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Length-Dependent Resistance of Thin Wires

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The electrical properties of wires with cross-sectional areas of order 1×10^{-11} cm^2 and lengths as small as 0.2 μm have been studied. At temperatures below about 10 K, the resistance of the wires increases with decreasing temperature, as found in previous studies of much longer wires. For wires shorter than about 5 μm, the resistance rise decreases as the length of the wire is decreased. From these results a characteristic length scale of approximately 0.2 μm at 1.5 K is found, in good agreement with the current theory.

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Several years ago Thouless[1] predicted that localization would cause all long wires to exhibit a thermally activated conductance at low temperatures and to be insulators at absolute zero. This prediction has stimulated a great deal of experimental[2-4] and theoretical[5,6] work. While the experiments are in qualitative agreement with the theory, there are quantitative discrepancies which have yet to be fully resolved.[2,4,5] It has also been shown[6] that electron-electron (e-e) interactions in the presence of disorder can lead to effects very similar to those caused by localization, and it is not yet clear whether localization, or e-e interactions, or some combination, are responsible for the behavior observed experimentally. Both theories predict that a wire will exhibit an extra resistance, ΔR, given by[7]

$$\Delta R/R_0 = L_i/L_0, \tag{1}$$

where R_0 is the impurity (i.e., temperature-independent) resistance, L_0 is the length of the wire which has an impurity resistance[8] of 36 500 Ω, and L_i is a characteristic length. According to localization theory, L_i is the distance which an electron diffuses between inelastic scattering events,[1] while for the e-e interaction mechanism L_i is a "cutoff" length given by $(\hbar D/k_BT)^{1/2}$, where D is the electronic diffusion constant, and T is the temperature.[6] Previous experimental measurements[2-4] of ΔR imply through (1) that $L_i \simeq 0.15$ μm at 1.5 K. The problem which we wish to address in this paper is what happens when the length of the wire, L_w, is comparable to L_i. One might expect that in this case

$$\Delta R/R_0 = L_{\rm eff}/L_0, \tag{2}$$

where $L_{\rm eff}$ is an appropriate combination of L_i and L_w, with $L_{\rm eff}\to L_i$ for L_w large (the limit appropriate for all previous experiments[2-4]) and $L_{\rm eff}\to L_w$ for L_w much smaller than L_i. We find experimentally that this type of behavior does indeed occur, and we have been able to determine the functional form of $L_{\rm eff}$.

The method of sample fabrication is shown schematically in Fig. 1. We began with wires of $Au_{40}Pd_{60}$ with diameters in the range 300–700 Å, which were made using techniques described previously.[2,9-11] Glass fibers[12] with diameters of order 1 μm were then laid across each wire as shown in Fig. 1(a). Next a Ag film was deposited at normal incidence [Fig. 1(b)]. The fiber was then carefully lifted off leaving a short length of

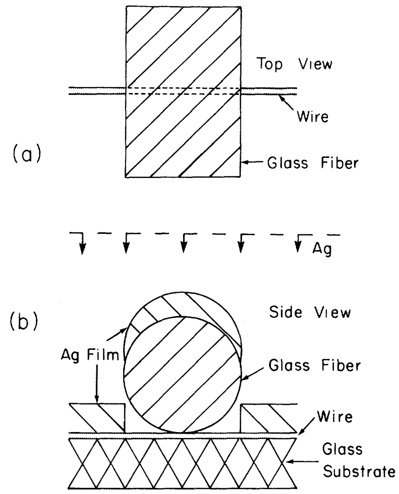

FIG. 1. Description of the method used to make the short wires studied in this work.

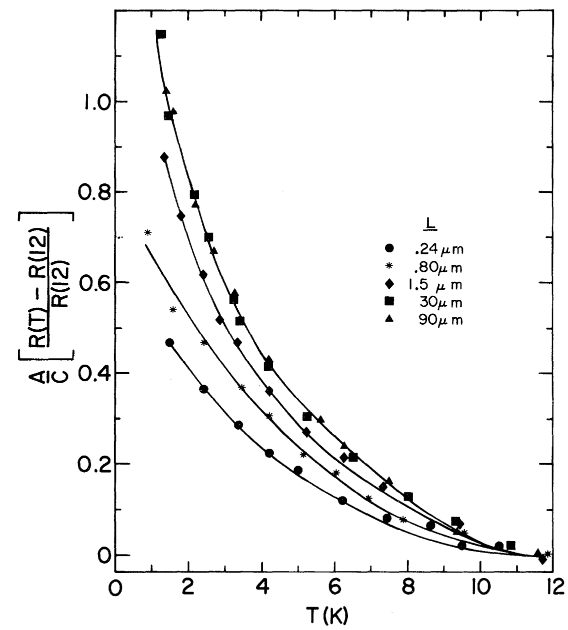

FIG. 2. Normalized resistance rise $(A/C)[R(T) - R(12\text{ K})]/R(12\text{ K})$ as a function of temperature for wires of various lengths. The wires had diameters of 590, 500, 430, 370, and 600 Å, where we have listed the value for the longest wire first, etc.

wire connecting the two regions covered with Ag. By varying the size of the glass fiber it was possible to make wires with lengths as small as 0.2 μm with this method. Examination with a scanning electron microscope (SEM) confirmed that the wires did appear as illustrated in Fig. 1, and provided a direct measurement of the length of each sample. The length-to-width ratio for the samples varied from 4 to 50, and our results indicate that this is sufficiently large that to within the experimental error the behavior is independent of this ratio.[11]

The resistance as a function of temperature was measured using standard ac bridge techniques.[2,11] Some typical results are shown in Fig. 2, where we plot the normalized resistance rise $(A/C)\Delta R/R_0$ as a function of temperature for several samples. Here A is the cross-sectional area of each wire, and C $(=1.2\times10^{-13}\text{ cm}^2)$ is a constant factor. We have normalized the results in this way in order to remove the known dependence of $\Delta R/R_0$ on area,[2,3] and the factor C simply makes the numerical values more convenient for the discussion which follows. Since our choice of normalization removes the dependence of $\Delta R/R_0$ on area, we would expect the results for *all* samples to fall on a common curve. This was, in fact, the case for wires longer than about 5 μm. Moreover, the results for these "long" wires agree well with those of previous workers.[2-4] However, as the length of the wire, L_w, is reduced below about 5 μm the normalized resistance rise falls systematically below the result for long wires. This suggests that as L_w is made smaller, L_{eff} in (2) is also reduced, in good qualitative agreement with the expected behavior discussed above.

In Fig. 3 we show the same results for the normalized resistance rise, but now as a function of $1/\sqrt{T}$. Previous work[2] has demonstrated that the results for long wires follow a $1/\sqrt{T}$ temperature dependence below about 5 K, and this is in good agreement with the results shown in Fig. 3. As the length of the wire is decreased below a few micrometers, the temperature dependence changes qualitatively. The quantity $\Delta R/R_0$ no longer follows a $1/\sqrt{T}$ dependence, but rather appears to be leveling off as $T\to0$. Again, this is in qualitative agreement with the theories described above. Both theories predict that L_i becomes large as $T\to0$, so that in this limit we would expect $L_{\text{eff}}\to L_w$, and hence that $\Delta R/R_0$ should approach a constant value.

In Fig. 4 we show the normalized resistance rise at 1.5 K as a function of the length of the wire. Since the normalized rise falls to half its "full" value when $L_w\simeq0.3$ μm, this is probably a good first approximation for L_i at 1.5 K. It is interesting to note that this value is in reasonable agreement with the value of the inelastic diffusion length measured by Chaudhari *et al.*[13] This lends support to their conclusion that L_i in (1)

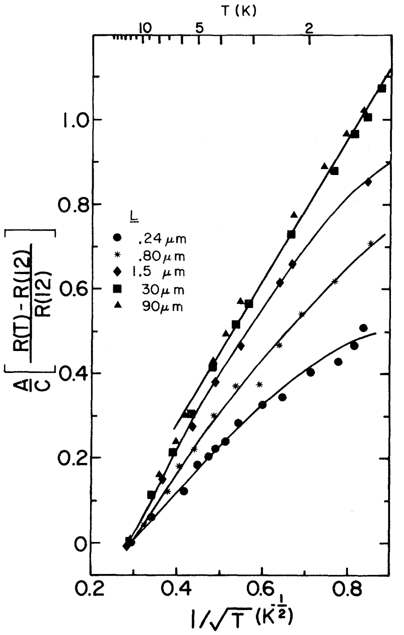

FIG. 3. Normalized resistance rise as a function of $1/\sqrt{T}$. Some of the data points have been omitted for clarity. Note that while the results for the 90-μm wire fall slightly above those for the 30-μm sample, this difference is within the uncertainties in the values of A used in the normalization and is therefore not significant.

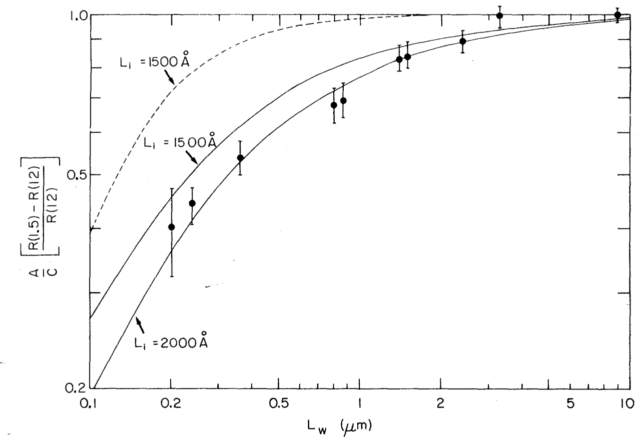

FIG. 4. Normalized resistance rise at 1.5 K as a function of the length of the wire, L_w. The solid and dashed lines are discussed in the text.

is the inelastic diffusion length.

To analyze our results further, it is necessary to consider the manner in which L_i and L_w combine to give $L_{\rm eff}$ in (2). Perhaps the simplest combination is

$$\frac{1}{L_{\rm eff}} = \frac{1}{L_i} + \frac{1}{L_w} . \tag{3}$$

Now, since Fig. 4 is normalized such that the rise $\Delta R/R_0$ approaches unity for long wires, it is basically a plot of the ratio of $(\Delta R/R_0)$ for a wire of length L_w to the value of $(\Delta R/R_0)$ appropriate for a long wire. From the arguments given above, we have $\Delta R/R_0 = L_{\rm eff}/L_0$ for a wire of length L_w, and $\Delta R/R_0 = L_i/L_0$ for a long wire, so this ratio is just $(L_{\rm eff}/L_0)/(L_i/L_0) = L_{\rm eff}/L_i$. Thus Fig. 4 essentially gives $L_{\rm eff}/L_i$ as a function of the length of the wire.[14] It is therefore possible to plot (3) directly in Fig. 4, and this is shown by the solid curves. One curve assumes $L_i = 0.15$ μm at 1.5 K, the value deduced from measurements[2] of ΔR, and this curve is in very reasonable agreement with our results. The other solid curve corresponds to $L_i = 0.20$ μm, and is seen to yield slightly better agreement.

Equation (3) is quite simple and one might ask if it has any theoretical basis. According to localization theory,[1] L_i is the inelastic diffusion length. If τ_i is the inelastic scattering time, and τ_w is the time it takes an electron to diffuse the length of the wire, then it is natural to obtain

$$\frac{1}{\tau_{\rm eff}} = \frac{1}{\tau_i} + \frac{1}{\tau_w} , \tag{4}$$

where $\tau_{\rm eff}$ is the "effective" diffusion time. Since the electronic motion is diffusive, $L_i \sim \sqrt{\tau_i}$, etc., for L_w and $L_{\rm eff}$, so that (4) implies

$$\frac{1}{L_{\rm eff}^2} = \frac{1}{L_i^2} + \frac{1}{L_w^2} . \tag{5}$$

The dashed curve in Fig. 4 is a plot of (5) where we have assumed $L_i = 0.15$ μm. For no value of L_i does (5) agree with the experimental results.[11] Somewhat more sophisticated treatments[11] of the electronic motion yield results similar to (5), and it is not clear at this time how to reconcile these arguments with the results shown in Fig. 4. We do not know of any simple way to estimate $L_{\rm eff}$ for the electron-electron interaction mechanism, and so it is not possible to make a comparison with this theory at present.[15]

In summary, we have found that the resistance of thin wires displays a length dependence which implies a characteristic length of approximately 0.2 μm at 1.5 K, in good qualitative agreement with current theory. While the length dependence is observed to follow a very simple form, a quantitative theoretical explanation of this form is not yet available. When theories of the length dependence are developed, our results may provide a means of determining the relative importance of localization versus electron-electron interaction effects in these systems.

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[14]It is also necessary to allow for the (small) resistance rise which occurs above 12 K. This has been taken into account, and affects the curves in Fig. 4 a small amount (see Ref. 11).

[15]In two dimensions, in the presence of a magnetic field, the electron-electron interaction mechanism predicts that the magnetic field "length" and the cutoff length L_i add according to a digamma function (Ref. 6), which is very similar to (5). The problem of the addition of lengths in two dimensions has also been considered by M. Kaveh, M. J. Uren, R. A. Davies, and M. Pepper [J. Phys. C 14, L413 (1981)], who predict an addition rule identical to (5).