Comment on "Upper-Hybrid Wave Collapse"

In a recent Letter,[1] Giles claimed to have shown the collapse of upper-hybrid waves. His study was motivated by a recent laboratory experiment.[2] For this purpose, Giles considered the interaction of upper-hybrid waves with ion-cyclotron waves. Such an interaction has been discussed before.[3,4]

The main purpose of this Comment is to clarify the physics which goes into the derivation of the slow plasma motion. It is our belief that Giles did not treat the ion-cyclotron dynamics nor the quasistatic plasma response correctly. Specifically, the ion-cyclotron waves have a nonvanishing parallel (to the external magnetic field $B_0\hat{z}$) phase velocity Ω/K_z which satisfies $v_{ti} < \Omega/K_z < v_{te}$, where $v_{ti,e} = (T_{i,e}/m_{i,e})^{1/2}$, so that the electrons can establish equilibrium by moving freely along $B_0\hat{z}$. Thus, in calculating the slowly varying electron density perturbation n_{es} driven by a low-frequency ponderomotive force of the upper-hybrid waves, one must allow for a small z component of the upper-hybrid wave electric field E_z in order to satisfy momentum conservation.[5] Giles ignored such a variation.

When a small parallel electric field of the upper-hybrid wave is taken into account, the slow electron-density variation is governed by the electron momentum equation

$$0 = -v_{te}^2\partial_z n_{es}/n_0 + (e/m_e)\partial_z\chi_s - \partial_z\Psi_z^e. \qquad (1)$$

Here, the ponderomotive potential Ψ_z^e is given by $(R^2|E_z|^2 + QR|E_x|^2)/4$, where Q and R are defined in Ref. 5. This ponderomotive force is transmitted to the slow ion motion via the ambipolar potential χ_s.[3]

For ion-cyclotron modulation $(\Omega, \vec{K})$, where $\Omega \sim \Omega_i$, we find[3,4] for $\partial_z^2 \ll \partial_x^2$, $|E_z|^2 \ll |E_x|^2$,

$$\left[\partial_t^2 + \Omega_i^2 - c_s^2\left(1 + \frac{\gamma T_i}{T_e}\right)\nabla^2\right]\frac{n_{es}}{n_0} = \frac{\omega_{pe}^2}{\omega_0^2 - \Omega_e^2}\, c_s^2\nabla^2\,\frac{|E_x|^2}{16\pi n_0 T_e}, \qquad (2)$$

where the quasineutrality condition $n_{es} = n_{is}$ has been used. Here, Ω_e and Ω_i are the electron and ion gyrofrequencies, ω_{pe} is the electron plasma frequency, $c_s \equiv (T_e/m_i)^{1/2}$ is the ion sound speed, $\nabla^2 = \partial_x^2 + \partial_z^2$, and ω_0 is the upper-hybrid frequency. Thus, the slow motion is two-dimensional in the (x,z) plane. Note that (2) differs significantly from Eq. (12) of Ref. 1, which was derived by neglecting the $\vec{v}\times\vec{B}_0$ force on the slow transverse electron motion. That this neglect is unjustified can most easily be seen from the *exact* linear relation[6]

$$\frac{n_{es}}{n_0} = \frac{(K_z^2 - K^2\Omega^2/\Omega_e^2)v_{te}^2}{K_z^2 v_{te}^2 - \Omega^2(1 - \Omega^2/\Omega_e^2 + K^2\rho_e^2)}\,\frac{e\chi_s}{T_e}, \qquad (3)$$

easily derivable for a warm electron fluid. Here $K^2 = K_\perp^2 + K_z^2$ and $\rho_e^2 = v_{te}^2/\Omega_e^2$. Mathematically, there are two ways to obtain from (3) the adiabatic response $n_{es}/n_0 = e\chi_s/T_e$. One, used here, is to assume $\Omega^2/\Omega_e^2 \ll K_z^2/K^2 \ll 1$, $\Omega \ll K_z v_{te}$, and $K^2\rho_e^2 \ll 1$. The other, adopted implicitly by Giles, is to use $1 \gg \Omega^2/\Omega_e^2 \gg K_z^2/K^2$ and $K^2\rho_e^2 \gg 1$, which is clearly inconsistent with the fluid theory, thus invalidating this approach.

On the other hand, for quasistatic slow plasma response, namely $\Omega/K_z \ll v_{ti}, v_{te}$, the slow ion-density perturbation is[5] $n_{is}/n_0 = -e\chi_s/T_i$. Combining this with (1) and using the quasineutrality condition, one readily finds

$$n_{es}/n_0 = -\omega_{pe}^2|E_x|^2/(\omega_0^2 - \Omega_e^2)16\pi n_0 T, \qquad (4)$$

where $T = T_e + T_i$. Thus, Giles has incorrectly treated the adiabatic response case.

Following Giles, we have implicitly assumed $\omega_{pe}^2 \gg \Omega_e^2$ in the above discussion. For the experiment of Christiansen, Jain, and Stenflo,[2] however, one has $\omega_{pe}^2 \approx \Omega_e^2$. In that case, the factor a^2 in Ref. 1 should be replaced by $v_{te}^2\omega_{pe}^2/(\omega_{\mathrm{UH}} - 4\Omega_e^2)$, leading to $I > 0$ and the absence of collapse even according to Eq. (13) of Ref. 1. This last issue is also discussed in Ref. 7.

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