Injection of a High-Current Beam into a Modified Betatron Accelerator

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A method is proposed for injecting and trapping a high-current, high-energy, nonneutral electron beam into a modified betatron accelerator. The injection is substantially simplified as a result of the balancing of two effects that are prominent in high-current beams.

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Recently, there has been increasing interest in the development of compact accelerators that are capable of generating ultrahigh-current beams. One of the most promising is the modified betatron accelerator.[1-4] This device consists of a conventional betatron[5] magnetic field configuration as well as a toroidal magnetic field. It has been shown both analytically and numerically that the toroidal field improves the stability[2,6] of the high-current beams. However, the injection and extraction of the electron beam is substantially more involved as a result of the toroidal field.

In this paper, we report on an injection scheme that is conceptually simple and rather easily realizable. Although the proposed scheme has some similarities with previous injection techniques of relativistic beams into toroidal geometries,[7-9] several of its key features are different.

The proposed injection scheme is closely related to two effects that are very important in high-current beams. The first effect is associated with the reduction of the kinetic energy of the injected beam (inductive effect) and the second is associated with the additional force that appears on the geometric center of the beam as a result of the finite radius of curvature of the circulating electron beam (toroidal effect). Either of these effects could drastically change the major radius of the electron ring and thus drive the injected beam to the wall of the vacuum chamber.

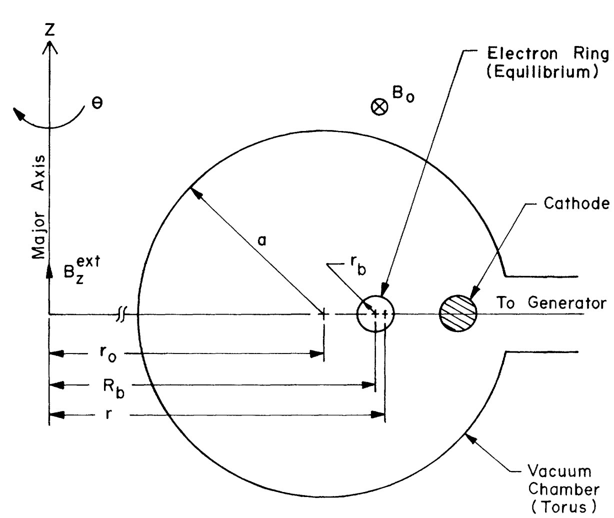

FIG. 1. System of coordinates used in the analysis.

Consider a nonneutral electron beam emitted from a diode that is located inside the torus, as shown in Fig. 1. During injection, the beam kinetic energy is reduced in order to provide the necessary energy to build up the electromagnetic fields inside the torus. The reduction of the beam's kinetic energy may be computed from the conservation of energy

$$N(\gamma_0 - 1)m_0c^2 = N\langle\gamma - 1\rangle m_0c^2 + (2c)^{-1}\int \vec{J}\cdot\vec{A}\,dV + \tfrac{1}{2}\int \rho\Phi\,dV, \quad (1)$$

where N is the total number of electrons in the beam, $(\gamma_0 - 1)m_0c^2$ is the kinetic energy of electrons at the anode, and $\langle\gamma - 1\rangle m_0c^2$ is the average kinetic energy of the electrons after equilibrium has been established. The last two terms in Eq. (1) represent the magnetic and electric field energies, respectively. For highly relativistic, large-aspect-ratio rings, it is shown later on that the two potentials are about equal and thus the field energy terms are also about equal. Thus, for J constant, Eq. (1) gives

$$\gamma_0 - \langle\gamma\rangle \approx -|e|\int A_\theta{}^s dV/Vm_0c^2,$$

where $A_\theta{}^s$ is the self-magnetic vector potential and V is the volume occupied by the beam. If we assume that in the equilibrium state all the electrons have the same canonical angular momentum, this equation becomes for a large-aspect-ratio (R_b/r_b) ring

$$\gamma_0 - \gamma(R_b) \approx (-|e|/mc^2)A_\theta{}^s(R_b), \tag{2}$$

where R_b is the equilibrium radius, r_b is the minor radius of the ring, and $[\gamma(R_b)-1]m_0c^2$ is the kinetic energy of a reference electron that is located at the center of the ring.

The equilibrium position of the beam is determined from the force balance equation

$$-\frac{v_\theta{}^2}{R_b} = -\frac{|e|}{\gamma(R_b)m_0}\left(E_r + \frac{v_\theta}{c}B_z\right), \tag{3}$$

where E_r is the radial electric field and B_z is the total axial magnetic field at R_b. Since $E_r = -\partial\Phi/\partial r$ and $B_z = r^{-1}\partial(rA_\theta)/\partial r$, where Φ is the electrostatic potential and A_θ is the total magnetic vector potential, and since $1/\gamma^2 \ll 1$, i.e., $v_\theta \simeq c$, Eq. (3) becomes

$$\frac{1}{R_b} = \frac{|e|}{\gamma(R_b)m_0c^2}\left(-\frac{\partial\Phi}{\partial r} + \frac{\partial A_\theta{}^s}{\partial r} + \frac{A_\theta{}^s}{r} + B_z{}^{\text{ext}}\right)_{r=R_b}, \tag{4}$$

where $B_z{}^{\text{ext}}$ is the external field at $r = R_b$.

For $R_b/a \gg 1$ and $v_\theta \approx c$, the two potentials Φ and $A_\theta{}^s$ satisfy the same equation and the same boundary conditions. Thus Eq. (4) becomes

$$\frac{1}{R_b} = \frac{|e|}{\gamma(R_b)m_0c^2}\left(\frac{A_\theta{}^s}{R_b} + B_z{}^{\text{ext}}(R_b)\right). \tag{5}$$

Substituting Eq. (2) into Eq. (5) yields

$$R_b = \frac{c\gamma_0}{V^{-1}\int(|e|B_z{}^{\text{ext}}/m_0c)dV} \simeq \frac{m_0c^2\gamma_0}{|e|B_z{}^{\text{ext}}}. \tag{6}$$

Therefore, when $\gamma^2 \gg 1$ and $a/R_b \ll 1$ inductive and toroidal effects balance each other, even when the beam is injected off axis. As a consequence, the injection process is considerably simplified and the local external magnetic field required to confine the ultrahigh-current electron beam in its equilibrium position is given by the simple relation of Eq. (6).

The balancing may be computed explicitly when the electron ring is located along the minor axis of the torus. In this special case the self-magnetic potential is given by

$$A_\theta{}^s(r_0,0,t) = [2I(t)/c][\tfrac{1}{2} + \ln(a/r_b)]. \tag{7}$$

The change in γ on the axis of the ring may be computed from Eqs. (2) and (7) and is

$$\gamma_0 - \gamma(r_0,0,t) = +2\nu[\tfrac{1}{2} + \ln(a/r_b)], \tag{8}$$

where ν is the Budker parameter.

Because of the toroidal effect neither the self-electric nor the self-magnetic field is equal to zero at the axis of the electron ring even when the axis of the ring lies along the minor axis of the torus. For a constant-current-density the fields at the center of the ring are[2,6]

$$E_r = -\pi|e|n_0\frac{r_b{}^2}{r_0}\ln\left(\frac{a}{r_b}\right), \tag{9}$$

and

$$B_z = -\pi|e|n_0\left(\frac{v}{c}\right)\frac{r_b{}^2}{r_0}\left[1+\ln\left(\frac{a}{r_b}\right)\right]. \tag{10}$$

By substituting Eqs. (8), (9), and (10) into the radial force balance equation, it is easy to show that the equilibrium radius of the beam remains constant, although the kinetic energy of the beam is substantially reduced.

The above treatment is based on an asymmetric injection, i.e., when the canonical angular momentum P_θ of the equilibrium state is not the same as that of the diode. The same results are obtained for a symmetric "injection." Although it is not practical, the symmetric "injection" can be easily analyzed and is realized when, for example, the current of an intially very weak ring increases rapidly with time. The main advantage of the symmetric "injection" is that it can be simulated with two-dimensional codes.

The balancing of inductive and toroidal effects both on axis and off axis was verified in several computer simulation runs. Excellent agreement was found between theory and simulation, provided the assumptions of the theoretical model were satisfied. Typical results from the computer simulation when the beam is injected on axis are shown in Fig. 2. The electron ring remained at the center of the torus, although its current increased from 0 to 10 kA. The betatron field $(B_{0z}{}^{\text{ext}})$ used in the simulation was the single-particle magnetic field corresponding to the diode energy.

If the assumptions of the theoretical model are not adequately satisfied, the lowest-order correction to the local, single-particle magnetic field is

$$\delta B_z \approx 0.17\times10^4\left(\frac{\Delta\gamma}{r_0{}^2} + \frac{2\nu}{\gamma_a{}^2a^2}\right)\Delta r\ \text{G},$$

where the displacement Δr, the major radius r_0, and the minor radius a of the torus are measured in centimeters.

As a result of the balancing of inductive and

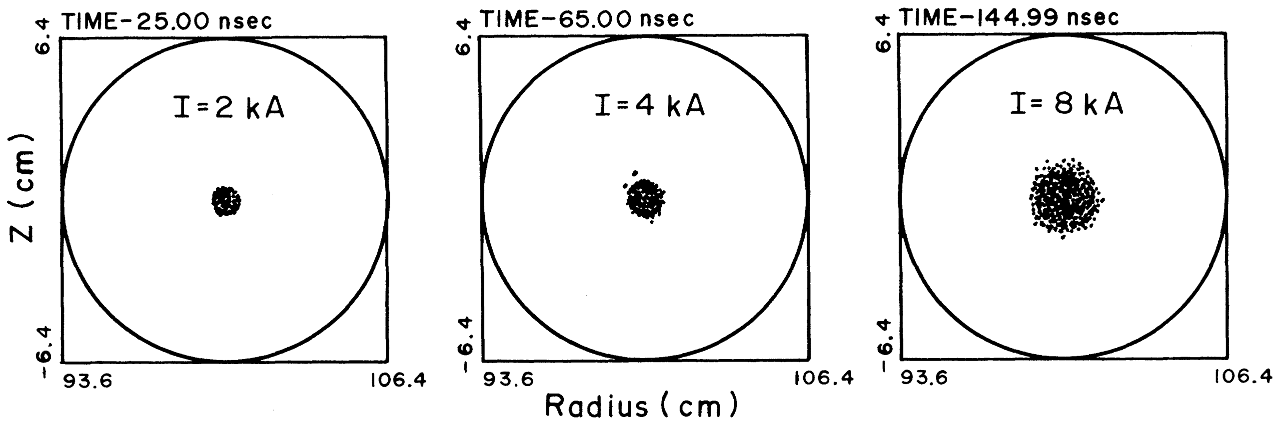

FIG. 2. Snapshots of the minor cross section of the electron ring injected along the minor axis of the torus.

toroidal effects, the center of the beam will remain stationary if the betatron field at the point of injection is equal to the equilibrium field defined in Eq. (6). Thus, after a revolution around the major axis of the torus, the beam will come back to hit the injector. However, if the value of the betatron field is a few precent different from its equilibrium value, the center of the beam will drift away from the injector as the beam propagates along the torus. If the betatron magnetic field remains constant in time or both the flux and the local magnetic field vary in sychronism,

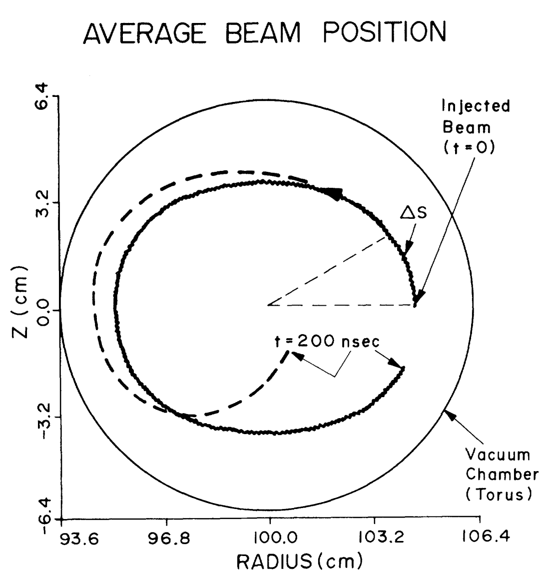

FIG. 3. Computer simulation results showing the projection r-z plane of the center of the 10-kA electron beam as a function of time. The various parameters are listed in Table I. During the bounce period the local betatron field was kept constant (solid line) or reduced by 2 G (dashed line).

the beam will drift and come back to the injector after a bounce period[2] (or approximately after ten revolutions around the major axis). The projection of the center of the rotating beam in the r-z plane as a function of time is shown by the solid line in Fig. 3. The values of the various parameters are listed in Table I. It is apparent that after about 250 nsec (one bounce period) the beam would return and strike the injector. However, when the local magnetic field is reduced slightly during the bounce period the beam drifts away from the injector. This is shown by the dashed line in Fig. 3. In this computer simulation run the value of the betatron field during a bounce period was reduced by 2 G, i.e., from 146 to 144 G. If the fields were held constant after a bounce period, the center of the beam would continue rotating around a fixed equilibrium position. However, during the acceleration both the field and the flux through the orbit increase at the same rate, the equilibrium position moves closer to the center of the minor cross section of the

TABLE I. Parameters for the computer run of Fig. 3.

Beam current	$I = 10$ kA
Beam energy at the diode	$E_0 = 3.84$ MeV ($\gamma_0 = 8.5$)
Beam energy after injection	$E = 3.05$ MeV ($\gamma = 6.97$)
Injection radius	$R_b = 104.5$ cm
Initial beam minor radius	$r_{bi} = 1.0$ cm
Final beam minor radius	$r_{bf} = 1.2$ cm
Major radius	$r_0 = 100$ cm
Torus minor radius	$a = 6.4$ cm
Toroidal magnetic field	$B_{0\theta} = 1415$ G
Equilibrium betatron field	$B_{0z}^{eq}(r_0, 0) = 152$ G
Betatron magnetic field	$B_{0z}(r_0, 0) = 146$ G

torus, and the beam rotates around it with a progressively smaller radius.

This paper reports on the injection and trapping of an ultrahigh-current electron beam into a modified betatron accelerator. For computational convenience the various parameters in the simulation were such that the induced fields dominate the external fields. This parameter regime should be avoided in an actual device because during the acceleration it is possible that the low frequency of rotation (bounce) will change sign and thus the beam will become unstable. However, the main features of the proposed injection are not sensitive to the relative magnitude of the fields but rather to the magnitude of their difference. Both analysis and simulation are based on the cold-beam approximation. Presently, work is in progress with finite-emittance beams. Finally, it has been assumed that a hard vacuum is continuously maintained inside the confining chamber and thus the plasma formation and its effect on the beam[10] were neglected.

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Sound Attenuation Measurements in Superfluid ^{3}He-A Well below T_c: An Anomalous Behavior at Low Pressure

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Sound attenuation has been measured at low and medium pressures down to $T/T_c \sim 0.4$ in the A phase of superfluid ^{3}He stabilized by a magnetic field. Damping is due to Cooper pair breaking and constitutes a useful probe of the superfluid gap. The gap parameter is found to correspond to the weak-coupling value at high pressure but to be anomalously small at low pressure.

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We have measured the attenuation of zero sound in the A phase of superfluid ^{3}He at low pressure down to temperatures less than $0.4T_c$. Since the dominant damping mechanism is Cooper pair breaking, these measurements yield direct information on the microscopic structure of the A phase.

As is now well established,[1] the A phase owes its existence above the polycritical pressure in zero magnetic field to strong-coupling effects and, more specifically, to the spin-fluctuation mechanism put forward by Brinkman and Anderson.[2] Its domain of existence is extended down to absolute zero by the application of magnetic fields of a few kilogauss[3] which suppress the $S_z = 0$ component of the B phase while leaving the gap param-