Comment on "One-Parameter Scaling of Localization Length and Conductance in Disordered Systems"

MacKinnon and Kramer[1] claim an accuracy which exceeds that of earlier work by orders of magnitude. The purpose of this Comment is to show that there are large systematic errors in the above work, and that when these are taken into account, the overall errors are comparable to those in previous work. Thus this calculation does not settle the issue of whether there are power-law localized states in the two-dimensional Anderson model, and there should be substantial error bars in the $\beta(g)$ function.

The basis of the claim of small errors is that the localization length, λ, computed for a strip (bar) of finite length is very close to that of the infinite strip (bar). Because of the one-dimensional nature of the system, the authors can calculate λ for strips (bars) of arbitrary length. However, this error estimate does not take into account the finite width of the strip (bar).

The systematic errors are due to the authors' use of only periodic, lateral boundary conditions on the strips (bars). A complete estimate of the error can be obtained by varying the lateral boundary conditions from periodic to antiperiodic, and to other phase relationships, varying along the length. In the cases where λ is comparable with M, the width of the strip (bar), there are significant interactions across the boundaries and thus the wave functions will depend on the boundary condition. As a result λ will also depend on the boundary condition, giving a much larger spread of values than the 1% quoted by the authors. Of course, as M increases, the effect of boundary conditions will decrease, and all systems will converge to the two-dimensional (three-dimensional) Anderson model.

It is not sufficient to consider the scaling curve for only one out of the infinite family of lateral boundary conditions. Indeed, they must all possess the same limit for large M; however, each curve approaches that limit differently. For finite M, no one of these curves is a better estimate of that limit than the others.

The larger errors affect the authors' conclusions in two ways. Firstly, it is no longer possible to say whether or not there is any kind of transition, from exponentially to power-law localized, or to extended states, in two dimensions. Secondly, all scaling curves should indicate increased errors for large λ. This implies that there should also be substantial errors in the $\beta(g)$ for large g.

The effect of the boundary conditions on these calculations is related to the use, by Licciardello and Thouless,[2] of the boundary conditions to determine the conductivity, and implicitly λ, for squares and solids. In a loose sense MacKinnon and Kramer's calculation performs a configurational average of λ over the squares (cubes) making up the strip (bar). Thus they have done a good job on the statistics of finite samples characterized by M. However, the finite size of the sample dominates the statistics even for Licciardello and Thouless's work, and so when the variation of boundary conditions is taken into account, MacKinnon and Kramer's results are of comparable quality.

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