the experimental procedure is very simple, and needs no auxiliary measurements as in previous experiments with single-channel polarizers. We are thus led to the rejection of realistic local theories if we accept the assumption that there is no bias in the detected samples: Experiments support this natural assumption.

Only two loopholes remain open for advocates of realistic theories without action at a distance.[10] The first one, exploiting the low efficiencies of detectors, could be ruled out by a feasible experiment.[11] The second one, exploiting the static character of all previous experiments, could also be ruled out by a "timing experiment" with variable analyzers[12] now in progress.

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Hopping Mechanism Generating $1/f$ Noise in Nonlinear Systems

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It is shown experimentally that a bistable system is driven by a suitable external modulation to a region of random intermittency which displays a low-frequency power-law divergence. This low-frequency divergence is associated with a topological alternation between two strange attractors in phase space, either unsymmetric or fully symmetric depending on whether the two potential valleys are differently or equally located. This picture seems sufficiently general to apply to most cases of low-frequency noise currently reported.

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We show evidence of a mechanism responsible for low-frequency excess noise in a nonlinear system. The mechanism seems to be sufficiently general to be a possible explanantion for many physical cases where excess, or $1/f$, noise appears, even though we do not pretend to claim full universality.

In order to study the routes to chaos in nonlinear nonequilibrium physical systems we have built a suitable electronic oscillator with a cubic nonlinearity imposed by a selected field-effect transistor device and driven by an external modulation. The internal force law corresponds to a two-valley potential. The dynamical equation for the normalized oscillator output $x(t)$ is

$$\frac{d^2x}{d\tau^2} + k\frac{dx}{d\tau} - x + 4x^3 = A\cos\omega\tau, \tag{1}$$

where $\tau = \omega_0 t$, ω_0 is the oscillator frequency, ω is the modulation frequency normalized to ω_0,

TABLE I. Values of the modulation amplitude V_n at the onset of the $f/2^n$ subharmonic, increments ΔV_n, and relative increments $\delta_n = (V_{n+1} - V_n)/(V_{n+2} - V_{n+1})$, for a modulation frequency of $f = 573.3 \pm 0.1$ Hz.

	V_n (mV)	ΔV_n (mV)	δ_n
$f/2$	572.15 ± 0.05	46.6 ± 0.1	4.65 ± 0.05
$f/4$	618.72 ± 0.05	10.0 ± 0.1	4.7 ± 0.3
$f/8$	628.75 ± 0.05	2.1 ± 0.1	
$f/16$	630.85 ± 0.05		

and $k = 0.154$ is the scaled damping rate. The modulation amplitude A is taken as control parameter. In the following it will be given in terms of the rms value V (volts) of the modulation signal. We set the modulation frequency f around 560 Hz against an intrinsic frequency $f_0 = 459.1$ Hz.

As long as the motion is confined within one valley, we have the standard sequence of subharmonic bifurcations[1-6] leading eventually to a strange attractor.

For some ranges of the modulation frequency and amplitude, the system can "hop" between the two valleys. Strong modulation induces a stable hopping with a power spectrum qualitatively not different from that associated with motion in one valley. However, for selected modulation ranges, this hopping is accompanied by a strong low-frequency branch of the continuous power spectrum.[7] This branch can be fitted by a $f^{-\alpha}$ law, with $\alpha \approx 0.6$ or $\alpha \approx 1.0$, depending on two distinct, but repeatable, modes of phase-space motion. In the first set of experiments we introduce a small bias in order to have the left valley, corresponding to negative values, preferred with respect to the right one. For small modulation amplitudes the motion is confined within that valley, so that we revisit situations analogous to those of Ref. 2, but eventually we break the similarity when the system explores the second valley as well. In the second set, we study a purely bistable situation (no bias) so that there is no *a priori* preference for one valley.

For increasing values of A, the sequence of plots reported in Fig. 1 is representative for the biased set of experiments. In each of the three parts [(a), (b), and (c)], we report the following experimental data: (i) the phase plot $x(t)$, $\dot{x}(t)$ recorded on an oscilloscope; (ii) the averaged power spectrum, measured with a Rockland spectrum analyzer.

Figure 1(a) shows the $f/8$ case of a standard subharmonic bifurcation sequence that we are able to follow up to $f/16$.[8] The critical values of the control parameter for which a new subharmonic appears are reported in Table I, and the experimentally meaningful values of $\delta_n = (V_{n+1} - V_n)/(V_{n+2} - V_{n+1})$ are seen to be in good agreement with Feigenbaum's value $\delta = 4.669\ldots$. Furthermore, as shown in the log representation of the spectrum in Fig. 1(a), the line connecting the peaks of $f/4$ and its harmonic $3f/4$ lies 16.2 dB above the line averaging over the four peaks $f/8$,

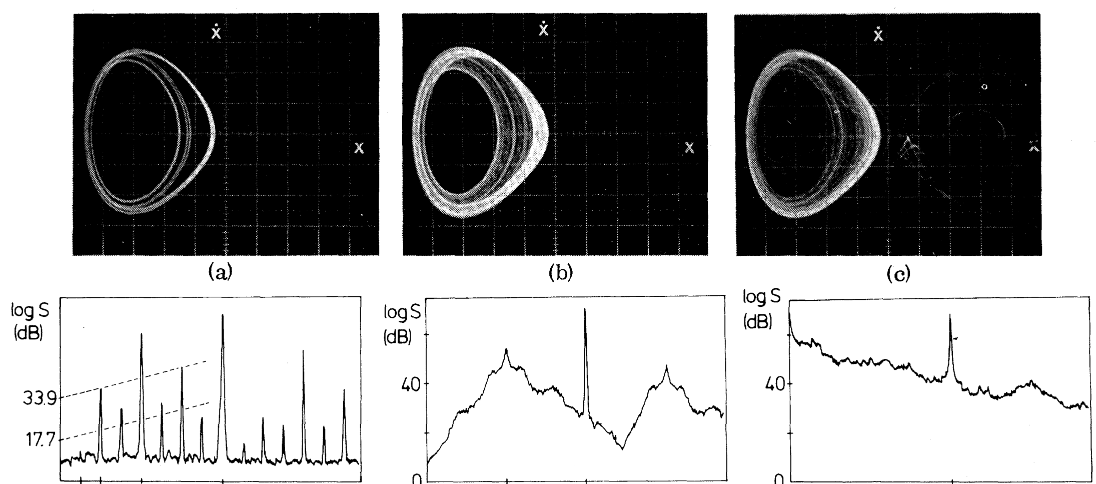

FIG. 1. Three examples of nonlinear motion for a modulation frequency $f = 575.2$ Hz. Row I: phase-space plots [horizontal $x(t)$]. Row II: power spectra in log (vertical)-linear (horizontal) scales. (a) $V = 642.76$ mV, line spectrum; (b) $V = 649.12$ mV, one strange attractor, broad spectrum with low-frequency cutoff; (c) $V = 685.56$ mV, two strange attractors, low-frequency divergence.

$3f/8$, $5f/8$, and $7f/8$, thus verifying that the μ value obeys the relation[1] $10\log_{10}\mu \approx 8.2$ dB. This agrees with previous measurements.[3-5]

As we further increase the critical parameter we get a strange-attractor regime confined around the left minimum of the nonlinear potential [Fig. 1(b)].

Notice that the broadened spectrum has a drastic cutoff at low frequencies. Indeed, the combination of our experimental device and the measuring apparatus is free from low-frequency noise contributions down to 0.5 Hz.

For still higher values of the control parameter, the system is allowed to explore also the right valley, even though less frequently than the left [Fig. 1(c)]. This corresponds to a sudden dramatic increase of the continuous spectrum at low frequencies showing a marked low-frequency divergence already in the linear scale. To explore such a new feature, we present in Fig. 2 a log-log plot of the spectrum corresponding to the low-frequency region, taken for the cases of Figs. 1(b) and 1(c). At the lowest measured frequency (about two decades below the modulation frequency) the divergent component is 80 dB above the previous spectrum.

We point out the following relevant features: (i) We show evidence of a low-frequency excess noise which diverges as $f^{-\alpha}$ with $\alpha \approx 0.6$. (ii) As shown at the top of Fig. 1(c), this low-frequency noise is associated with a random hopping between two types of attractors, one confined in the left valley, the other intruding into the right valley.

We have explored a series of cases where, qualitatively, the phase plot was showing the same ratio of occupation of the left and right sides. We have consistently measured a power-law spectrum $S(f) = f^{-\alpha}$ with α values ranging between 0.53 and 0.66 for pairs of control parameters (modulation amplitude and frequency) ranging, respectively, between 0.6260 and 0.8598 V and 558.8 and 624.0 Hz. Whether in the space of control parameters there is a point where the values merge to a unique universal value is still an open question that would require a much larger body of experiments.

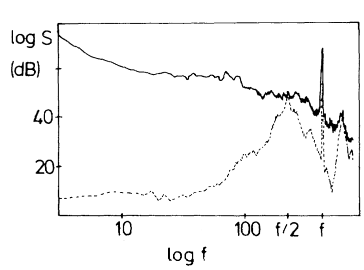

FIG. 2. Comparison of the low-frequency parts of the spectra (b) (dashed line) and (c) (solid line) of Fig. 1, in a log-log representation.

By far more interesting is the behavior in the unbiased, purely bistable, set of experiments. For increasing values of the control parameter (precisely, above 1.8 eV), the system rapidly evolves towards a random hopping between the two types of attractors, one bifurcated on the left, the other on the right. The two alternating phase plots reported in Fig. 3(a) show a complete symmetry with respect to the origin. The power spectrum [Fig. 3(b)] shows subharmonics down to $f/8$, plus a low-frequency divergence accounting for the random hopping between the two basins of attraction. Notice the accurate $1/f$ law in the log-log plot. For convenience we show also a sample of the time behavior of $x(t)$ [Fig. 3(c)].

The two $f^{-\alpha}$ spectra of Figs. 2 and 3 seem different, insofar as the first is accompanied by a broadened high-frequency spectrum, whereas the second does not show other significant chaotic features besides the $f/8$ broadening. In the first case, the two basins of attraction have unequal extension. Indeed [as qualitatively shown by the traces of Fig. 1(c)] the attractor confined in the left valley occurs more frequently than the one hopping over the two valleys and hence crossing $x=0$. Thus this low-frequency divergence is associated with a randomness in zero crossing. In the second case the two attractors are mirror-

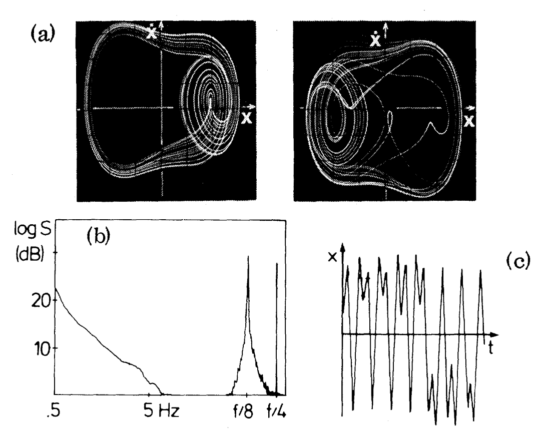

FIG. 3. Hopping between two attractors and associated $1/f$ spectrum in the purely bistable case. Modulation values: $f = 558.4$ Hz, $V = 1.451\,51$ V. (a) Symmetric phase-space plots; (b) log-log spectrum showing the $1/f$ branch, a broadened $f/8$ line, and a narrow $f/4$ line; (c) a sample of the $x(t)$ plot.

like and thus have a stable rate of zero crossing. However, the bifurcated loop moves randomly from left to right, thus inducing the $1/f$ divergence.

To specify this difference we have considered a new type of measurement, namely, the probability distribution $P(t)$ of the times of zero crossing of the amplitude $x(t)$. These are obtained by classifying the time distances between successive zero crossings with positive slope on a multiscalar analyzer. Figure 4 shows that in the first case we have a long tail, while in the second case we have a distribution $P(t)$ confined to a single channel and hence a stable frequency of zero crossings. This appears also from the oscilloscope plot of Fig. 3(c).

It is suggestive to think that the old problem of $1/f$ noise[9-11] may have a heuristic explanation as due to long-time jumps between different basins of attraction within each of which the motion has correlation functions with relatively short lifetimes. It is clear that an added external noise may bridge the two basins, drastically changing the $1/f$ features. This fact will be dealt with elsewhere.

A relation between random intermittency and $1/f$ spectrum has already been observed by a computer model,[12] without, however, connecting such a feature with the detailed dynamical motion as we have clearly displayed in our phase-space plots.

If we extrapolate our picture to other situations with a multiplicity of stable valleys and a driving external force, we can think of, e.g., the flicker noise of electrons in a resistor driven by a current as due to random jumps from one surface trap to another.[10]

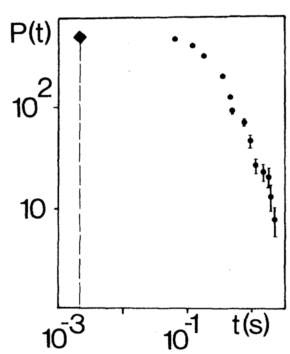

FIG. 4. Statistical distributions of time intervals between zero crossings of $x(t)$. Circles: broad distribution associated with unsymmetric attractors. Lozenge: single-channel line associated with symmetric attractors.

In quantum optics, phenomena like optical bistability[13] should easily display such features by a suitable modulation of the input laser field. Another example can be suggested in selective photochemistry of large molecules by infrared lasers. Here the multiphoton excitation of a vibrational mode is hampered by the intramolecular coupling of that mode with the thermal bath of all other modes.[14] The associated relaxation occurs in the picosecond region. If we consider two-valley potential systems as in isomerization processes,[15] we anticipate that a suitable choice of the frequency and amplitude of the exciting laser should yield a long-time $1/f$ tail.

These are just qualitative hints, but we feel allowed to take such a bold attitude by the fact that, for the first time in the history of low-frequency noise, we are not limited to giving a model without experimental counterpart, nor to an empirical measurement without an understanding of the underlying phenomena, as has been done thus far.[9]

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Neutrino Flux and Total Charged-Current Cross Sections in High-Energy Neutrino-Deuterium Interactions

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From 12 000 charged-current $\nu_\mu D$ events obtained in an exposure of the Fermilab 15-ft bubble chamber to a high-energy wide-band neutrino beam, the absolute neutrino flux is determined using the reaction $\nu_\mu n \to \mu^- p$. For the total charged-current cross section, $\sigma_t = kE_\nu$, $k = (0.68 \pm 0.04 \pm 0.10) \times 10^{-38}$ cm^2/GeV is obtained for E_ν between 10 and 200 GeV. No clear energy dependence of the slope parameter k is observed.

PACS numbers: 13.15.Em

In a naive quark-parton model, the total cross sections for neutrino and antineutrino interactions are expected to show a linear rise with the beam energy,[1]

$$\sigma_t = kE_\nu. \tag{1}$$

Deviations from the linear rise indicate a scaling violation and may suggest new effects such as neutrino oscillations. Previous measurements of σ_t suggested such deviations[2,3]; however, accurate determination of the slope parameter k is difficult because of uncertainties in the neutrino flux measurements. In this paper, we present the first measurement of the neutrino flux in the Fermilab wide-band beam using the neutrino quasielastic reaction,

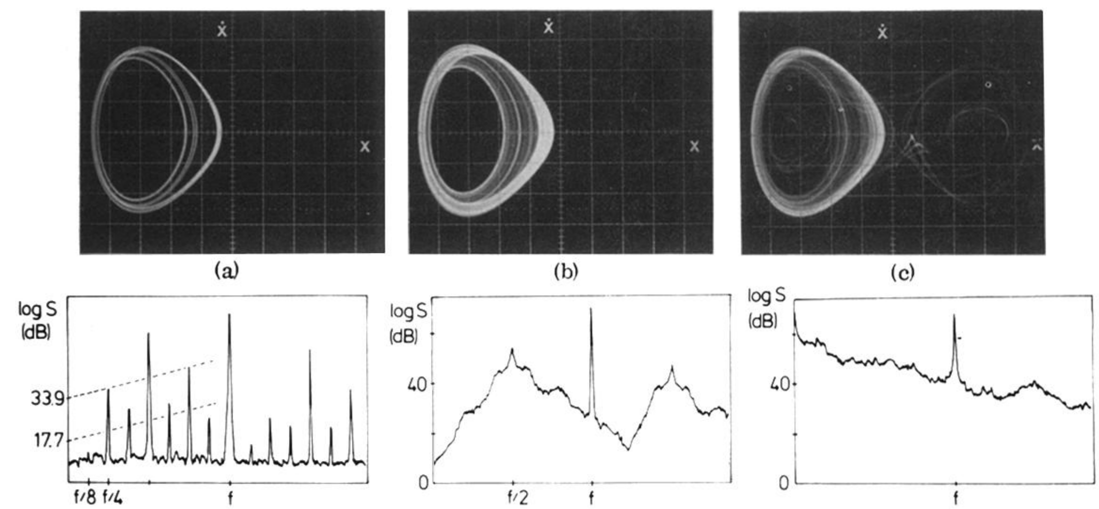
$$\nu_\mu + n \to \mu^- + p. \tag{2}$$

The flux is then used to obtain the total cross section for the charged-current reaction on an isoscalar target,

$$\nu_\mu + D_2 \to \mu^- + X. \tag{3}$$

Other results concerning reaction (3) obtained in this experiment have been reported elsewhere.[4-6]

The data sample comes from a 328 000-frame exposure of the Fermilab 15-ft deuterium-filled bubble chamber, with a two-plane external muon



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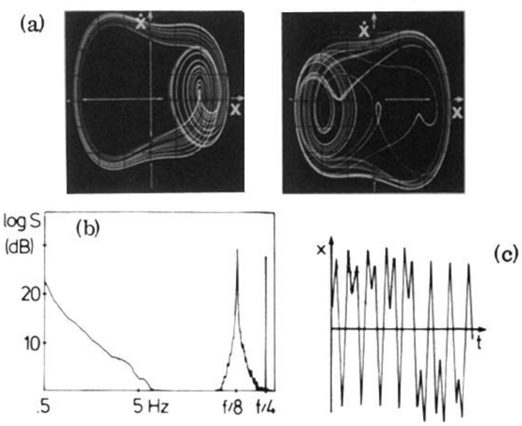

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