Quark Model of the Deuteron

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A model of the two-nucleon interaction derived from the potential model of the quark structure of baryons is proposed. This model leads to a nonlocal Schrödinger equation for the deuteron which has been solved numerically. The model parameters are consistent with those fitted to baryon spectroscopy and the predicted deuteron properties are in even better agreement with experiment than the authors had dared hope.

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The idea that the nuclear force may be "bonding" of some kind is a very old idea.[1] In 1977, Liberman,[2] and independently Barry,[2] interpreted the bonding in terms of quark exchange arising from the Pauli principle, in a manner analogous to covalent bonding in molecules. This idea has been further developed by several authors,[3-8] most of whom adopt some variant of the resonating group method. Closest in spirit to our approach is the work reported in Ref. 5. We reported a preliminary version of our work,[4] but that early work was marred by subtle errors, several of which are also found in the work of Warke and Shanker.[5] Our present treatment is sketched briefly in Ref. 6, and a detailed account will be submitted for publication shortly.[7]

Our treatment of the nucleon-nucleon force problem differs significantly from others in the following respects: (i) We use the Breit potential for the one-gluon exchange potential, as presented by De Rújula, Georgi, and Glashow,[9] in its full gory detail, including the momentum-dependent terms omitted by almost all other authors. (ii) Since the Breit potential contains relativistic corrections to order $(v/c)^2$, we include corrections to the quark kinetic energies to the same order. (iii) The separation of the relative motion of two clusters is nontrivial when relativistic corrections to quark motions are included, but we treat these complications in a physically consistent way. In addition to the one-gluon exchange Breit potential, we include a linear confining potential. Our detailed theory[7] shows that the effective two-nucleon potential should be rather insensitive to the form chosen for the confining potential, and our detailed deuteron calculations confirm this.

The advantages of using the Breit one-gluon exchange potential are that it incorporates many of the desirable potential forms, such as tensor and spin orbit, with only a single strength parameter α_s, and it has at least tentative justification from QCD. Since the constituent quark model must be regarded as a quasiparticle approximation to a full QCD treatment, one might be surprised to find the Breit potential better than an heuristic guide to possible contributions to the potential. However, Kiefer and Williams[10] have shown that the exact Breit potential together with a linear confining potential and relativistic corrections to the quark kinetic energies can yield respectable agreement with the mass spectrum of the nonstrange baryons, the baryon magnetic moments, and the proton charge radius. The parameters used in the present paper differ from those found by Kiefer and Williams by only a few percent.

We assume for momentum transfer of the order of 100–500 MeV/c that the underlying strong-interaction field theory, quantum chromodynamics, yields up-down quasi quarks in a nucleon having an effective mass of about 330 MeV which interact semirelativistically via a two-body potential V_{ij}, which to a good approximation is

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given by[9]

$$V_{ij}=\vec{\lambda}_i\cdot\vec{\lambda}_j(\alpha_s V_{\mathrm{OGE}}+V_{\mathrm{conf}}). \quad (1)$$

In Eq. (1) the $\vec{\lambda}$ are the vector generators of color SU(3) and α_s is the quark-gluon coupling constant. V_{OGE} is all of the vector exchange potential expanded to order v^2/c^2 as in Ref. 9, and we take

$$V_{\mathrm{conf}}=-k|\vec{r}_i-\vec{r}_j|. \quad (2)$$

We will call V_{ij} the OGEC potential (one-gluon exchange plus confining).

Our goal has been to use a six-quark Hamiltonian to derive a dinucleon Schrödinger equation and the accompanying dinucleon potential. The six-quark Hamiltonian has the symbolic form

$$H=\sum_{i=1}^{6}T_i+\sum_{i=1}^{6}T_i'+\sum_{i<j}^{6}V_{ij}. \quad (3)$$

In Eq. (3), T_i' is the v^2/c^2 correction to the quark kinetic energy T_i as in (ii). Equation (3) is rewritten by a change of variables in terms of internal Jacobi coordinates, the relative two-nucleon coordinate, and the center-of-mass coordinate of the nucleon pair. When this change of variables is made one observes that the term in H corresponding to center-of-mass motion and to the relative three-quark–three-quark (dinucleon) motion involves $3m$, where m is the quark mass. Of course this should involve M, the nucleon mass. The nucleon mass follows from the three-quark Hamiltonian [Eq. (3) truncated at 3] and involves the three-quark dynamics. We want to work in the center-of-mass frame, but before doing so, we add and subtract from Eq. (3) P^2/M, where P is the two-nucleon relative momentum. The subtracted term, $-P^2/M$ is rewritten in terms of expectation values from the three-quark problem. This yields corrections to the internal-coordinate problem which properly ascribe to the relative motion the reduced two-nucleon mass, as in (iii).

The Hamiltonian of Eq. (3) then (in center-of-mass coordinates) takes the form

$$H=T_R+H_A+H_B+V_{AB} \quad (4a)$$

in which

$$H_A=T(\vec{\rho}_A,\vec{\lambda}_A)+\sum_{i<j}^{3}V_{ij}, \quad (4b)$$

$$H_B=T(\vec{\rho}_B,\lambda_B)+\sum_{4\,i<j}^{6}V_{ij},$$

and

$$V_{AB}=\sum_{i<j}^{6}V_{ij}-(V_A+V_B). \quad (4c)$$

The $\vec{\rho}$, $\vec{\lambda}$ are internal coordinates and the V_{AB} involves both internal coordinates and the relative coordinate $\vec{R}$, whose conjugate momentum is $\vec{P}$; T_R, the relative-motion kinetic energy, now involves the physical nucleon mass M. This partitioning of H assigns quarks 1, 2, 3 to nucleon A and 4, 5, 6 to nucleon B. Of course, any other 3-3 partitioning is equivalent, reflecting the S_6 invariance of H. We have partitioned H in the manner indicated in anticipation of the form of the variation wave function which we use.

We start with a trial wave function of the form

$$\psi^{\pi}(\vec{R})\varphi_A\varphi_B|STC\rangle, \quad (5a)$$

which is constructed to be explicitly antisymmetric under

$$[S_3(1,2,3)\times S_3(4,5,6)]\overset{\sigma}{\times}S_2(A,B),$$

where $\overset{\sigma}{\times}$ denotes a semidirect product. In Eq. (5a), φ_A and φ_B are functions describing the internal motions of the pair of nucleons and $|STC\rangle$ is the spin-isospin, color singlet state for the six-quark system. To insure antisymmetry under S_2, $(-1)^{S+T+1}=\pi$. The form of Eq. (5a) is motivated by the desire to have a wave function which clearly describes two interacting nucleons. Additional support comes from noting that the three-quark color singlet bags have lower energy than a single six-quark color singlet bag.[11] To make the trial function completely antisymmetric under S_6 we use an idempotent operator and write

$$\Psi^{STC\pi}=\mathcal{O}\psi^{\pi}(\vec{R})\varphi_A\varphi_B|STC\rangle, \quad (5b)$$

where $\mathcal{O}$ is the antisymmetrizing operator for the six-quark system,[12] so that Ψ^{STC} completely satisfies the Pauli principle.

There is nothing special, nor more importantly physical, about the partitioning $A\supset(1,2,3)$, $B\supset(4,5,6)$. Any 3-3 partitioning of H together with the corresponding partitioning of Ψ^{STC} and the use of the appropriate form of $\mathcal{O}$ would be fully equivalent.

The action is

$$A=\int(\Psi^{STC\pi})^*H\Psi^{\pi STC}+\lambda[1-\int(\Psi^{STC\pi})^*\Psi^{STC\pi}], \quad (6)$$

where the appropriate integration over all six quark coordinates is written in terms of integration over the internal coordinates and the relative coordinate. The Lagrange multiplier λ is written as $E+E_A+E_B$, where E is the two-nucleon relative energy, and $E_A+E_B=\langle H_A+H_B\rangle$, where the internal-coordinate expectation is taken. The internal functions φ_A and φ_B are fixed and $\psi^{\pi}(\vec{R})$

is the varied function. In principle, φ_A and φ_B are the lowest eigenstates of H_A and H_B. In practice, we choose them to be the lowest s-state approximation to that eigenstate. This retains the essential physical features and makes mathematical simplicity.

The variation principle then yields

$$\delta A/\delta\psi^{\pi *}(\vec{R}) = 0 \text{ and } \delta A/\delta\lambda = 0, \quad (7)$$

and these lead to the *nonlocal* Schrödinger equation

$$\int H(\vec{R}, \vec{R}')\psi_j{}^{\pi}(\vec{R}') = E_j \int K(\vec{R}, \vec{R}')\psi_j{}^{\pi}(R') \quad (8a)$$

and normalization

$$\int \psi_i{}^{\pi *}(\vec{R}) K(\vec{R}, \vec{R}')\psi_j{}^{\pi}(\vec{R}') = \delta_{ij}. \quad (8b)$$

The exact forms of $H(\vec{R}, \vec{R}')$ and $K(\vec{R}, \vec{R}')$, which are fairly complicated, will be given in Ref. 7. The important feature of the kernel $K(\vec{R}, \vec{R}')$ which arises out of $\mathcal{O}$ is that K is positive definite, so that one may (if desired) suitably transform H into an equivalent Hamiltonian (also nonlocal) whose eigenfunctions have the usual normalization. Although H is nonlocal, it contains all the phenomenological two-nucleon forms such as spin orbit, tensor, central, etc., and in addition contains the elusive L^2 term.

We have solved Eq. (8a) for the deuteron bound state by expansion of $\psi^{\pi}(\vec{R})$ in terms of a complete set of isotropic harmonic-oscillator functions. The calculation involves four parameters: α_s, m, k, and β, which is the reciprocal of the intrinsic length of the internal functions, φ. The quark mass m is fixed by the nucleon magnetic moments to be 325 MeV. Similarly, β is fixed by the proton charge radius to be 243 MeV. The parameters α_s and k should be taken from hadron spectroscopy. From Kiefer and Williams[10] $\alpha_s \sim 1.8$ and $k \sim 40\,000$ MeV^2, but we allow a small amount of latitude in these parameters. In the end, the deuteron results were quite insensitive to k (as we had expected[7,13]) and our final value of α_s was 1.785. Details of the numerical procedure will be given in Ref. 13 and here we only quote the results in Table I. We have estimated the values to which the results would converge for an infinitely large oscillator basis. The binding energy, in particular, is quite sensitive to small changes in α_s and even more so to small changes in β and m. We would stress that the latter two were fixed by single-nucleon data and were not varied in the deuteron calculation.

Unlike the conclusion presented in previous papers, our conclusion is that the semirelativistic, one-gluon exchange potential yields a two-nucleon potential which predicts low-energy two-nucleon properties in decent accord with experimental data. The biggest discrepancy is in the quadrupole moment which is of course directly traceable to the percentage of D state in the deuteron wave function. The OGEC prediction is about 4% D state, which is somewhat too small. In Ref. 13 we have examined the D-state contribution which arises from the tensor part of the two-nucleon interaction by adding a regularized, one-pion exchange potential form with no new parameters. We find that this increases the D-state probability to about 6% and predicts a quadrupole moment very near to experiment. This does not of course address in any way the fundamental nature of the relative weakness of the tensor part of the OGEC quark interaction *vis à vis* the stronger central term. It is of course possible that additional terms in the quark potential exist which have not yet been seen in hadron spectroscopy calculations and which could increase the tensor strength.

TABLE I. Experimental and OGEC quark predictions of the deuteron properties; binding energy (BE) $= -E(J = 1)$. The uncertainties in the theoretical values arise since these are estimates of the converged values for an infinite oscillator basis. We do not quote the experimental uncertainties.

Quantity	Experimental value	OGEC prediction
BE (MeV)	2.226	$2.1 < \text{BE} < 2.28$
Q ($e^2 \cdot$ fm^2)	0.282	0.13 ± 0.005
μ (μ_N)	0.857	0.90 ± 0.01
r_{rms} (fm)	2.1	1.8 ± 0.2
E ($J = 0$)	> 0	> 0

In conclusion then, regardless of whatever doubts one may have concerning the potential approach to hadron spectroscopy, nonrelativistic quark approximation, etc., the *fact* is that the quark potential of Eq. (1) with Hamiltonian Eq. (4a) and six-quark trial function Eq. (5b) gives rise to a nonlocal dinucleon Schrödinger equation (8a) whose predictions are in very reasonable agreement with experimental data.

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[1]See, for example, R. G. Sachs, *Nuclear Theory* (Addison-Wesley, Cambridge, Mass., 1953).

[2]D. A. Liberman, Phys. Rev. D 16, 1542 (1977); G. W. Barry, Phys. Rev. D 16, 2886 (1977).

[3]M. B. Kislinger, Phys. Lett. 79B, 474 (1978); D. Robson, Nucl. Phys. A308, 381 (1978); C. DeTar, Phys. Rev. D 17, 323 (1978).

[4]P. D. Morley, D. L. Pursey, and S. A. Williams, Bull. Am. Phys. Soc. 24, 574 (1979); D. L. Pursey, P. D. Morley, and S. A. Williams, Bull. Am. Phys. Soc. 24, 574 (1979).

[5]C. S. Warke and R. Shanker, Phys. Rev. C 21, 2643 (1980); M. Oka and K. Yazaki, Phys. Lett. 90B, 41 (1980); H. Toki, Z. Phys. A 294, 173 (1980); J. E. F. T. Ribeiro, Z. Phys. C 5, 27 (1980).

[6]An incomplete overview is given by P. D. Morley, D. L. Pursey, and S. A. Williams, in *Statistical Mechanics of Quarks and Hadrons*, edited by H. Satz (North-Holland, Amsterdam, 1981).

[7]P. D. Morley, D. L. Pursey, and S. A. Williams, to be published.

[8]T. D. Babutsidze, T. I. Kopaleishvili, and I. Z. Machabeli, Yad. Fiz. 33, 1406 (1981) [Sov. J. Nucl. Phys. 33, 754 (1981)]; Fan Wang and Yin He, Chin. Phys. 1, 1016 (1981); Yin He, Yu-Ming Zhang, Zhao-gi Lu, and Fan Wang, Chin. Phys. 2, 189 (1982).

[9]A. De Rújula, Howard Georgi, and S. L. Glashow, Phys. Rev. D 12, 147 (1975).

[10]M. Kiefer and S. A. Williams, private communication (1982).

[11]T. DeGrand, R. L. Jaffe, K. Johnson, and J. Kiskis, Phys. Rev. D 12, 2060 (1975).

[12]S. A. Williams and D. L. Pursey, J. Math. Phys. 17, 1383 (1976).

[13]S. A. Williams, F. J. Margetan, D. L. Pursey, and P. D. Morley, to be published.