Comment on "Proton Single-Particle States above $Z = 64$"

Recently Nagai *et al.*[1] reported the lowest four positive-parity states in ${}^{147}_{65}\mathrm{Tb}$. The authors obtained the proton single-particle (s.p.) energies as a function of A through the standard BCS calculations (with blocking and fixed pairing strength of 0.2 MeV) by varying the s.p. energies for each nucleus so as to reproduce (within 20 keV) the observed energies of low-lying states which are assumed to be pure quasiparticle (qp) states, of $N = 82$ odd-A ($A \simeq 137-147$) nuclei. Here I emphasize that there exists a well-defined method for extracting s.p. energies and the related quantities, using the observed odd-even mass difference (P) and the experimental excitation energies of the low-lying states of single-closed-shell odd-A nuclei. This procedure, known as the inverse gap equation (IGE)[2] (also the improved IGE) method, assumes that the low-lying states of single-closed-shell odd-A nuclei are described in terms of single- (a mixture of one and three) qp excitations. This, together with P, then yields G, the strength of the effective interaction, the occupation probabilities, s.p. energies, etc., without any adjustable parameter. I include here the results of the IGE calculation for $N = 82$, odd-A ($A = 137-147$) nuclei.

Some of the IGE (the correction due to the three-qp admixtures, $\Delta E_a = 0$) results with s-δ interaction are shown in Table I. The resulting proton energies and the experimental energies $E_a{}^*$ (taken from Wildenthal *et al.*[4]) used in the calculation are shown in Fig. 1. The results reveal the same features as those of Ref. 1 though the individual numbers differ even by 0.2 MeV. To examine the effect of blocking, the IGE number equation is solved with an additional term corresponding to the lowest qp blocked state. The maximum fractional difference up to 10% arises for the $d_{5/2}$ state, while for other states it is 5% or less. To exemplify the sensitivity of the calculated results upon ΔE_a I repeated the calculations with $\Delta E_{1/2^+}$ taken as 4%–8% of the observed $E_{1/2^+}{}^*$ for the ${}^{141}_{59}\mathrm{Pr}$ nucleus. The particle energy $E_{1/2^+}$ changes by $\simeq 2\%$ and the remaining states are practically unaltered.

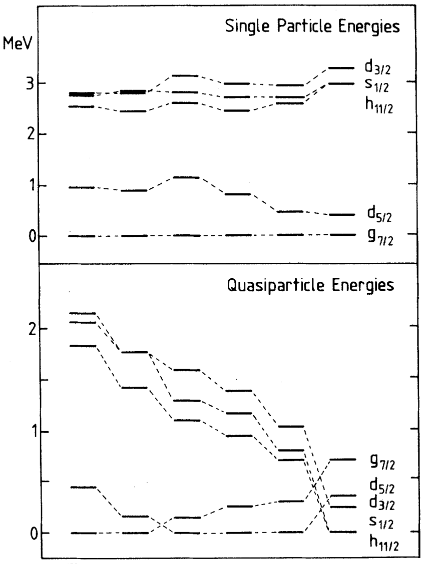

FIG. 1. The calculated proton s.p. energies and the qp energies (taken from Ref. 4) used in the **calculation,** for $N = 82$, odd-A ($A = 137-147$) isotones.

TABLE I. The calculated G and the fractional uncertainty Δn in the particle number n. P is obtained from the observed binding energies (Ref. 3) by use of Eq. (11) of Ref. 2.

n	P (MeV)	G (MeV)	Δn
5	1.578	0.186	0.012
7	1.892	0.189	0.003
9	1.994	0.189	0.002
11	1.996	0.187	0.036
13	2.008	0.182	0.001
15	2.670	0.186	0.001

In view of the uncertainties arising due to the number nonconservation in the qp theory, I do not emphasize the finer corrections due to the blocking and three-qp admixtures. However, the present results, which agree with that of Nagai *et al.*,[1] are fair representation of the variation of the proton s.p. energies with mass number A in this region. I obtain a gap of 2.56 MeV in the s.p. spectrum at $Z = 64$, supporting the semimagic nature of the ${}^{146}_{64}\mathrm{Gd}$ nucleus.

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Received 4 January 1982
PACS numbers: 21.10.Pc, 25.70.-z, 27.60.+j

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