Density-of-States Anomalies in a Disordered Conductor: A Tunneling Study

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Results of single-particle tunneling experiments performed on films of indium oxide are presented. The zero-bias anomalies change character with decreasing thickness from a three-dimensional behavior to a logarithmic energy dependence. Theoretical consideration suggest a crossover from a $\ln V$ to a $V^{1/2}$ correction to the density of states at a thickness-dependent energy. In the $V^{1/2}$ range, where the sample exhibits a *three-dimensional* density-of-states anomaly, its transport properties may still be *two dimensional*. These expectations are experimentally confirmed.

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Effects of electron-electron Coulomb interactions can be quite prominent in disordered conductors. The breakdown of momentum conservation[1] makes the scattering rate considerably larger than the value expected by the standard Fermi-liquid theory.[2] Such effects should also contribute to the real part of the self-energy,[3-6] $\Sigma(E)$, and lead to anomalies in the density of states (DOS) $n(E)$ near the Fermi energy, E_F. These anomalies are believed[6] to be a percursor of the insulating phase "Coulomb gap"[7] that would be obtained when the disorder is increased until the states of E_F become localized.[8-10] In the conducting phase, square-root and logarithmic anomalies in $n(E)$ near E_F have been predicted for three-dimensional[3] (3D) and two-dimensional[4] (2D) systems, respectively. Such effects (including those associated with the transition to the insulating phase) were experimentally observed in a tunneling experiment on 3D granular metals by Abeles *et al.*[11] and recently by others.[12,13]

In this Letter we report on DOS anomalies observed in tunneling experiments made on indium-oxide (InO_x) films. Our main results include a demonstrated correlation between the system dimensionality and the character of the DOS anomalies as well as a first observation of a new, energy related dimensional crossover.

The InO_x films were prepared by the previously described[14] method. To form the tunneling junctions the following procedure was used: After the InO_x film was deposited and allowed to cool down (in vacuum) to room temperature, a coating of 25–30 Å of SiO was applied on top of it. The edges of the film were then masked by 600–800 Å of SiO leaving out a slit (~1 mm wide) along the sample. Finally, 3500–4000-Å cross strips (~0.1 mm wide) of pure lead were deposited to act as the counterelectrode for a four-probe measurement.

Junctions prepared in this way had resistances in the range 8–45 kΩ, were stable up to voltages of ±1 V, and survived temperature recycling. Conductance versus voltage data were obtained from the (digitized) I-V plots in the presence of ~1 kOe magnetic field to suppress the superconductivity in the Pb electrode (see inset of Fig. 1). Higher magnetic fields (up to 4 kOe) had no observable effect on the I-V characteristics. (This was also checked by a standard ac derivative technique.) Except for voltages of the order of k_BT, the I-V curves were temperature independent and the low-bias anomaly was clearly observed even at 77 K. For each sample, at least two different junctions were studied and (upon normalization) their conductances versus voltage were within 10% of each other in the voltage range reported. This range (±20 meV) was chosen because up to these energies we can make sure that $\sigma_T(V)$ is mainly due to tunneling by observing the Pb phonon-induced structure. Resistivity and Hall-constant measurements were made in the same way previously described[14] except that fields in the range 1 to 2 kOe were employed for the Hall-voltage determination and only the 1–4.2 K temperature range was studied. These measurements were performed in each case along the *same* strip used for the tunneling measurement.

Experimental results for films of various thick-

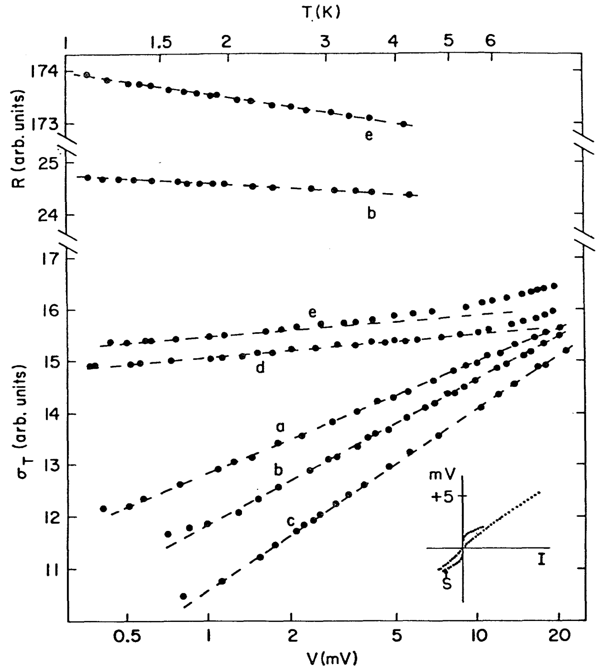

FIG. 1. Tunneling conductance vs voltage for five 2D samples (measured at 1.2 K) and typical resistance vs temperature for two of these samples (see Table I for sample identification). Inset depicts the low-bias I-V characteristics of sample b with the Pb electrode superconducting (curve s) and in the presence of 1 kOe magnetic field.

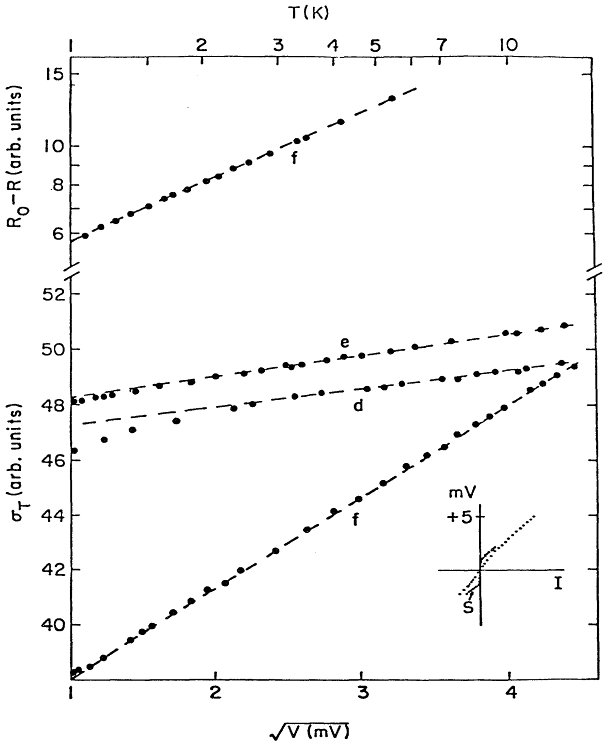

FIG. 2. Tunneling conductance vs voltage of various samples in the 3D range (measured at 1.2 K) and a log-log plot of the resistance change vs voltage for a bulk sample (see Table I for sample identification). Inset depicts the low-bias I-V characteristics of sample f (compare with Fig. 1).

nesses are given in Figs. 1 and 2 [for the resistance versus temperature, $R(T)$, and the tunneling conductance versus voltage, $\sigma_T(V)$]. The relevant parameters of these films, thickness, d, sheet resistance, $R_\square$ (measured at 4.2 K), and resistivity, ρ, are listed in Table I.

First, we note that the five thin samples ($d \leq 460$ Å, two of which are shown in Fig. 1) show a $\ln T$ dependence of ρ, while the thick "reference" sample Fig. 2, f, exhibits a power-law temperature dependence [i.e., $R(T) \simeq R_0 - AT^{1/2}$ with A and R_0 constants]. This behavior is consistent with the conjecture that samples a through e are effectively 2D at this range of temperature and that the 2600-Å film is[3] 3D. This observation is further supported by the negative magnetoresistance (MR) results (Refs. 14, 15, and to be published); the functional dependence of the MR is characteristically 2D and 3D, respectively (this distinction manifests itself clearly at high enough fields where[16-18] the MR goes like $\ln H$ for 2D and as $H^{1/2}$ for 3D).

We turn now to the tunneling results. For the bulk sample (f) we clearly observe a $V^{1/2}$ anomaly in $\sigma_T(V)$ in agreement with results on other 3D systems[11-13] (Fig. 2). Since the present system is a single-phase, crystalline material[19] (as opposed to previous works) the above result adds considerable credence to the universality of the mechanism underlying these anomalies.

The tunneling data for the five thin films are

TABLE I. Relevant parameters for the studied films. The parameter αP was obtained from the logarithmic slopes of $R(T)$. The constants R_0 and A have the same units as in Fig. 2. The density of free electrons in these samples is $(0.95 \pm 0.1) \times 10^{20}/\mathrm{cm}^3$.

Sample	d(Å)	$R_\square$ (Ω)	$10^4\rho$ (Ω cm)	αP	R_0	A
a	160	570	9.1	1.53	⋯	⋯
b	190	560	10.6	1.74	⋯	⋯
c	210	665	14.0	1.70	⋯	⋯
d	310	400	12.4	1.50	⋯	⋯
e	460	200	9.2	1.67	⋯	⋯
f	2600	135	35.5	⋯	1316	5.65

substantially different; for each sample $\sigma_T(V)$ shows a low-bias logarithmic singularity (Fig. 1). An even more interesting feature reflected in these data is a crossover from a $\ln V$ to a $V^{1/2}$ DOS correction at higher voltages. This crossover can be seen in Figs. 1 and 2 for examples e and d. As we shall see, this feature is related to the existence of an energy-dependent characteristic length in the problem.

We shall analyze our results in the context of the simplest theoretical picture[3-6] for the modification of the DOS, $n(E)$, due to the exchange part of the self-energy with statically screened Coulomb interactions. (More elaborate calculations including, e.g., dynamic screening, introduce changes of order unity in the numerical coefficients.[5]) $n(E)$ is given in terms of the free-electron density of states, $n_0(E)$, by $n(E)=n_0(E)/[1+d(\mathrm{Re}\Sigma)/dE]$, while the exchange contribution is given by[3-6]

$$\frac{d\Sigma_{\mathrm{ex}}}{dE}=\frac{1}{(2\pi)^3}\int d^3q\,\frac{Dq^2}{(Dq^2)^2+(E/\kappa)^2}\,\frac{1}{\pi n(0)}\,, \qquad (1)$$

where E is measured from the Fermi energy, D is the electronic diffusion coefficient, and the $q\to 0$ limit of the (screened) Coulomb interaction was taken [since the dominant contributions to (1) come from values of q which are much less than the inverse screening length]. In 3D, as long as the correction to the DOS is small, it is given by $\delta n(E)/n(0)=(\sqrt{2}/4\pi^2)D^{-3/2}E^{1/2}/n(0)$, which is smaller by a factor of 2 than the result of Ref. 3. In the 2D case of a film whose thickness, d, in the z direction is so small that only one state can be taken in the sum over q_z, one finds

$$n(E)=n_0(E)\left[1+\frac{3}{8(k_{\mathrm{F}}d)(E_{\mathrm{F}}\tau_{\mathrm{el}}/\kappa)}\ln\left(\frac{E}{E_0}\right)\right]^{-1}, \qquad (2)$$

where E_0 is a suitable cutoff energy, e.g., $\kappa/\tau_{\mathrm{el}}$. This again agrees to within a numerical factor of order unity with the result of Ref. 4. To connect the coefficient in (2) with readily observable quantities we note that the resistance per square of the film is given by $R_\square=\rho/d=\frac{3}{2}\pi^2(\hbar^2/e^2E_{\mathrm{F}}\tau_{\mathrm{el}})\times(1/k_{\mathrm{F}}d)$. At a given distance, E, from the Fermi energy the range of q which contributes dominantly to Eq. (1) is given by $Dq^2\sim E/\hbar$. Clearly, the condition for the film to behave two or three dimensionally is that the next q_z value, namely $2\pi/d$, be much larger or much smaller, respectively, than $(E/D\hbar)^{1/2}$. An equivalent way to state this condition is in terms of the ratio between the length $l_E=(\hbar D/E)^{1/2}$ and the film thickness d. A length similar to l_E arises as a function of frequency ω, when the frequency dependent conductivity is considered. For our InO_x samples, the order of magnitude of D is ~ 10 cm^2/sec, which yields a crossover energy on the order of ~ 10 meV for a film thickness of 400 Å, i.e., in the easily measurable range.

As concerns the coefficients of the power-law and logarithmic anomalies in the DOS, we find that the coefficients of $\sqrt{E}$ for the 3D samples agree to within a factor of 2 with the theoretical prediction. The coefficients of $\ln E$ for the 2D samples (Fig. 1), while showing some scatter, appear to correlate better with R than with ρ. The average value is larger by a factor of about 5 than the one given by Eq. (2). It is not clear whether these discrepancies are due to problems commonly associated with tunneling experiments[13] or to the naive treatment of screening in this simple theory.[4,6,20]

The tunneling data for samples d and e (which behave strictly two dimensionally as far as temperature- and magnetic-field–dependent transport properties are concerned) display a dimensional crossover which is consistent with the existence of an energy-dependent length alluded to above. In fact, sample d behaves *two dimensionally* for $E \leqslant E_c \sim 8$ meV (Fig. 1) and *three dimensionally* for $E \geqslant E_c$ (Fig. 2). Sample e, which is thicker, exhibits the same crossover around $E_c\sim 3$ meV. These crossover energies are in agreement with the rough theoretical estimate $E_c\sim\hbar D(2\pi/d)^2$ (in which the numerical factor may vary due to boundary conditions). A more meaningful check of consistency is obtained by comparing the ratio of D/d^2 of the samples with the ratio of their respective E_c's. These are found to be ~1.8 and ~2.5, respectively, which should be judged as a reasonable agreement.

This crossover is a further corroboration of the basic ideas of the theory; it may be qualitatively interpreted as being due to the inability of the quasiparticle diffusing during a time $\hbar/E$ to experience the restricted dimension of the sample. We have observed similar crossovers in $\rho(T)$ in Ref. 15.

To summarize, our results support the universal aspects to the density-of-states anomalies in disordered conductors in 2D and 3D. Two types of dimensional crossover were demonstrated: (1) the theoretically expected change in the effective dimension as a function of sample thickness both in the transport properties[15] and in the density of states; (2) a novel dimensional crossover for a given sample as a function of energy. The observed DOS anomalies are thus electron-electron interaction effects. It is noteworthy that these anomalies are not directly reflected in the transport properties (note, e.g., the absence of temperature dependence in the Hall constant[14]). Our results demonstrate that disorder and Coulomb interactions may coexist while manifesting themselves in different physical properties.

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