini. One of us (J.S.) is a recipient of a fellowship granted by the Comisión Nacional de Energia Atómica.

[1]P. G. de Gennes, C. R. Acad. Sci. Ser. II, 292, 9, 279 (1981).

[2]S. Alexander, to be published.

[3]J. Riess, J. Phys. (Paris), Lett. 43, L277 (1982); H. J. Fink, A. López, and R. Maynard, Phys. Rev. B (to be published); R. Rammal, T. C. Lubensky, and G. Toulouse, to be published. These authors considered finite strands similar to those dealt with here.

[4]B. Giovannini, D. H. Sánchez, and J. L. Berchier, in *Inhomogeneous Superconductors—1979*, edited by T. L. Francavilla, D. V. Gubser, J. R. Leibowitz, and S. A. Wolff, AIP Conference Proceedings No. 58 (American Institute of Physics, New York, 1979); R. F. Voss and R. A. Webb, Phys. Rev. B 25, 3446 (1982).

[5]H. J. Fink, Phys. Rev. 177, 732 (1969); H. A. Schultens, Z. Phys. 232, 430 (1970).

[6]S. Aubry, in *Bifurcation Phenomena in Mathematical Physics and Related Topics*, edited by C. Bardos and C. Bessis (Reidel, New York, 1980), pp. 163–184.